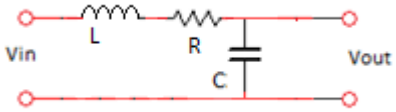
Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 4

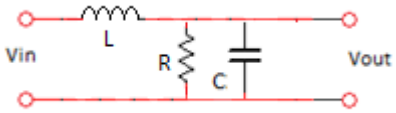
Dr. Waheed Ahmad Younis
Do not submit this homework. There will be a quiz from this homework on Thursday (Oct 07, 2021).

Topics covered in this week:
Second order passive filters

- LPF:



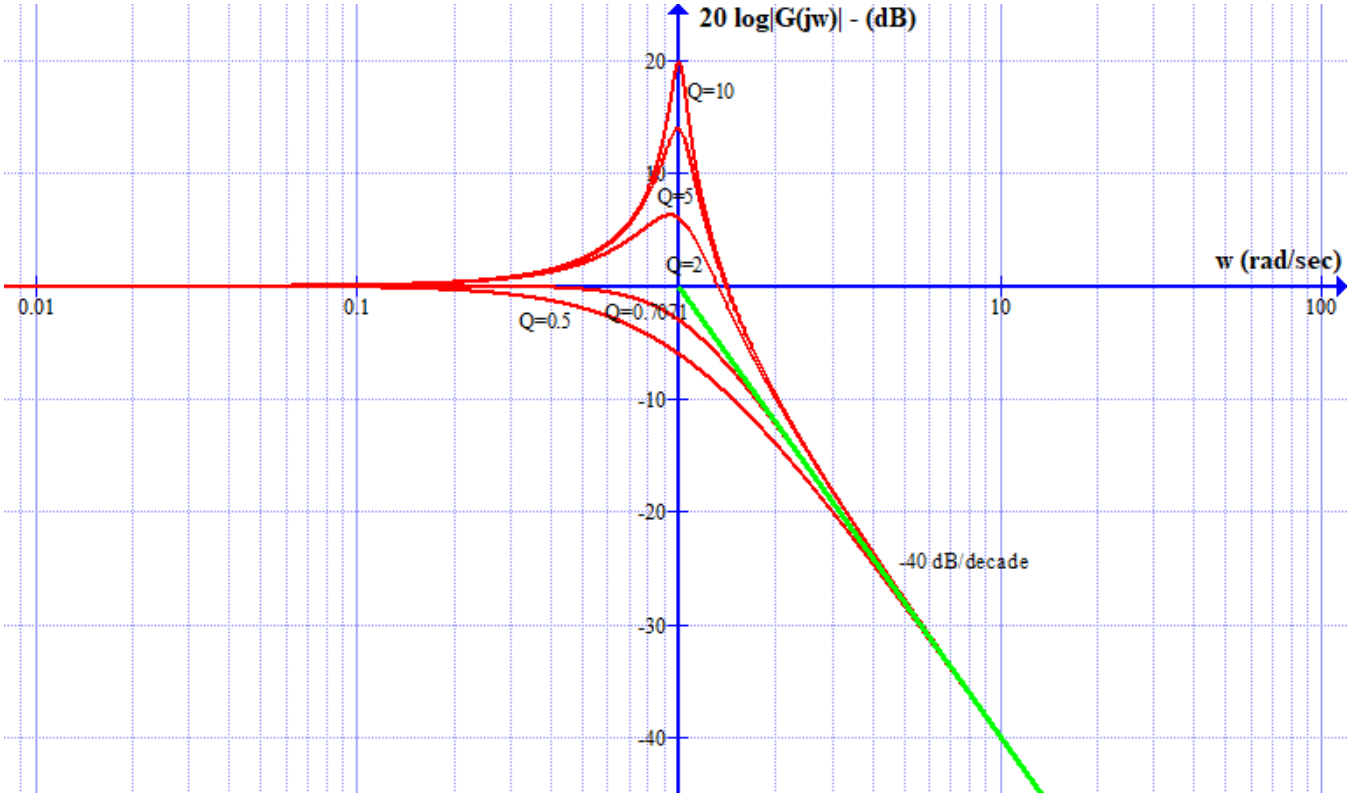
$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$



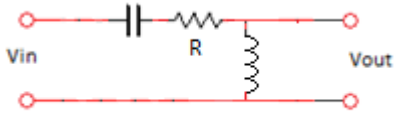
$$Q = R\sqrt{\frac{C}{L}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$G(s) = \frac{{\omega_0}^2}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$

