Umm Al-Qura Universtiy, Makkah
Department of Electrical Engineering
Special Topics in Electronics and Communications (8024990)
Term 1; 2021/2022
Homework 3

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Do not submit this homework. There will be a quiz from this homework on Wednesday (Sep 30, 2021).

Topics covered in this week:

- First order filters

  LPF: $G(s) = \frac{\omega_0}{s+\omega_0}$ HPF: $G(s) = \frac{s}{s+\omega_0}$ Where $\omega_0 = \frac{R}{L}$ or $\frac{1}{RC}$ Pole location: $s = -\omega_0$

  Roll off = 20 dB/decade
- Second order filters

  LPF: $G(s) = \frac{{\omega_0}^2}{s^2+(\omega_0/Q)s+{\omega_0}^2}$ Where $\omega_0 = \frac{1}{\sqrt{LC}}$ and $Q = R\sqrt{\frac{C}{L}}$ or $\frac{1}{R}\sqrt{\frac{L}{C}}$

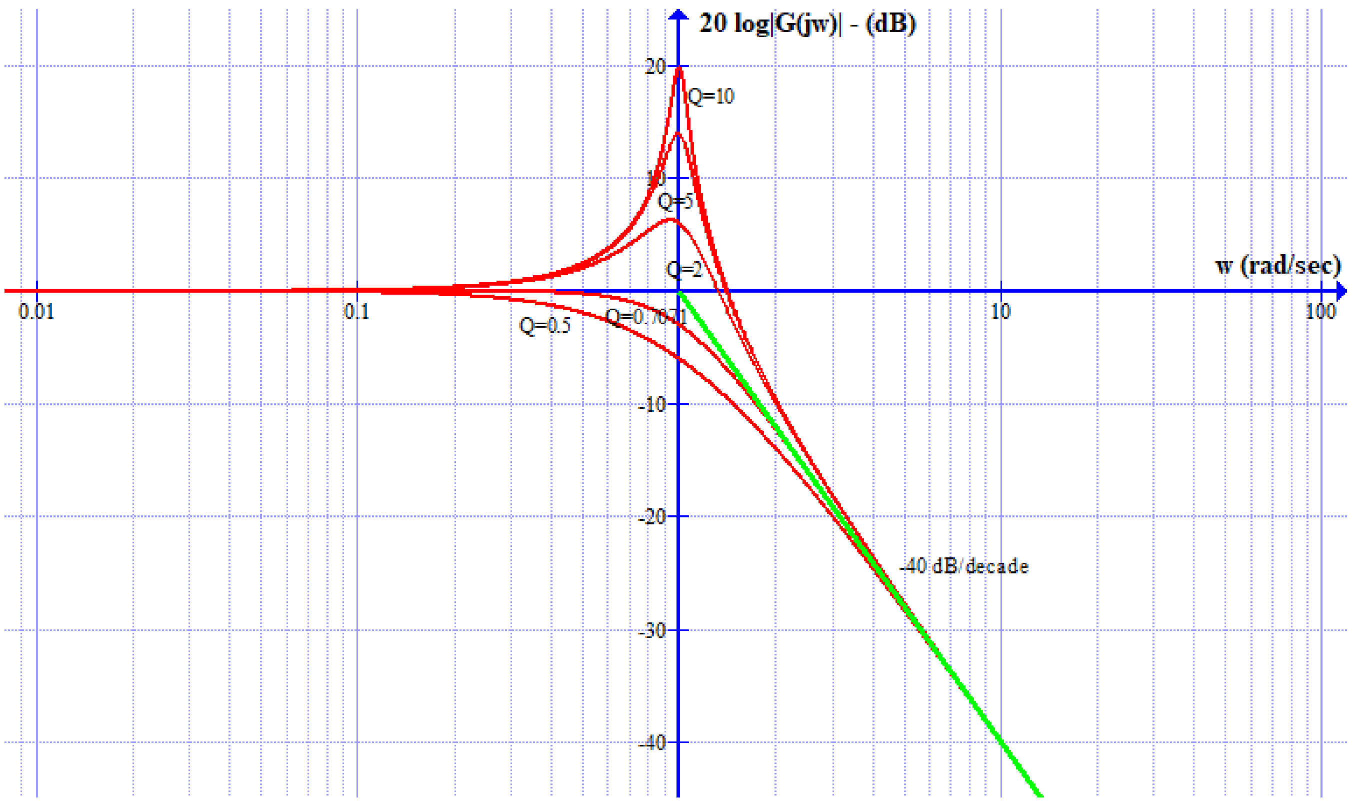
  $G(j\omega) = \frac{{\omega_0}^2}{{\omega_0}^2-\omega^2+j\omega(\omega_0/Q)} = \frac{1}{1-(\omega/\omega_0)^2+j(\omega/\omega_0 Q)}$

