Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 8

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Do not submit this homework. This will be part of the midterm exam on Monday (Nov 01, 2021).

Topics covered in this week:

- Concept of linear phase
  Why linear phase is important to avoid signal distortion.
- Chebyshev LP filter
  - Finding the filter order from the given specifications:
    $$n = \frac{\cosh^{-1}\sqrt{\frac{10^{-G_{max}/10}-1}{10^{-G_{min}/10}-1}}}{\cosh^{-1}\left(\omega_s/\omega_p\right)}$$
    Compare it with Butterworth case: $n = \frac{\log\left[\frac{10^{-G_{max}/10}-1}{10^{-G_{min}/10}-1}\right]}{2\log(\omega_s/\omega_p)}$
  - Finding the pole locations:
    1. Find n and ϵ
       $$\epsilon = \sqrt{10^{-G_{min}/10}-1}$$
    2. Find a:
       $$a = \frac{1}{n}\sinh^{-1}\frac{1}{\epsilon}$$
    3. Find the location of k-th pole:
       $\sigma_k = -\sinh a \,.\, \sin\left(\frac{2k-1}{2n}\pi\right)$ - real part of the k-th pole
       $\omega_k = \cosh a \,.\, \cos\left(\frac{2k-1}{2n}\pi\right)$ - imaginary part of the k-th pole
       Where $k = 1, 2, \dots, n$

Q1. The following specifications are given for a Chebyshev lowpass filter
$G_{min} = -0.5\ dB$, $G_{max} = -22\ dB$, $\omega_p = 1{,}000$ rad/s, $\omega_s = 2{,}330$ rad/s. Find "n".

Solution:

$$n = \frac{\cosh^{-1}\sqrt{\frac{10^{-G_{max}/10}-1}{10^{-G_{min}/10}-1}}}{\cosh^{-1}\left(\omega_s/\omega_p\right)} = \frac{\cosh^{-1}\sqrt{\frac{10^{22/10}-1}{10^{0.5/10}-1}}}{\cosh^{-1}(2{,}330/1{,}000)} = 2.87$$

So, we need a 3rd order Chebyshev filter. Compare it with Butterworth:

$$n = \frac{\log\left[\frac{10^{-G_{max}/10}-1}{10^{-G_{min}/10}-1}\right]}{2\log\left(\omega_s/\omega_p\right)} = \frac{\log\left[\frac{10^{22/10}-1}{10^{0.5/10}-1}\right]}{2\log(2{,}330/1{,}000)} = 4.23$$

We need a 5th order Butterworth filter for the same specifications.

Q2. Determine the location of Chebyshev poles for n = 3 and $G_{min} = -1.0\ dB$.

Solution:

1. Find $n\ and\ \epsilon$:

   $n = 3$ (Given)

   $\epsilon = \sqrt{10^{-G_{min}/10} - 1} = \sqrt{10^{1/10} - 1} = 0.508847$
2. Find a:

   $a = \frac{1}{n}\sinh^{-1}\frac{1}{\epsilon} = \frac{1}{3}\sinh^{-1}\frac{1}{0.508847} = 0.476$
3. Find the location of k-th pole:

   $\sigma_k = -\sinh a\,.\sin\left(\frac{2k-1}{2n}\pi\right) = -(0.49417)\sin\left(\frac{2k-1}{6}\pi\right)$

   $\omega_k = \cosh a\,.\cos\left(\frac{2k-1}{2n}\pi\right) = (1.11544)\cos\left(\frac{2k-1}{6}\pi\right)$

   $\sigma_1 = -(0.49417)\sin\left(\frac{1}{6}\pi\right) = -0.24709$

   $\omega_1 = (1.11544)\cos\left(\frac{1}{6}\pi\right) = 0.96600$

   $\sigma_2 = -(0.49417)\sin\left(\frac{1}{2}\pi\right) = -0.49417$

   $\omega_2 = (1.11544)\cos\left(\frac{1}{2}\pi\right) = 0$

   $\sigma_3 = -(0.49417)\sin\left(\frac{5}{6}\pi\right) = -0.24709$

   $\omega_3 = (1.11544)\cos\left(\frac{5}{6}\pi\right) = -0.96600$

Hence the three poles are:

