Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 8

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Do not submit this homework. This will be part of the midterm exam on Monday (Nov 01, 2021).

Topics covered in this week:

- Loss less Ladder filters
  - Analysing a given ladder circuit (finding the transfer function from a given ladder circuit).
  - Synthesising the circuit (finding the ladder circuit for a given transfer function).
    - Given $D(s)$ the denominator polynomial (coming from Butterworth or Chebyshev expressions)
    - Find the reflection co-efficient: $\rho(s) = \frac{s^n}{D(s)}$
    - Find the input impedance: $Z_{in} = \left[\frac{1-\rho(s)}{1+\rho(s)}\right]^{\pm 1}$

Q1. Design a 5th order Butterworth filter using LC ladder filter topology.

Solution:

The denominator polynomial for a 5th order Butterworth filter:

$$D(s) = (s+1)(s^2 + 2\cos 36 + 1)(s^2 + 2\cos 72 + 1)$$
$$= s^5 + (2\cos 36 + 2\cos 72 + 1)s^4 + (4\cos 36\cos 72 + 2\cos 36 + 2\cos 72 + 2)s^3$$
$$+ (4\cos 36\cos 72 + 2\cos 36 + 2\cos 72 + 2)s^2 + (2\cos 36 + 2\cos 72 + 1)s + 1$$
$$= s^5 + 3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1$$

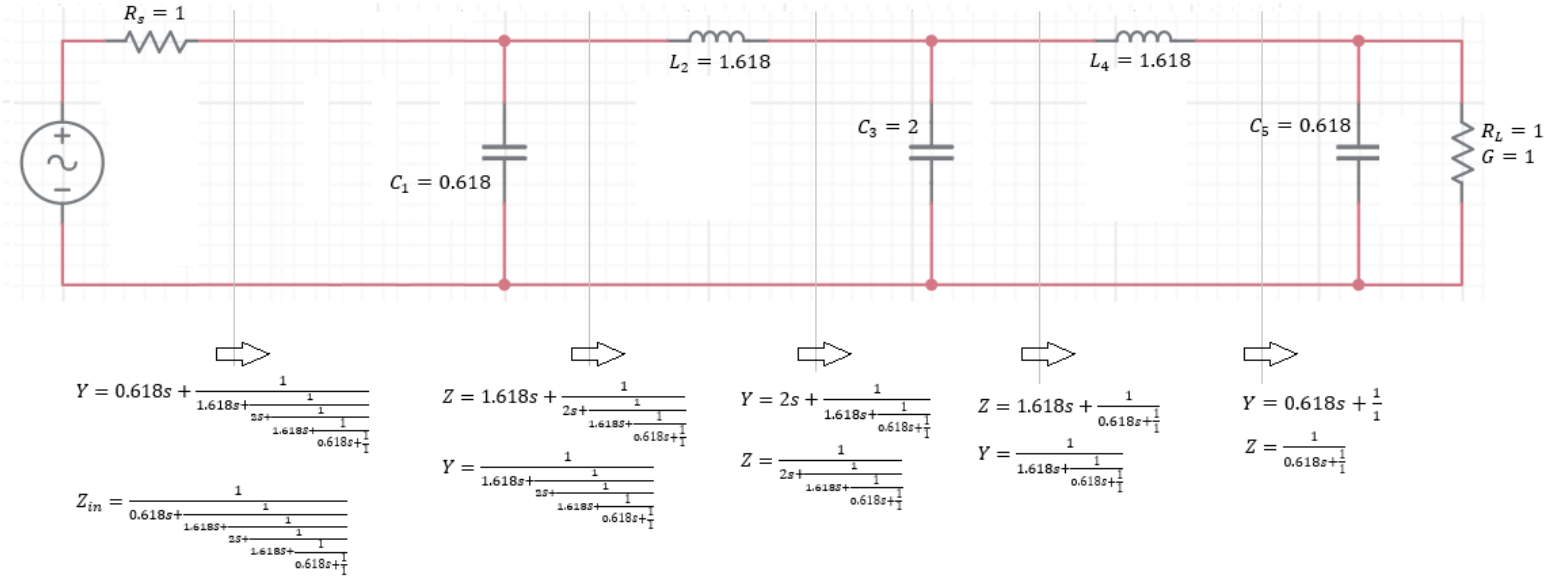
Input impedance of the ladder filter:

$$Z_{in} = \frac{3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}{2s^5 + 3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}$$
$$= \frac{1}{\frac{2s^5 + 3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}{3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}}$$
$$= \frac{1}{0.618s + \frac{2s^3 + 3.2361s^2 + 2.618s + 1}{3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}}$$
$$= \frac{1}{0.618s + \frac{1}{\frac{3.2361s^4 + 5.2361s^3 + 5.2361s^2 + 3.2361s + 1}{2s^3 + 3.2361s^2 + 2.618s + 1}}}$$
$$= \frac{1}{0.618s + \frac{1}{1.618s + \frac{s^2 + 1.618s + 1}{2s^3 + 3.2361s^2 + 2.618s + 1}}}$$
$$= \frac{1}{0.618s + \frac{1}{1.618s + \frac{1}{\frac{2s^3 + 3.2361s^2 + 2.618s + 1}{s^2 + 1.618s + 1}}}} = \frac{1}{0.618s + \frac{1}{1.618s + \frac{1}{2s + \frac{0.618s + 1}{s^2 + 1.618s + 1}}}}$$

$$= \frac{1}{0.618s + \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{\dfrac{s^2 + 1.618s + 1}{0.618s + 1}}}}} = \frac{1}{0.618s + \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + 1}}}}}$$

$$= \frac{1}{0.618s + \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}}}$$

This input impedance can be realized by the following LC ladder circuit:



$Y = 0.618s + \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}}$

$Z_{in} = \dfrac{1}{0.618s + \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}}}$

$Z = 1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}$

$Y = \dfrac{1}{1.618s + \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}}$

$Y = 2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}$

$Z = \dfrac{1}{2s + \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}}$

$Z = 1.618s + \dfrac{1}{0.618s + \frac{1}{1}}$

$Y = \dfrac{1}{1.618s + \dfrac{1}{0.618s + \frac{1}{1}}}$

$Y = 0.618s + \dfrac{1}{1}$

$Z = \dfrac{1}{0.618s + \frac{1}{1}}$