CHAPTER 16

POWER SYSTEM STABILITY

When ac generators were driven by reciprocating steam engines, one of the major problems in the operation of machinery was caused by sustained oscillations in speed or *hunting* due to the periodic variations in the torque applied to the generators. The resulting periodic variations in voltage and frequency were transmitted to the motors connected to the system. Oscillations of the motors caused by the variations in voltage and frequency sometimes caused the motors to lose synchronism entirely if their natural frequency of oscillation coincided with the frequency of oscillation caused by the engines driving the generators. Damper windings were first used to minimize hunting by the damping action of the losses resulting from the currents induced in the damper windings by any relative motion between the rotor and the rotating field set up by the armature current. The use of turbines has reduced the problem of hunting although it is still present where the prime mover is a diesel engine. Maintaining synchronism between the various parts of a power system becomes increasingly difficult, however, as the systems and interconnections between systems continue to grow.

16.1 THE STABILITY PROBLEM

Stability studies which evaluate the impact of disturbances on the electromechanical dynamic behavior of the power system are of two types—*transient* and *steady state*. Transient stability studies are very commonly undertaken by electric utility planning departments responsible for ensuring proper dynamic performance of the system. The system models used in such studies are extensive because present-day power systems are vast, heavily interconnected systems with hundreds of machines which can interact through the medium of their extra-high-voltage and ultra-high-voltage networks. These machines have associated excitation systems and turbine-governing control systems which in some but not all cases are modeled in order to reflect properly correct dynamic performance of the system. If the resultant nonlinear differential and algebraic equations of the overall system are to be solved, then either direct methods or iterative step-by-step procedures must be used. In this chapter we emphasize transient stability considerations and introduce basic iterative procedures used in transient stability studies. Before doing so, however, let us first discuss certain terms commonly encountered in stability analysis.[1]

A power system is in a *steady-state operating condition* if all the measured (or calculated) physical quantities describing the operating condition of the system can be considered *constant* for purposes of analysis. When operating in a steady-state condition if a sudden change or sequence of changes occurs in one or more of the parameters of the system, or in one or more of its operating quantities, we say that the system has undergone a *disturbance* from its steady-state operating condition. Disturbances can be large or small depending on their origin. A *large disturbance* is one for which the nonlinear equations describing the dynamics of the power system cannot be validly linearized for purposes of analysis. Transmission system faults, sudden load changes, loss of generating units, and line switching are examples of large disturbances. If the power system is operating in a steady-state condition and it undergoes change which can be properly analyzed by linearized versions of its dynamic and algebraic equations, we say that a *small disturbance* has occurred. A change in the gain of the automatic voltage regulator in the excitation system of a large generating unit could be an example of a small disturbance. The power system is *steady-state stable* for a particular steady-state operating condition if, following a small disturbance, it returns to essentially the same steady-state condition of operation. However, if following a large disturbance, a significantly different but acceptable steady-state operating condition is attained, we say that the system is *transiently stable*.

Steady-state stability studies are usually less extensive in scope than transient stability studies and often involve a single machine operating into an infinite bus or just a few machines undergoing one or more small disturbances. Thus, steady-state stability studies examine the stability of the system under small *incremental* variations in parameters or operating conditions about a steady-state equilibrium point. The nonlinear differential and algebraic equations of the system are replaced by a set of linear equations which are then

[1]For further discussion, see "Proposed Terms and Definitions for Power System Stability," A Task Force Report of the System Dynamic Performance Subcommittee, *IEEE Transactions on Power Apparatus and Systems*, vol. PAS 101, July 1982, pp. 1894–1898.

solved by methods of linear analysis to determine if the system is steady-state stable.

Since transient stability studies involve large disturbances, linearization of the system equations is not permitted. Transient stability is sometimes studied on a *first-swing* rather than a *multiswing* basis. First-swing transient stability studies use a reasonably simple generator model consisting of the transient internal voltage E_i' behind transient reactance X_d'; in such studies the excitation systems and turbine-governing control systems of the generating units are not represented. Usually, the time period under study is the first second following a system fault or other large disturbance. If the machines of the system are found to remain essentially in synchronism within the first second, the system is regarded as being transiently stable. Multiswing stability studies extend over a longer study period, and therefore the effects of the generating units' control systems must be considered since they can affect the dynamic performance of the units during the extended period. Machine models of greater sophistication are then needed to properly reflect the behavior of the system.