$s_1 = -0.24709 + j0.966$
$s_2 = -0.49417$
$s_3 = -0.24709 - j0.966$

And the denominator polynomial:

$$(s - s_1)(s - s_2)(s - s_3) = (s + 0.24709 - j0.966)(s + 0.49417)(s + 0.24709 + j0.966)$$
$$= (s + 0.49417)(s + 0.24709 - j0.966)(s + 0.24709 + j0.966)$$
$$= (s + 0.49417)[(s + 0.24709)^2 - (0.966)^2] = (s + 0.49417)(s^2 + 0.49417s + 0.99421)$$

Q3. Find the Chebyshev transfer function to meet the requirements n = 5 and $G_{min} = -0.5\ dB$ in $0 \le \omega \le 1{,}000$ rad/s. Passband gain is 0 dB.

Solution:

1. Find $n\ and\ \epsilon$:

   $n = 5$ (Given)

   $\epsilon = \sqrt{10^{-G_{min}/10} - 1} = \sqrt{10^{0.5/10} - 1} = 0.349311$
2. Find a:

$$a = \frac{1}{n}\sinh^{-1}\frac{1}{\epsilon} = \frac{1}{5}\sinh^{-1}\frac{1}{0.349311} = 0.354827$$

3. Find the location of k-th pole:

$$\sigma_k = -\sinh a\,.\sin\left(\frac{2k-1}{2n}\pi\right) = -(0.36232)\sin\left(\frac{2k-1}{10}\pi\right)$$

$$\omega_k = \cosh a\,.\cos\left(\frac{2k-1}{2n}\pi\right) = (1.06361)\cos\left(\frac{2k-1}{10}\pi\right)$$

$$\sigma_1 = -(0.36232)\sin\left(\frac{\pi}{10}\right) = -0.111963$$

$$\omega_1 = (1.06361)\cos\left(\frac{\pi}{10}\right) = 1.011553$$

$$\sigma_2 = -(0.36232)\sin\left(\frac{3\pi}{10}\right) = -0.293123$$

$$\omega_2 = (1.06361)\cos\left(\frac{3\pi}{10}\right) = 0.625174$$

$$\sigma_3 = -(0.36232)\sin\left(\frac{\pi}{2}\right) = -0.36232$$

$$\omega_3 = (1.06361)\cos\left(\frac{\pi}{2}\right) = 0$$

$$\sigma_4 = -(0.36232)\sin\left(\frac{7\pi}{10}\right) = -0.293123$$

$$\omega_4 = (1.06361)\cos\left(\frac{7\pi}{10}\right) = -0.625174$$

$$\sigma_5 = -(0.36232)\sin\left(\frac{9\pi}{10}\right) = -0.111963$$

$$\omega_5 = (1.06361)\cos\left(\frac{9\pi}{10}\right) = -1.011553$$

Hence the five poles are:

$$s_1 = -0.111963 + j1.011553$$
$$s_2 = -0.293123 + j0.625174$$
$$s_3 = -0.36232$$
$$s_4 = -0.293123 - j0.625174$$
$$s_5 = -0.111963 - j1.011553$$

And the denominator polynomial:

$$(s-s_1)(s-s_2)(s-s_3)(s-s_4)(s-s_5)$$
$$= (s+0.111963-j1.011553)(s+0.293123-j0.625174)(s+0.36232)(s+0.293123+j0.625174)$$
$$(s+0.111963-j1.011553)$$
$$= (s+0.36232)(s+0.111963-j1.011553)(s+0.111963-j1.011553)(s+0.293123-j0.625174)$$
$$(s+0.293123+j0.625174)$$
$$= (s+0.36232)(s^2+0.22392s+1.03577)(s^2+0.586245s+0.47676)$$

Finally, the transfer function would be

$$T(s) = \frac{(0.36232)(1.03577)(0.47676)}{(s+0.36232)(s^2+0.22392s+1.03577)(s^2+0.586245s+0.47676)}$$
$$= \frac{0.17892}{(s+0.36232)(s^2+0.22392s+1.03577)(s^2+0.586245s+0.47676)}$$