  $|G(j\omega)| = \frac{1}{\sqrt{\{1-(\omega/\omega_0)^2\}^2+(\omega/\omega_0\, Q)^2}}$

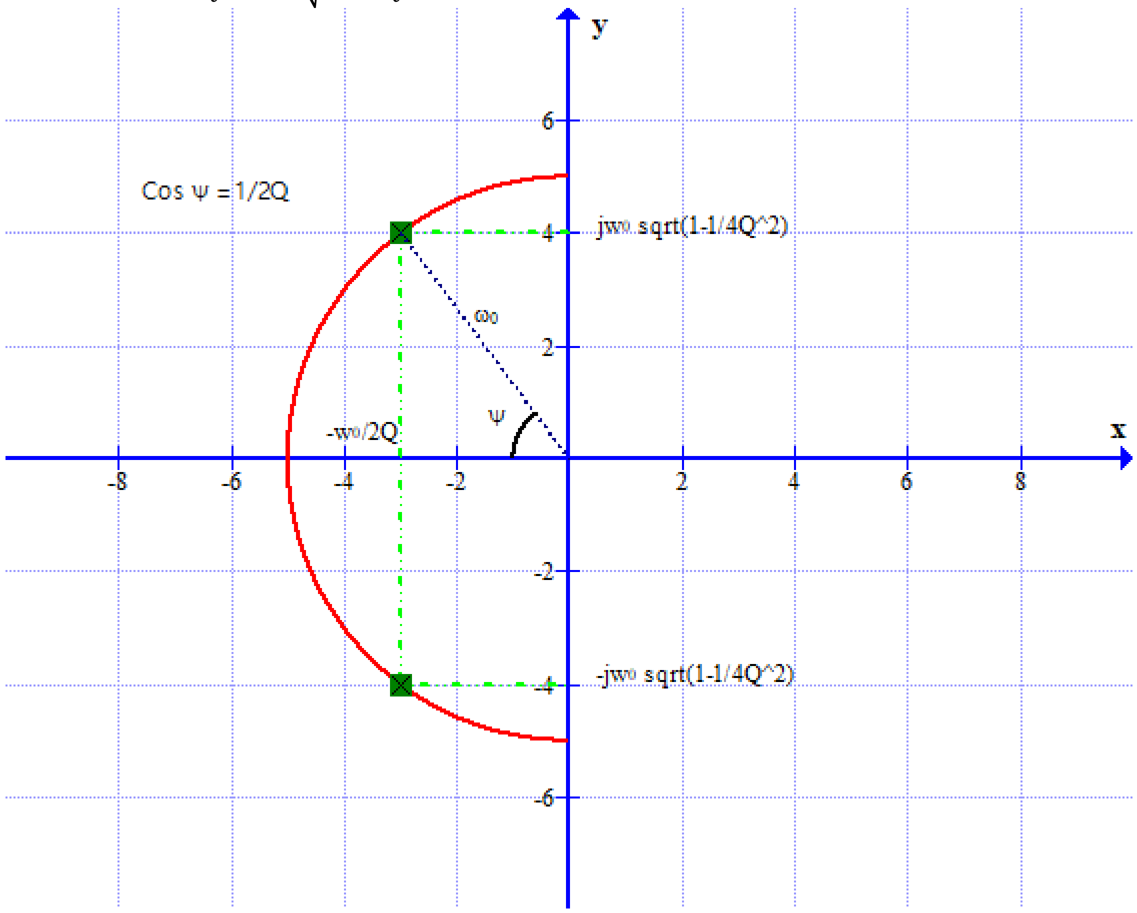
  $20\log|G(j\omega)| = -10\log[\{1-(\omega/\omega_0)^2\}^2+(\omega/\omega_0\, Q)^2] = \begin{cases} 0\ dB & if\ \omega \ll \omega_0 \\ -40\log\frac{\omega}{\omega_0}\ dB & if\ \omega \gg \omega_0 \end{cases}$