Thus, excitation systems and turbine-governing control systems may or may not be represented in steady-state and transient stability studies depending on the objectives. In all stability studies the objective is to determine whether or not the rotors of the machines being perturbed return to constant speed operation. Obviously, this means that the rotor speeds have departed at least temporarily from synchronous speed. To facilitate computation, three fundamental assumptions therefore are made in *all* stability studies:

1. Only synchronous frequency currents and voltages are considered in the stator windings and the power system. Consequently, dc offset currents and harmonic components are neglected.
2. Symmetrical components are used in the representation of unbalanced faults.
3. Generated voltage is considered unaffected by machine speed variations.

These assumptions permit the use of phasor algebra for the transmission network and solution by power-flow techniques using 60-Hz parameters. Also, negative- and zero-sequence networks can be incorporated into the positive-sequence network at the fault point. As we shall see, three-phase balanced faults are generally considered. However, in some special studies circuit-breaker clearing operation may be such that consideration of unbalanced conditions is unavoidable.[2]

[2]For information beyond the scope of this book, see P. M. Anderson and A. A. Fouad, *Power System Control and Stability*, The Iowa State University Press, Ames, IA, 1977.

16.2 ROTOR DYNAMICS AND THE SWING EQUATION

The equation governing rotor motion of a synchronous machine is based on the elementary principle in dynamics which states that accelerating torque is the product of the moment of inertia of the rotor times its angular acceleration. In the MKS (meter-kilogram-second) system of units this equation can be written for the synchronous generator in the form

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \text{ N-m} \tag{16.1}$$

where the symbols have the following meanings:

- J the total moment of inertia of the rotor masses, in kg-m^2
- θ_m the angular displacement of the rotor with respect to a stationary axis, in mechanical radians (rad)
- t time, in seconds (s)
- T_m the mechanical or shaft torque supplied by the prime mover less retarding torque due to rotational losses, in N-m
- T_e the net electrical or electromagnetic torque, in N-m
- T_a the net accelerating torque, in N-m

The mechanical torque T_m and the electrical torque T_e are considered positive for the synchronous generator. This means that T_m is the resultant shaft torque which tends to accelerate the rotor in the positive θ_m direction of rotation, as shown in Fig. 16.1(a). Under steady-state operation of the generator T_m and T_e are equal and the accelerating torque T_a is zero. In this case there is no acceleration or deceleration of the rotor masses and the resultant constant speed is the *synchronous speed.* The rotating masses, which include the rotor of the generator and the prime mover, are said to be *in synchronism* with the other machines operating at synchronous speed in the power system. The prime

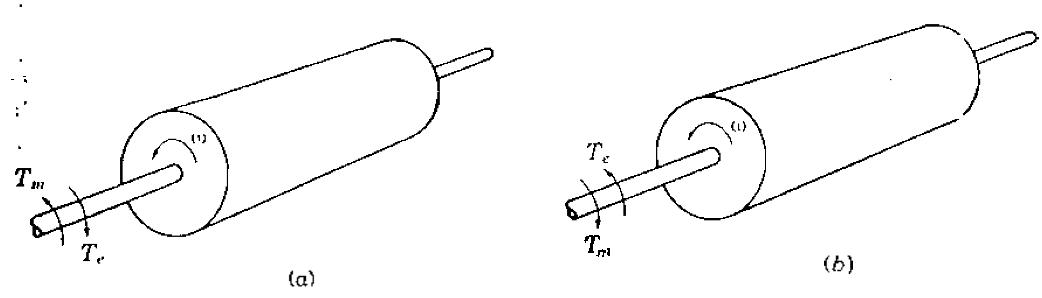


FIGURE 16.1
Representation of a machine rotor comparing direction of rotation and mechanical and electrical torques for: (a) a generator; (b) a motor.

mover may be a hydroturbine or a steam turbine, for which models of different levels of complexity exist to represent their effect on T_m. In this text T_m is considered constant at any given operating condition. This assumption is a fair one for generators even though input from the prime mover is controlled by governors. Governors do not act until after a change in speed is sensed, and so they are not considered effective during the time period in which rotor dynamics are of interest in our stability studies here. The electrical torque T_e corresponds to the net air-gap power in the machine, and thus accounts for the total output power of the generator plus $|I|^2R$ losses in the armature winding. In the synchronous motor the direction of power flow is opposite to that in the generator. Accordingly, for a motor both T_m and T_e in Eq. (16.1) are reversed in sign, as shown in Fig. 16.1(*b*). T_e then corresponds to the air-gap power supplied by the electrical system to drive the rotor, whereas T_m represents the counter torque of the load and rotational losses tending to retard the rotor.