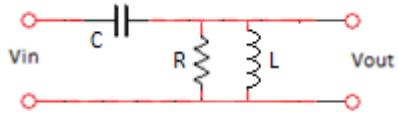
$$20\log|G(j\omega)| = -10\log[\{1-(\omega/\omega_0)^2\}^2 + (\omega/\omega_0\, Q)^2] = \begin{cases} 0\ dB & if\ \omega \ll \omega_0 \\ -40\log\frac{\omega}{\omega_0}\ dB & if\ \omega \gg \omega_0 \\ 20\log Q\ dB & if\ \omega = \omega_0 \end{cases}$$



- HPF:



$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

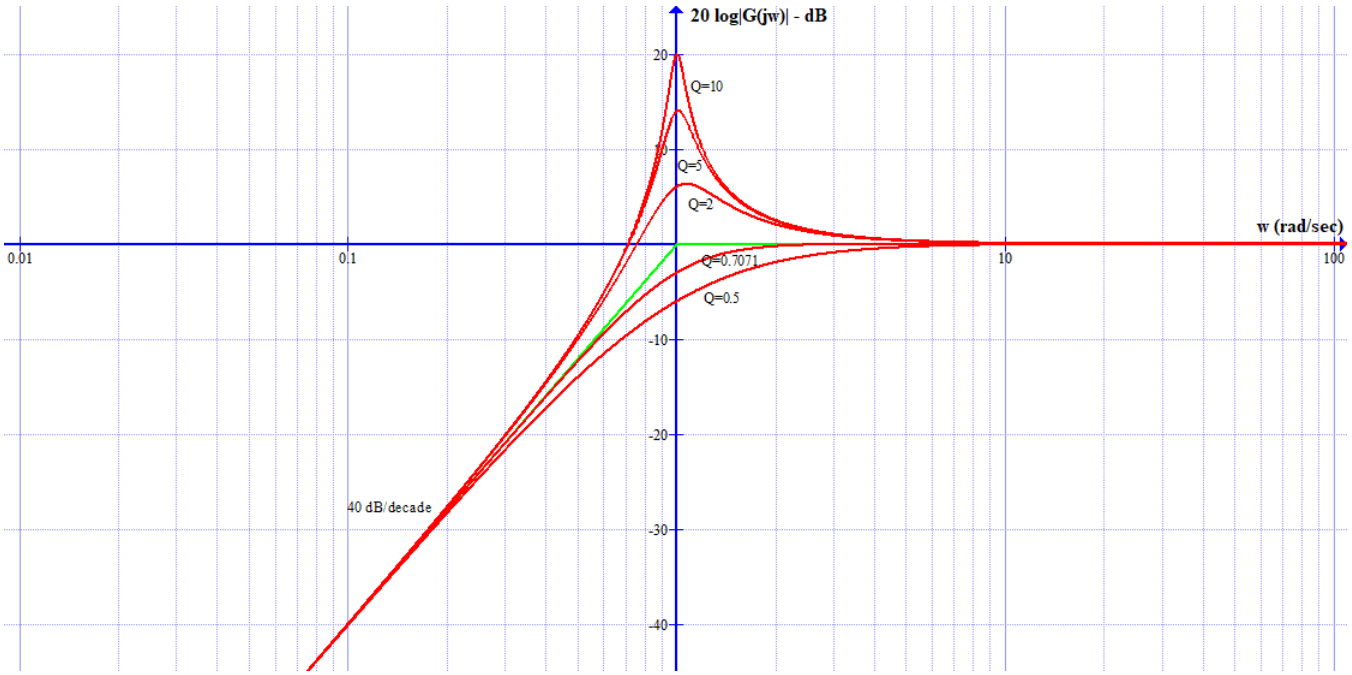


$$Q = R\sqrt{\frac{C}{L}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

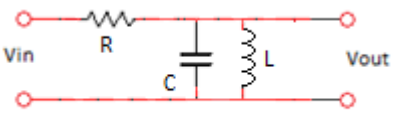
$$G(s) = \frac{s^2}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$

$$20\log|G(j\omega)| = 40\log(\omega/\omega_0) - 10\log[\{1-(\omega/\omega_0)^2\}^2 + (\omega/\omega_0\, Q)^2] =$$

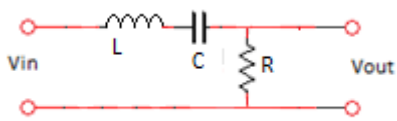
$$\begin{cases} 40\log\frac{\omega}{\omega_0}\ dB & if\ \omega \ll \omega_0 \\ 0\ dB & if\ \omega \gg \omega_0 \\ 20\log Q\ dB & if\ \omega = \omega_0 \end{cases}$$



