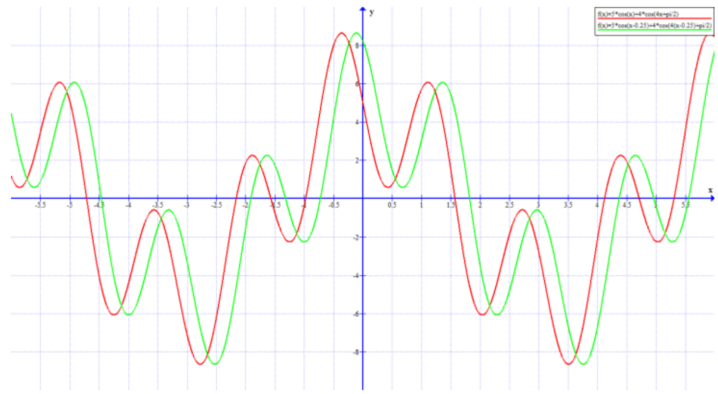
Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 10

Dr. Waheed Ahmad Younis

Do not submit this homework. There will be a quiz from this homework on Thursday (Nov 25, 2021).

Topics covered in this week:



- A serious problem with filters (and many other systems/circuits): phase distortion.

  Here, all frequencies are delayed by the same amount of time. Hence no distortion.

- All systems add some delay. Amount of delay is frequency dependant. It is

$$Delay(\omega) = D(\omega) = -\frac{d\theta}{d\omega}$$

For example, consider the first order LPF: $H(j\omega) = \frac{\omega_0}{\omega_0 + j\omega} \quad \Rightarrow \quad \angle H(j\omega) = \theta(\omega) = -\tan^{-1}\frac{\omega}{\omega_0}$

$D(\omega) = -\frac{d\theta}{d\omega} = \frac{1}{1+(\omega/\omega_0)^2}\frac{1}{\omega_0} = \frac{\omega_0}{{\omega_0}^2+\omega^2}$

The delay function for $\omega_0 = 1{,}000$ rad/s is shown below.

- Consider the 3[rd] order Butterworth LP filter

$H(s) = \frac{1}{(s+1)(s^2+s+1)} = \frac{1}{s^3+2s^2+2s+1} \quad \Rightarrow \quad H(j\omega) = \frac{1}{-j\omega^3-2\omega^2+j2\omega+1} = \frac{1}{(1-2\omega^2)+j(2\omega-\omega^3)}$

$\Rightarrow \quad \angle H(j\omega) = \theta(\omega) = -\tan^{-1}\left(\frac{2\omega-\omega^3}{1-2\omega^2}\right)$

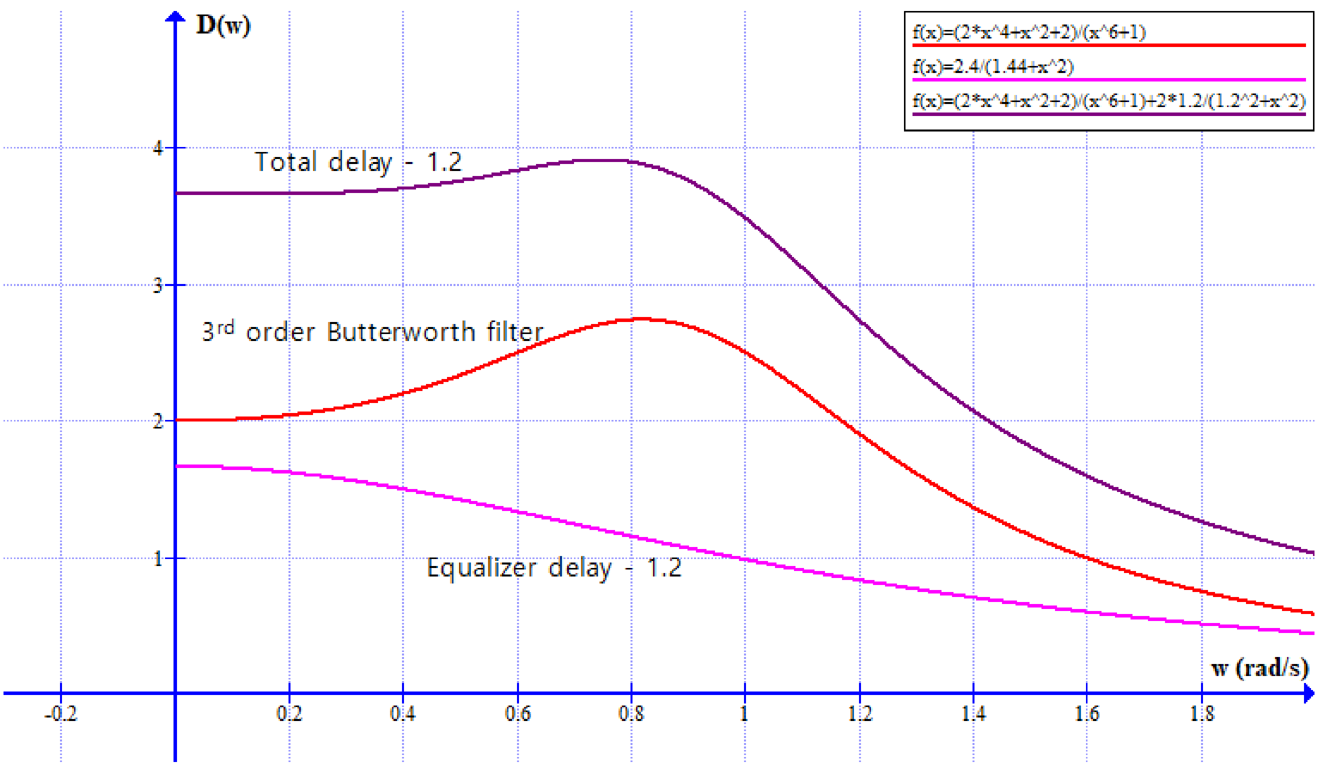
$$D(\omega) = -\frac{d\theta}{d\omega} = \frac{1}{1+[(2\omega-\omega^3)/(1-2\omega^2)]^2}\frac{(1-2\omega^2)[2-3\omega^2]-(2\omega-\omega^3)[-4\omega]}{(1-2\omega^2)^2} = \frac{2\omega^4+\omega^2+2}{\omega^6+1}$$

This delay function is plotted below.

In order to equalize this delay, a 1st order all pass filter $\left[H_{AP}(s) = \frac{s-\sigma}{s+\sigma}\right]$ is cascaded.

$$H_{AP}(j\omega) = \frac{j\omega-\sigma}{j\omega+\sigma} \quad \Rightarrow \quad \angle H_{AP}(j\omega) = \theta_{AP}(\omega) = -2\tan^{-1}\frac{\omega}{\sigma} \quad \Rightarrow \quad D_{AP}(\omega) = -\frac{d\theta}{d\omega} = \frac{2\sigma}{\sigma^2+\omega^2}$$

The delay caused by this all-pass filter (for $\sigma = 1.2$) and the total delay due to the LPF and HPF is plotted below.



It can be noted that

Un-compensated LP filter: $\Delta D = 2.75 - 2 = 0.75$

Compensated LP filter: $\Delta D = 3.9 - 3.7 = 0.2$