Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 6

Dr. Waheed Ahmad Younis
Do not submit this homework. There will be a quiz from this homework on Thursday (Oct 21, 2021).

Topics covered in this week:

- Butterworth (maximally flat) LP filter design
  - Identifying the pole location
  - Forming the proper denominator polynomial (1st and 2nd order expressions)
  - Designing the filter using op-amps and resistors, capacitors
- Magnitude function of Butterworth filter of order n
  $|G(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_0)^{2n}}}$ $\qquad 20\log|G(j\omega)| = G_{dB} = -10\log[1+(\omega/\omega_0)^{2n}]$
- Filter design based on given specifications
  Given: $\omega_p, \omega_s, G_{min}\ and\ G_{max}$ $\qquad$ find $\omega_0\ and\ n$

$$\omega_0 = \frac{\omega}{[10^{-G_{dB}/10}-1]^{\,1/2n}} \qquad and \qquad n = \frac{\log\left[{(10^{-G_{min}/10}-1)}\Big/{(10^{-G_{max}/10}-1)}\right]}{2\log\left({\omega_p}/{\omega_s}\right)}$$

- Filter designing using frequency scaling and impedance scaling.
  - Design the filter for a normalized frequency of 1 rad/s
  - Frequency scale by dividing L and C by ω_0.
  - Impedance scale by multiplying R and L by some factor Z and divide C by same factor Z.
- Design of Butterworth HP filter
  - Design the corresponding LP filter first
  - Interchange Rs and Cs with appropriate values
  - Properly scale (frequency scale and impedance scale) to get the real circuit

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Q1. You have to design a Butterworth LP filter. Passband should be up to 2.5 kHz. Stopband should be above 3.5 kHz. In the passband, filter gain should not be below -0.4 dB and in stopband, the filter gain should not be more than -30 dB. Find the order of the required filter and a proper choice of ω_0.

Solution:

Given:

$\omega_p = 2\pi(2{,}500)$ rad/s

$\omega_s = 2\pi(3{,}500)$ rad/s

$G_{min} = -0.4\ dB$

$G_{max} = -30\ dB$

$$n = \frac{\log\left[{(10^{-G_{min}/10}-1)}\Big/{(10^{-G_{max}/10}-1)}\right]}{2\log\left({\omega_p}/{\omega_s}\right)} = \frac{\log\left[{(10^{0.4/10}-1)}\Big/{(10^{30/10}-1)}\right]}{2\log\left({2.5}/{3.5}\right)} = 13.74$$ So, n = 14.

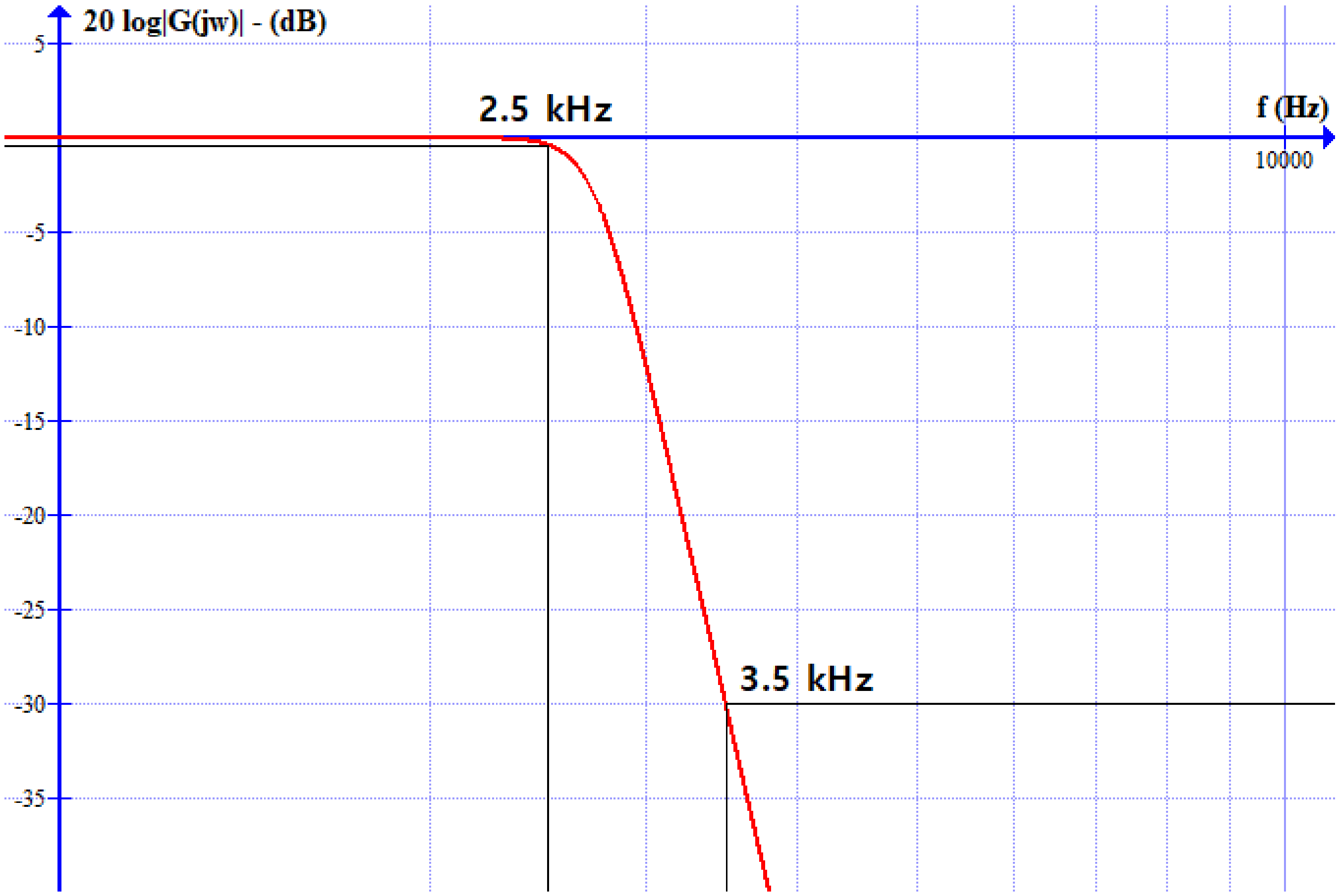
Using ω_p:

$$f_0 = \frac{f}{[10^{-G_{dB}/10}-1]^{\,1/2n}} = \frac{f_p}{[10^{-G_{min}/10}-1]^{\,1/2n}} = \frac{2{,}500}{[10^{0.4/10}-1]^{\,1/28}} = 2.718 \text{ kHz}$$

Using ω_s:

$$f_0 = \frac{f}{[10^{-G_{dB}/10}-1]^{\,1/2n}} = \frac{f_s}{[10^{-G_{max}/10}-1]^{\,1/2n}} = \frac{3{,}500}{[10^{30/10}-1]^{\,1/28}} = 2.735 \text{ kHz}$$

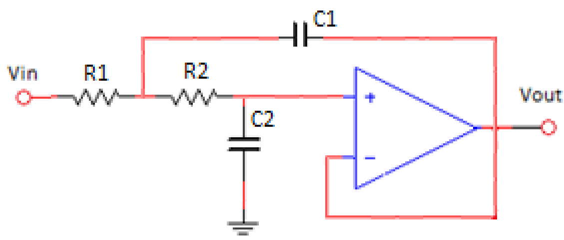
We recommend using: $f_0 = 2.726$ kHz



Q2. Consider a normalized Sallen-Key circuit with the following component values:

$R_1 = R_2 = 1\,\Omega$

$C_1 = 2Q\ F, \quad C_2 = \dfrac{1}{2Q} F$



Perform proper scaling and design a 2nd order LP filter with Q = 1.2 and cut-off frequency 2.3 kHz.

Solution:

Initial (normalized values):

$$R_1 = R_2 = 1\,\Omega$$
$$C_1 = 2.4\ F, \quad C_2 = 0.4167F$$

After frequency scaling:

$$R_1 = R_2 = 1\,\Omega$$
$$C_1 = 166\ \mu F, \quad C_2 = 28.832\ \mu F$$

After impedance scaling:

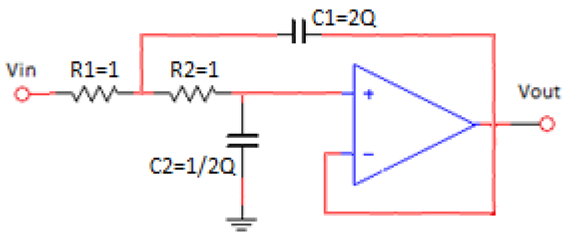
$$R_1 = R_2 = 1\ k\Omega$$
$$C_1 = 0.166\ \mu F, \quad C_2 = 0.0288\ \mu F$$

Q3. Design a 2nd order HP filter with Q = 0.8 and cut-off frequency 1.3 kHz.

Solution:

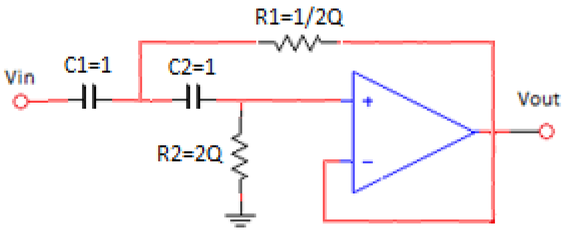
Consider the LP Sallen-Key circuit normalized to $\omega_0 = 1$.

Transfer function for this circuit is $G(s) = \frac{1}{s^2+\frac{s}{Q}+1}$



Interchanging Cs and Rs, we get the circuit for HP filter with transfer function

$G(s) = \frac{s^2}{s^2+\frac{s}{Q}+1}$



Let us calculate the component values for this HP filter:

Initial (normalized values):

$$C_1 = C_2 = 1 \text{ F}$$
$$R_1 = 0.625\ \Omega, \quad R_2 = 1.6\ \Omega$$

After frequency scaling:

$$C_1 = C_2 = 122.43\ \mu\text{F}$$
$$R_1 = 0.625\ \Omega, \quad R_2 = 1.6\ \Omega$$

After impedance scaling:

$$C_1 = C_2 = 0.122\ \mu\text{F}$$
$$R_1 = 625\ \Omega, \quad R_2 = 1.6\ k\Omega$$