Since θ_m is measured with respect to a stationary reference axis on the stator, it is an absolute measure of rotor angle. Consequently, it continuously increases with time even at constant synchronous speed. Since the rotor speed relative to synchronous speed is of interest, it is more convenient to measure the rotor angular position with respect to a reference axis which rotates at synchronous speed. Therefore, we define

$$\theta_m = \omega_{sm}t + \delta_m \tag{16.2}$$

where ω_{sm} is the synchronous speed of the machine in mechanical radians per second and δ_m is the angular displacement of the rotor, in mechanical radians, from the synchronously rotating reference axis. The derivatives of Eq. (16.2) with respect to time are

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \tag{16.3}$$

and

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \tag{16.4}$$

Equation (16.3) shows that the rotor angular velocity $d\theta_m/dt$ is constant and equals the synchronous speed only when $d\delta_m/dt$ is zero. Therefore, $d\delta_m/dt$ represents the deviation of the rotor speed from synchronism and the units of measure are mechanical radians per second. Equation (16.4) represents the rotor acceleration measured in mechanical radians per second-squared.

Substituting Eq. (16.4) in Eq. (16.1), we obtain

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \text{ N-m} \tag{16.5}$$

It is convenient for notational purposes to introduce

$$\omega_m = \frac{d\theta_m}{dt} \tag{16.6}$$

for the angular velocity of the rotor. We recall from elementary dynamics that power equals torque times angular velocity, and so multiplying Eq. (16.5) by ω_m, we obtain

$$J\omega_m \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \text{ W} \tag{16.7}$$

where P_m = shaft power input to the machine less rotational losses
P_e = electrical power crossing its air gap
P_a = accelerating power which accounts for any unbalance between those two quantities

Usually, we neglect rotational losses and armature $|I|^2R$ losses and think of P_m as power supplied by the prime mover and P_e as the electrical power output.

The coefficient $J\omega_m$ is the angular momentum of the rotor; at synchronous speed ω_{sm} it is denoted by M and called the *inertia constant* of the machine. Obviously, the units in which M is expressed must correspond to those of J and ω_m. A careful check of the units in each term of Eq. (16.7) shows that M is expressed in joule-seconds per mechanical radian, and we write

$$M\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \text{ W} \tag{16.8}$$

While we have used M in this equation, the coefficient is not a constant in the strictest sense because ω_m does not equal synchronous speed under all conditions of operation. However, in practice, ω_m does not differ significantly from synchronous speed when the machine is stable, and since power is more convenient in calculations than torque, Eq. (16.8) is preferred. In machine data supplied for stability studies another constant related to inertia is often encountered. This is the so-called *H constant*, which is defined by

$$H = \frac{\text{stored kinetic energy in megajoules at synchronous speed}}{\text{machine rating in MVA}}$$

and
$$H = \frac{\frac{1}{2}J\omega_{sm}^2}{S_{\text{mach}}} = \frac{\frac{1}{2}M\omega_{sm}}{S_{\text{mach}}} \text{ MJ/MVA} \tag{16.9}$$

where S_{mach} is the three-phase rating of the machine in megavoltamperes. Solving for M in Eq. (16.9), we obtain

$$M = \frac{2H}{\omega_{sm}} S_{\text{mach}} \text{ MJ/mech rad} \tag{16.10}$$

and substituting for M in Eq. (16.8), we find

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{S_{\text{mach}}} = \frac{P_m - P_e}{S_{\text{mach}}} \tag{16.11}$$

This equation leads to a very simple result.