- BPF:



$$Q = R\sqrt{\frac{C}{L}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

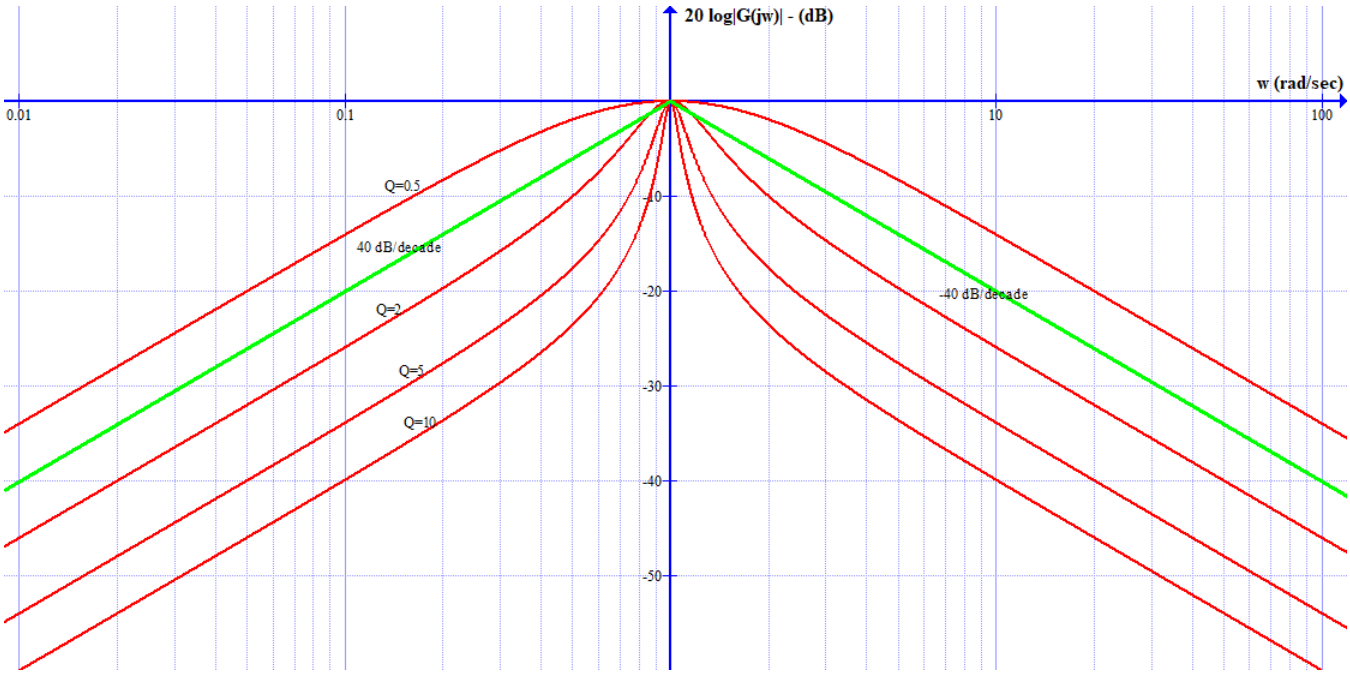


$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$G(s) = \frac{(\omega_0/Q)s}{s^2+(\omega_0/Q)s+{\omega_0}^2}$$

$$20\log|G(j\omega)| = 20\log(\omega/\omega_0\, Q) - 10\log[\{1-(\omega/\omega_0)^2\}^2 + (\omega/\omega_0\, Q)^2] =$$

$$\begin{cases} 20\log(\omega/\omega_0\, Q) & if\ \omega \ll \omega_0 \\ -20\log(\omega Q/\omega_0) & if\ \omega \gg \omega_0 \\ 0\ dB & if\ \omega = \omega_0 \end{cases}$$

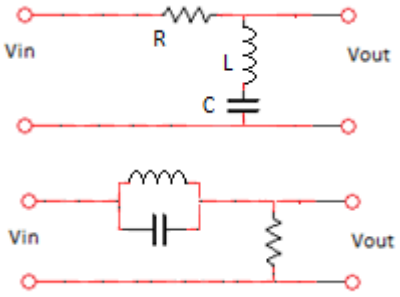


Upper cut-off frequency: $\omega_u = \omega_0\sqrt{1+\frac{1}{4Q^2}} + \frac{\omega_0}{2Q}$

Lower cut-off frequency: $\omega_l = \omega_0\sqrt{1+\frac{1}{4Q^2}} - \frac{\omega_0}{2Q}$

