Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 5

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Do not submit this homework. There will be a quiz from this homework on Thursday (Oct 14, 2021).

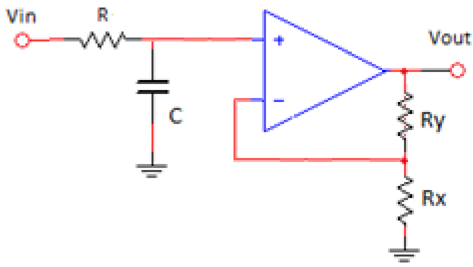
Topics covered in this week:

Active filters

- First order:
  - LPF

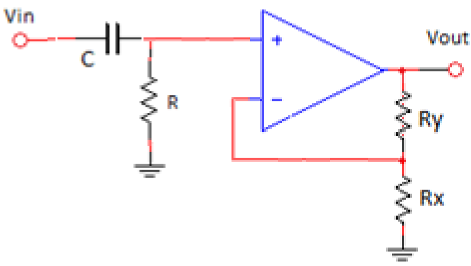
    Gain of the non-inverting amplifier = K = $1 + \frac{R_y}{R_x}$

    Transfer function: $G(s) = \frac{K\omega_0}{s+\omega_0}$ where $\omega_0 = \frac{1}{RC}$



  - HPF

    Transfer function: $G(s) = \frac{Ks}{s+\omega_0}$ where $\omega_0 = \frac{1}{RC}$



- Sallen-Key circuit (2nd order filter)

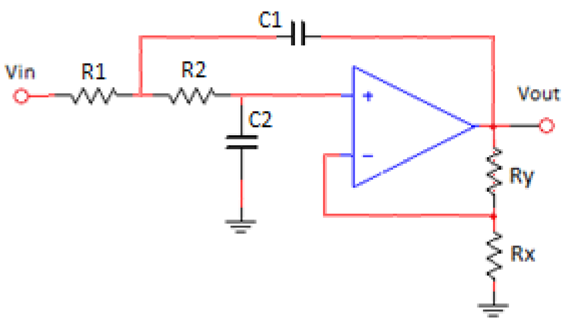
$$G(s) = \frac{K/R_1R_2C_1C_2}{s^2+\left(\frac{1}{R_1C_1}+\frac{1}{R_2C_2}+\frac{1}{R_2C_1}-\frac{K}{R_2C_2}\right)s+1/R_1R_2C_1C_2}$$

Where $K = 1 + \frac{R_y}{R_x}$

Replacing ${\omega_0}^2 = \frac{1}{R_1R_2C_1C_2}$

$$and \quad \frac{1}{Q} = \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}} + \sqrt{\frac{R_1C_1}{R_2C_2}}(1-K)$$

$$G(s) = \frac{K\,{\omega_0}^2}{s^2+\left(\frac{\omega_0}{Q}\right)s+{\omega_0}^2}$$



- Butterworth (maximally flat) LP filter

Q1. Design a 1st order LP filter with a gain of 5 at zero frequency. Cut-off frequency is 1 kHz.

Solution:

Transfer function of the active LP first order filter: $G(s) = \frac{K\omega_0}{s+\omega_0}$

$G(j\omega) = \frac{K\omega_0}{j\omega+\omega_0} = \frac{K}{1+j\omega/\omega_0}$ $\quad |G(j\omega)| = \frac{K}{\sqrt{1+(\omega/\omega_0)^2}}$ $\quad |G(j0)| = K$

$K = 1 + \frac{R_y}{R_x} = 5$ $\quad R_y = 4R_x$ $\quad$ If $R_x = 2\ k\Omega$ then $R_y = 8\ k\Omega$

$\frac{1}{RC} = 2\pi(10^3)$ $\quad if\ C = 0.1\ \mu F,$ $\quad R = \frac{1}{2{,}000\pi(0.1\times10^{-6})} = 1592\ \Omega = 1.59\ \mathrm{k}\Omega$

Q2. Design a 2nd order LP filter with Q=0.7071 and cut-off frequency 1 kHz. Gain is not specified.