Note that δ_m is expressed in mechanical radians in the numerator of Eq. (16.11), whereas ω_{sm} is expressed in mechanical radians per second in the denominator. Therefore, we can write the equation in the form

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ per unit} \tag{16.12}$$

provided both δ and ω_s have consistent units, which may be mechanical or electrical degrees or radians. H and t have consistent units since megajoules per megavoltampere is in units of time in seconds and P_a, P_m, and P_e must be in per unit on the same base as H. When the subscript m is associated with ω, ω_s, and δ, it means *mechanical* units are being used; otherwise, *electrical* units are implied. Accordingly, ω_s is the synchronous speed in electrical units. For a system with an electrical frequency of f hertz Eq. (16.12) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ per unit} \tag{16.13}$$

when δ is in electrical radians, while

$$\frac{H}{180f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \text{ per unit} \tag{16.14}$$

applies when δ is in electrical degrees.

Equation (16.12), called the *swing equation* of the machine, is the fundamental equation which governs the rotational dynamics of the synchronous machine in stability studies. We note that it is a second-order differential equation, which can be written as the two first-order differential equations

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_m - P_e \text{ per unit} \tag{16.15}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{16.16}$$

in which ω, ω_s, and δ involve electrical radians or electrical degrees.

We use the various equivalent forms of the swing equation throughout this chapter to determine the stability of a machine within a power system. When the swing equation is solved, we obtain the expression for δ as a function of time. A graph of the solution is called the *swing curve* of the machine and inspection of the swing curves of all the machines of the system will show whether the machines remain in synchronism after a disturbance.

16.3 FURTHER CONSIDERATIONS OF THE SWING EQUATION

The megavoltampere (MVA) base used in Eq. (16.11) is the machine rating S_{mach} which is introduced by the definition of H. In a stability study of a power system with many synchronous machines only one MVA base common to all parts of the system can be chosen. Since the right-hand side of the swing equation for each machine must be expressed in per unit on this common system base, it is clear that H on the left-hand side of each swing equation must also be consistent with the system base. This is accomplished by converting H for each machine based on its own individual rating to a value determined by the system base, S_{system}. Equation (16.11), multiplied on each side by the ratio (S_{mach}/S_{system}), leads to the conversion formula

$$H_{system} = H_{mach}\frac{S_{mach}}{S_{system}} \tag{16.17}$$

in which the subscript for each term indicates the corresponding base being used. In industry studies the system base that is usually chosen is 100 MVA.

The inertia constant M is rarely used in practice and the forms of the swing equation involving H are more often encountered. This is because the value of M varies widely with the size and type of the machine, whereas H assumes a much narrower range of values, as shown in Table 16.1. Machine manufacturers also use the symbol WR^2 to specify for the rotating parts of a generating unit (including the prime mover) the weight in pounds multiplied by the square of the radius of gyration in feet. Hence, $WR^2/32.2$ is the moment of inertia of the machine in slug-feet squared.

TABLE 16.1
Typical inertia constants of synchronous machines†

Type of machine	Inertia constant, H‡ MJ / MVA
Turbine generator:	
Condensing, 1800 r/min	9–6
3600 r/min	7–4
Noncondensing, 3600 r/min	4–3
Waterwheel generator:	
Slow-speed, < 200 r/min	2–3
High-speed, > 200 r/min	2–4
Synchronous condenser;§	
Large	1.25
Small	1.00
Synchronous motor with load varies from 1.0 to 5.0 and higher for heavy flywheels	2.00

†Reprinted by permission of the ABB Power T & D Company, Inc. from *Electrical Transmission and Distribution Reference Book*.

‡Where range is given, the first figure applies to machines of smaller megavoltampere rating.

§Hydrogen-cooled, 25% less.

Example 16.1. Develop a formula to calculate the H constant from WR^2 and then evaluate H for a nuclear generating unit rated at 1333 MVA, 1800 r/min with $WR^2 = 5{,}820{,}000$ lb-ft^2.