Bandwidth: $BW = \omega_u - \omega_l = \frac{\omega_0}{Q} \quad \Rightarrow \quad Q = \frac{\omega_0}{BW}$ Also: $\omega_0 = \sqrt{\omega_u\omega_l}$

- BRF:

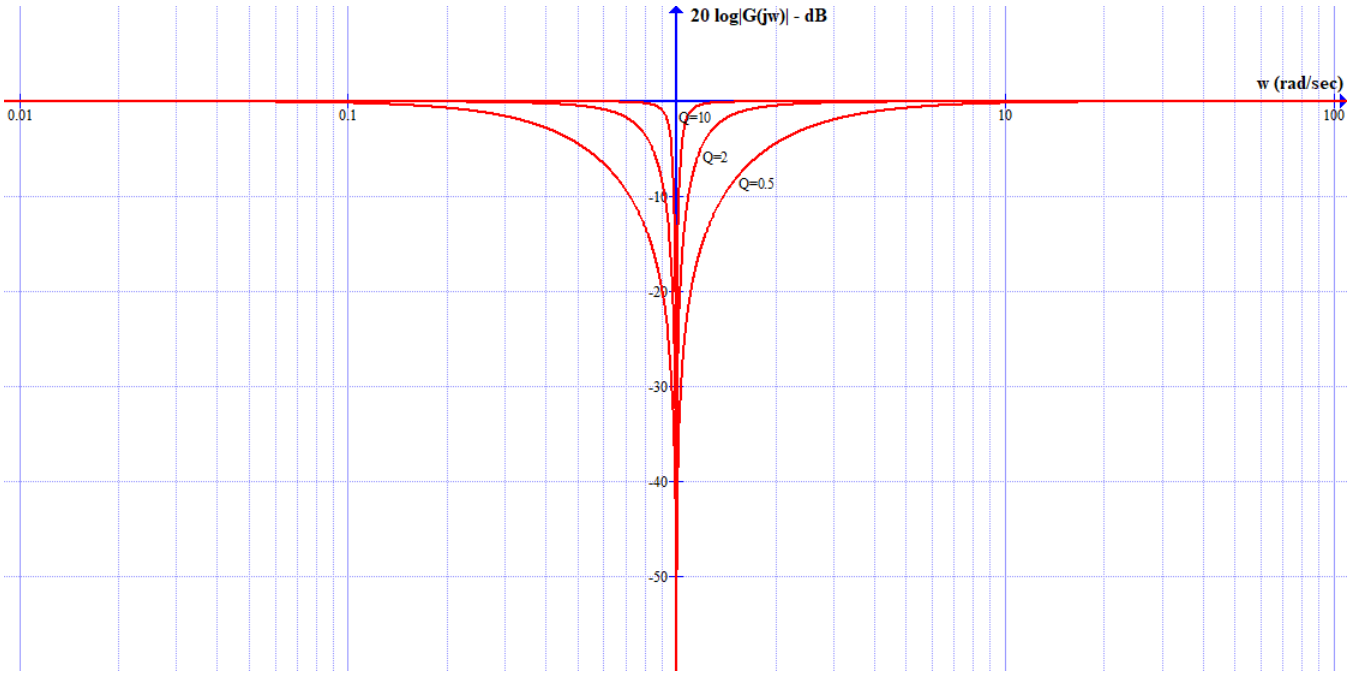


$$Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$Q = R\sqrt{\frac{C}{L}} \qquad \omega_0 = \frac{1}{\sqrt{LC}}$$

$$G(s) = \frac{s^2+{\omega_0}^2}{s^2+(\omega_0/Q)s+{\omega_0}^2}$$

$$20\log|G(j\omega)| = 20\log\left\{\left|1-\left(\frac{\omega}{\omega_0}\right)^2\right|\right\} - 10\log\left[\left\{1-\left(\frac{\omega}{\omega_0}\right)^2\right\}^2 + \left(\frac{\omega}{\omega_0 Q}\right)^{2^2}\right] = \begin{cases} 0\ dB & if\ \omega \ll \omega_0 \\ 0\ dB & if\ \omega \gg \omega_0 \\ -\infty\ dB & if\ \omega = \omega_0 \end{cases}$$



$$G_{LP}(s) = \frac{{\omega_0}^2}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$

$$G_{HP}(s) = \frac{s^2}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$

$$G_{BP}(s) = \frac{(\omega_0/Q)s}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$

$$G_{BR}(s) = \frac{s^2 + {\omega_0}^2}{s^2 + (\omega_0/Q)s + {\omega_0}^2}$$