  $20\log|G(j\omega)| = 20\log Q$ if $\omega = \omega_0$

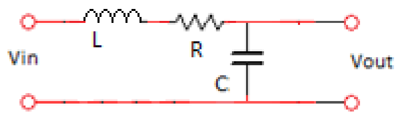
  Roll off = 40 db/decade



Pole location: $s_1, s_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}}$



Q1. Consider the following passive LPF:



If C=0.05μF, L=16mH and R=120Ω

a. Drive the transfer function.
b. Find the poles and zeros of the filter.
c. Calculate ω_0 *and* Q. What is the peak gain and at which frequency is it observed?
d. Redesign the circuit to raise its Q to 12 without changing ω_0.
e. Confirm your answers by plotting the frequency responses of the original and redesigned circuits.

Solution:

a. Transfer function: $G(s) = \frac{1/LC}{s^2+(R/L)s+1/LC} = \frac{\omega_0{}^2}{s^2+(\omega_0/Q)s+\omega_0{}^2}$

Where $\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(16\times10^{-3})(0.05\times10^{-6})}} = 35{,}355\ rad/s$

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{120}\sqrt{\frac{16\times10^{-3}}{0.05\times10^{-6}}} = 4.714$

b. Poles: $s_1, s_2 = -\frac{\omega_0}{2Q} \pm j\omega_0\sqrt{1-\frac{1}{4Q^2}} = -\frac{35{,}355}{2(4.714)} \pm j35{,}355\sqrt{1-\frac{1}{4(4.714)^2}} = -3{,}750 \pm j35{,}156$