Solution. The kinetic energy (KE) of rotation in foot-pounds at synchronous speed

$$\text{KE} = \frac{1}{2}\frac{WR^2}{32.2}\left[\frac{2\pi(\text{r/min})}{60}\right]^2 \text{ ft-lb}$$

Since 550 ft-lb/s equals 746 W, it follows that 1 ft-lb equals 746/550 J. Hence, converting foot-pounds to megajoules and dividing by the machine rating in megavoltamperes, we obtain

$$H = \frac{\left(\frac{746}{550} \times 10^{-6}\right)\frac{1}{2}\frac{WR^2}{32.2}\left[\frac{2\pi(\text{r/min})}{60}\right]^2}{S_{\text{mach}}}$$

which yields upon simplification

$$H = \frac{2.31 \times 10^{-10} WR^2(\text{r/min})^2}{S_{\text{mach}}}$$

Inserting the given machine data in this equation, we obtain

$$H = \frac{2.31 \times 10^{-10}(5.82 \times 10^{6})(1800)^2}{1333} = 3.27 \text{ MJ/MVA}$$

Converting H to a 100-MVA system base, we obtain

$$H = 3.27 \times \frac{1333}{100} = 43.56 \text{ MJ/MVA}$$

In a stability study for a large system with many machines geographically dispersed over a wide area it is desirable to minimize the number of swing equations to be solved. This can be done if the transmission-line fault, or other disturbance on the system, affects the machines within a power plant so that their rotors swing together. In such cases the machines within the plant can be combined into a single equivalent machine just as if their rotors were mechanically coupled and only one swing equation must be written for them. Consider a power plant with two generators connected to the same bus which is electrically remote from the network disturbances. The swing equations on the common system base are

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ per unit} \tag{16.18}$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ per unit} \tag{16.19}$$

Adding the equations together and denoting δ_1 and δ_2 by δ since the rotors swing together, we obtain

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ per unit} \tag{16.20}$$

where $H = (H_1 + H_2)$, $P_m = (P_{m1} + P_{m2})$, and $P_e = (P_{e1} + P_{e2})$. This single equation, which is in the form of Eq. (16.12), can be solved to represent the plant dynamics.

Example 16.2. Two 60-Hz generating units operate in parallel within the same power plant and have the following ratings:

Unit 1: 500 MVA, 0.85 power factor, 20 kV, 3600 r/min

$$H_1 = 4.8 \text{ MJ/MVA}$$

Unit 2: 1333 MVA, 0.9 power factor, 22 kV, 1800 r/min

$$H_2 = 3.27 \text{ MJ/MVA}$$

Calculate the equivalent H constant for the two units on a 100-MVA base.

Solution. The total kinetic energy (KE) of rotation of the two machines is

$$\text{KE} = (4.8 \times 500) + (3.27 \times 1333) = 6759 \text{ MJ}$$

Therefore, the H constant for the equivalent machine on 100-MVA base is

$$H = 67.59 \text{ MJ/MVA}$$

and this value can be used in a single swing equation, provided the machines swing together so that their rotor angles are in step at each instant of time.

Machines which swing together are called *coherent* machines. It is noted that when both ω_s and δ are expressed in electrical degrees or radians, the swing equations for coherent machines can be combined together even though, as in the example, the rated speeds are different. This fact is often used in stability studies involving many machines in order to reduce the number of swing equations which need to be solved.

For *any pair* of noncoherent machines in a system swing equations similar to Eqs. (16.18) and (16.19) can be written. Dividing each equation by its left-hand-side coefficient and subtracting the resultant equations, we obtain

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left(\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2}\right) \tag{16.21}$$

Multiplying each side by $H_1H_2/(H_1 + H_2)$ and rearranging, we find that

$$\frac{2}{\omega_s}\left(\frac{H_1H_2}{H_1 + H_2}\right)\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{P_{m1}H_2 - P_{m2}H_1}{H_1 + H_2} - \frac{P_{e1}H_2 - P_{e2}H_1}{H_1 + H_2} \tag{16.22}$$

which also may be written more simply in the form of the basic swing equation, Eq. (16.12), as follows:

$$\frac{2}{\omega_s}H_{12}\frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12} \tag{16.23}$$

Here the relative angle δ_{12} equals $\delta_1 - \delta_2$, and an equivalent inertia and weighted input and output powers are defined by

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2} \tag{16.24}$$

$$P_{m12} = \frac{P_{m1} H_2 - P_{m2} H_1}{H_1 + H_2} \tag{16.25}$$

$$P_{e12} = \frac{P_{e1} H_2 - P_{e2} H_1}{H_1 + H_2} \tag{16.26}$$

A noteworthy application of these equations concerns a two-machine system having only one generator (machine one) and a synchronous motor (machine two) connected by a network of pure reactances. Whatever change occurs in the generator output is thus absorbed by the motor, and we can write

$$\begin{aligned} P_{m1} &= -P_{m2} = P_m \\ P_{e1} &= -P_{e2} = P_e \end{aligned} \tag{16.27}$$

Under these conditions, $P_{m12} = P_m$, $P_{e12} = P_e$, and Eq. (16.22) reduces to

$$\frac{2H_{12}}{\omega_s} \frac{d^2\delta_{12}}{dt^2} = P_m - P_e$$

which is also the format of Eq. (16.12) for a single machine.