Solution:

For Sallen-Key circuit, if $R_1 = R_2 = R$ and $C_1 = C_2 = C$, then $\omega_0 = \frac{1}{RC}$ and $Q = \frac{1}{3-K}$
If Q=0.7071, K=3-1/Q=3-1/0.7071=1.586

$K = 1 + \frac{R_y}{R_x} = 1.586 \qquad R_y = 0.586 R_x \qquad$ If $R_x = 10\ k\Omega$ then $R_y = 5.86\ k\Omega$

$C = 0.1\ \mu F, \quad and \quad R = 1.59\ \text{k}\Omega$ will give $f_0 = 1\ kHz$

Q3. Design a 2nd order LP filter with Q=0.7071 and cut-off frequency 1 kHz. Gain is unity at zero frequency.
Solution:

For Sallen-Key circuit, if K=1, $\frac{1}{Q} = \sqrt{\frac{R_2C_2}{R_1C_1}} + \sqrt{\frac{R_1C_2}{R_2C_1}}$. Now assuming $R_1 = R_2 = R$, $Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}}, \quad C_1 = 2C_2$

If $C_2 = 0.1\ \mu F, \quad C_1 = 0.2\ \mu F$

$\omega_0{}^2 = \frac{1}{R^2C_1C_2} \qquad \omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad R = \frac{1}{2{,}000\pi\sqrt{(0.1\times10^{-6})(0.2\times10^{-6})}} = 1125\ \Omega = 1.125\ \text{k}\Omega$

K=1 if $R_y = 0$ and $R_x = \infty$.

Q4. Design a 5th order Butterworth LP filter with cut-off frequency 1 kHz.
Solution:
For 5th order filter, the poles are located on unit circle at: $0°, \pm36°, \pm72°$. Pole locations would be $s_1 = -1, \quad s_2, s_3 = -\cos 36 \pm j \sin 36, \quad s_4, s_5 = -\cos 72 \pm j \sin 72$ [note that the critical frequency is normalized to 1 rad/s]
The denominator would be
$(s+1)(s^2 + 2\cos 36 + 1)(s^2 + 2\cos 72 + 1) = (s+1)(s^2 + 1.618s + 1)(s^2 + 0.618s + 1)$
We can make the 5th order filter by cascading 3 filters:
$\left(\frac{1}{s+1}\right)\left(\frac{1}{s^2+1.618s+1}\right)\left(\frac{1}{s^2+0.618s+1}\right)$
For all the filters, K=1.

For 1st filter (1st order): $C = 0.1\ \mu F, \quad and \quad R = 1.59\ \text{k}\Omega$

For 2nd filter (2nd order): Q=1/1.618=0.618 $\quad R_1 = R_2 = R \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} = 0.618 \quad C_1 = 1.53C_2$

If $C_2 = 0.1\ \mu F, \quad C_1 = 0.153\ \mu F$

$\omega_0{}^2 = \frac{1}{R^2C_1C_2} \qquad \omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad R = \frac{1}{2{,}000\pi\sqrt{(0.1\times10^{-6})(0.153\times10^{-6})}} = 1287\ \Omega = 1.287\ \text{k}\Omega$

K=1 if $R_y = 0$ and $R_x = \infty$.

For 3rd filter (2nd order): Q=1/0.618=1.618 $\quad R_1 = R_2 = R \quad Q = \frac{1}{2}\sqrt{\frac{C_1}{C_2}} = 1.618 \quad C_1 = 10.47C_2$

If $C_2 = 0.1\ \mu F, \quad C_1 = 1.047\ \mu F$

$\omega_0{}^2 = \frac{1}{R^2C_1C_2} \qquad \omega_0 = \frac{1}{R\sqrt{C_1C_2}} \qquad R = \frac{1}{2{,}000\pi\sqrt{(0.1\times10^{-6})(1.047\times10^{-6})}} = 492\ \Omega$

K=1 if $R_y = 0$ and $R_x = \infty$.