c. $\omega_0 = 35{,}355\ rad/s$

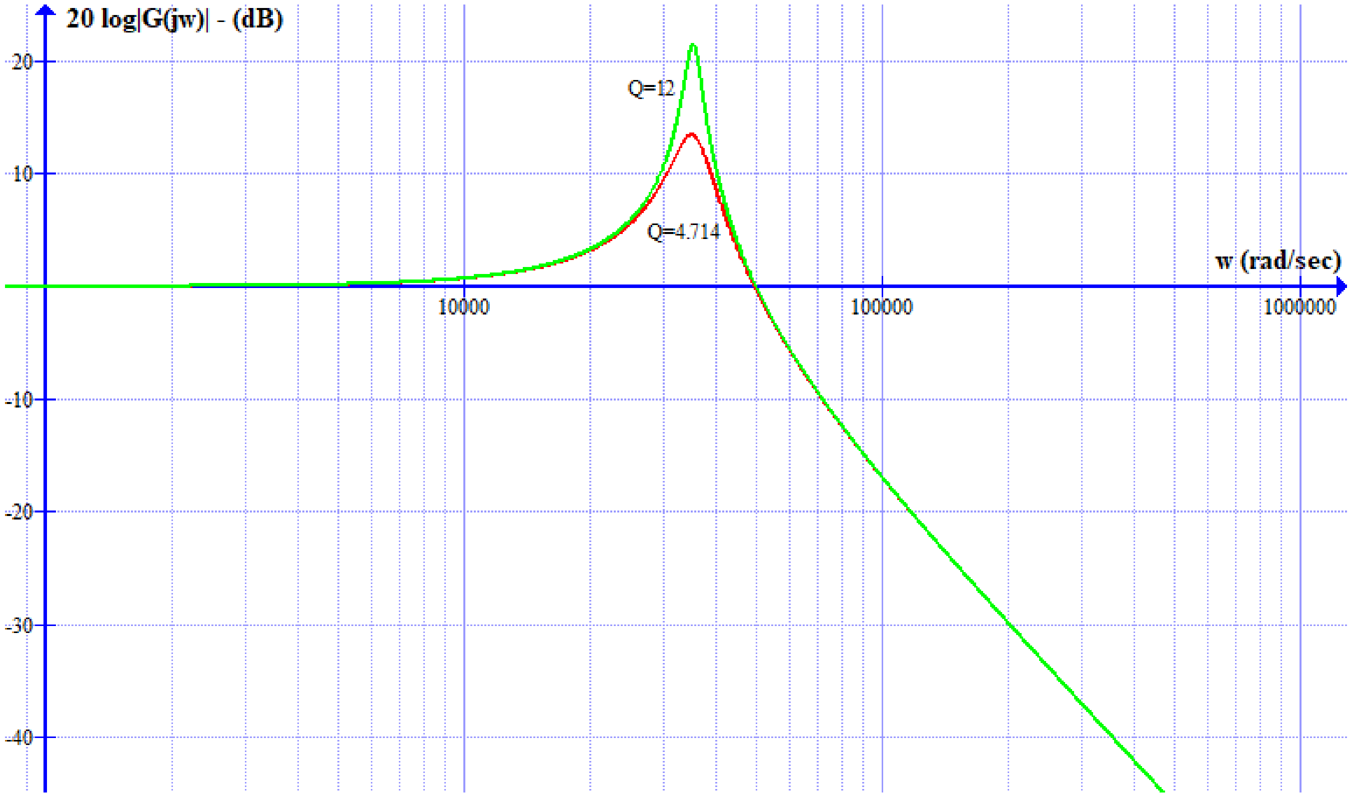
$Q = 4.714$

Peak gain $= Q\sqrt{1-\frac{1}{4Q^4}} = 4.714\sqrt{1-\frac{1}{4(4.714)^2}} = 4.713$

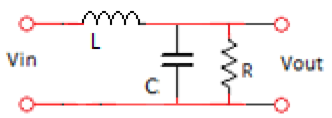
Frequency of peak gain $= \omega_0\sqrt{1-\frac{1}{2Q^2}} = 34{,}955\ rad/s$

d. Q will be 12 if: $R = \frac{1}{Q}\sqrt{\frac{L}{C}} = \frac{1}{12}\sqrt{\frac{16\times10^{-3}}{0.05\times10^{-6}}} = 47.14\ \Omega$

e.



Q2. Consider the following passive LPF circuit:



It is required that the poles should be located at $s_1, s_2 = 1{,}000e^{\pm j\frac{3\pi}{4}}$

a. Calculate $\omega_0\ and\ Q$.

b. Find the values of R, L & C.

Solution:

a. $\omega_0 = 1{,}000\ rad/s$ $\qquad Q = \frac{1}{2\cos\psi} = \frac{1}{2\cos 45°} = \frac{1}{\sqrt{2}} = 0.7071$

b. $\omega_0 = \frac{1}{\sqrt{LC}} = 1{,}000$ and $Q = R\sqrt{\frac{C}{L}} \Rightarrow \sqrt{\frac{C}{L}} = \frac{Q}{R} = \frac{0.7071}{R}$

Let R=70.71Ω

$$\sqrt{\frac{C}{L}} = \frac{Q}{R} = \frac{0.7071}{70.71} = \frac{1}{100}$$

$$\frac{1}{\sqrt{LC}}\sqrt{\frac{C}{L}} = (1{,}000)\left(\frac{1}{100}\right) \Rightarrow L = 0.1\ H$$

$$C = 10^{-4}(L) = 10^{-4}(0.1) = 100\ \mu F$$