Equation (16.22) demonstrates that stability of a machine within a system is a relative property associated with its dynamic behavior with respect to the other machines of the system. The rotor angle of one machine, say, δ_1, can be chosen for comparison with the rotor angle of each other machine, symbolized by δ_2. In order to be stable the angular differences between all machines must decrease after the final switching operation—such as the opening of a circuit breaker to clear a fault. Although we may choose to plot the angle between a machine's rotor and a synchronously rotating reference axis, it is the relative angles between machines which are important. Our discussion above emphasizes the relative nature of the system stability property and shows that the essential features of stability study are revealed by consideration of two-machine problems. Such problems are of two types: those having one machine of finite inertia swinging with respect to an infinite bus and those having two finite inertia machines swinging with respect to each other. An infinite bus may be considered for stability purposes as a bus at which there is located a machine of constant internal voltage, having zero impedance and infinite inertia. The point of connection of a generator to a large power system may be regarded as such a bus. In all cases the swing equation assumes the form of Eq. (16.12), each term of which must be explicitly described before it can be solved. The equation for P_e is essential to this description, and we now proceed to its characterization for a general two-machine system.

16.4 THE POWER-ANGLE EQUATION

In the swing equation for the generator the input mechanical power from the prime mover, P_m, will be considered constant. As we have mentioned previously, this is a reasonable assumption because conditions in the electrical network can be expected to change before the control governor can cause the turbine to react. Since P_m in Eq. (16.12) is constant, the electrical power output P_e will determine whether the rotor accelerates, decelerates, or remains at synchronous speed. When P_e equals P_m, the machine operates at steady-state synchronous speed; when P_e changes from this value, the rotor deviates from synchronous speed. Changes in P_e are determined by conditions on the transmission and distribution networks and the loads on the system to which the generator supplies power. Electrical network disturbances resulting from severe load changes, network faults, or circuit-breaker operations may cause the generator output P_e to change rapidly, in which case electromechanical transients exist. Our fundamental assumption is that the effect of machine speed variations upon the generated voltage is negligible so that the manner in which P_e changes is determined by the power-flow equations applicable to the state of the electrical network and by the model chosen to represent the electrical behavior of the machine. Each synchronous machine is represented for transient stability studies by its transient internal voltage E_i' in series with the transient reactance X_d', as shown in Fig. 16.2(a) in which V_t is the terminal voltage. This corresponds to the steady-state representation in which syn-

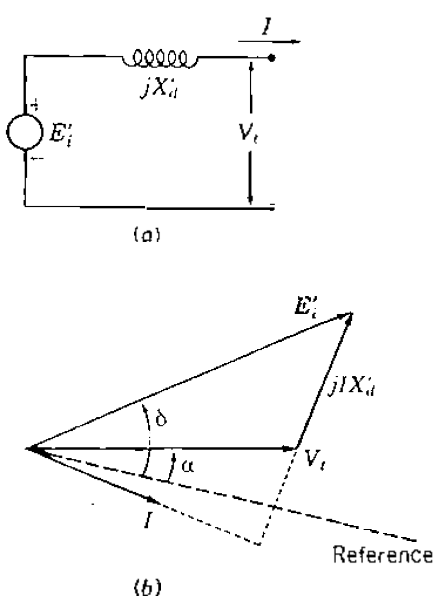


FIGURE 16.2
Phasor diagram of a synchronous machine for transient stability studies.

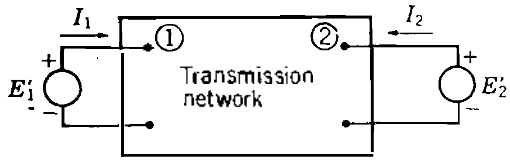


FIGURE 16.3
Schematic diagram for stability studies. Transient reactances associated with E_1' and E_2' are included in the transmission network.

chronous reactance X_d is in series with the synchronous internal or no-load voltage E_i. Armature resistance is negligible in most cases so that the phasor diagram of Fig. 16.2(*b*) applies. Since each machine must be considered relative to the system of which it is part, the phasor angles of the machine quantities are measured with respect to the common system reference.

Figure 16.3 schematically represents a generator supplying power through a transmission system to a receiving-end system at bus ②. The rectangle represents the transmission system of linear passive components, such as transformers, transmission lines, and capacitors, and includes the transient reactance of the generator. Therefore, the voltage E_1' represents the transient internal voltage of the generator at bus ①. The voltage E_2' at the receiving end is regarded here as that of an infinite bus or as the transient internal voltage of a synchronous motor whose transient reactance is included in the network. Later we shall consider the case of two generators supplying constant-impedance loads within the network. The bus admittance matrix for the network reduced to two nodes in addition to the reference node is

$$\mathbf{Y}_{\text{bus}} = \begin{array}{c} \\ ① \\ ② \end{array}\begin{array}{c} \begin{array}{cc} ① & ② \end{array} \\ \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \end{array} \tag{16.28}$$

From Eq. (9.4) we have

$$P_k + jQ_k = V_k \sum_{n=1}^{N} (Y_{kn}V_n)^* \tag{16.29}$$

Letting k and N equal 1 and 2, respectively, and substituting E' for V, we obtain

$$P_1 + jQ_1 = E_1'(Y_{11}E_1')^* + E_1'(Y_{12}E_2')^* \tag{16.30}$$

If we define

$$E_1' = |E_1'|\angle\delta_1 \qquad E_2' = |E_2'|\angle\delta_2$$

$$Y_{11} = G_{11} + jB_{11} \qquad Y_{12} = |Y_{12}|\angle\theta_{12}$$

Eq. (16.30) yields

$$P_1 = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}|\cos(\delta_1 - \delta_2 - \theta_{12}) \tag{16.31}$$

$$Q_1 = -|E_1'|^2 B_{11} + |E_1'||E_2'||Y_{12}|\sin(\delta_1 - \delta_2 - \theta_{12}) \tag{16.32}$$

Similar equations apply at bus ① by substituting subscripts 1 for 2 and 2 for 1 in the two preceding equations.

If we let

$$\delta = \delta_1 - \delta_2$$

and define a new angle γ such that

$$\gamma = \theta_{12} - \frac{\pi}{2}$$

we obtain from Eqs. (16.31) and (16.32)

$$P_1 = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}|\sin(\delta - \gamma) \tag{16.33}$$

$$Q_1 = -|E_1'|^2 B_{11} - |E_1'||E_2'||Y_{12}|\cos(\delta - \gamma) \tag{16.34}$$

Equation (16.33) may be written more simply as

$$P_e = P_c + P_{\max}\sin(\delta - \gamma) \tag{16.35}$$

where

$$P_c = |E_1'|^2 G_{11} \qquad P_{\max} = |E_1'||E_2'||Y_{12}| \tag{16.36}$$

Since P_1 represents the electric power output of the generator (armature loss neglected), we have replaced it by P_e in Eq. (16.35), which is often called the *power-angle equation*; its graph as a function of δ is called the *power-angle curve*. The parameters P_c, $P_{\max}$, and γ are constants for a given network configuration and constant voltage magnitudes $|E_1'|$ and $|E_2'|$. When the network is considered without resistance, all the elements of $\mathbf{Y}_{\text{bus}}$ are susceptances, and then G_{11} and γ are both zero. The power-angle equation which then applies for the pure reactance network is simply the familiar equation

$$P_e = P_{\max}\sin\delta \tag{16.37}$$

where $P_{\max} = |E_1'||E_2'|/X$ and X is the transfer reactance between E_1' and E_2'.

Example 16.3. The single-line diagram of Fig. 16.4 shows a generator connected through parallel transmission lines to a large metropolitan system considered as an infinite bus. The machine is delivering 1.0 per-unit power and both the terminal voltage and the infinite-bus voltage are 1.0 per unit. Numbers on the diagram

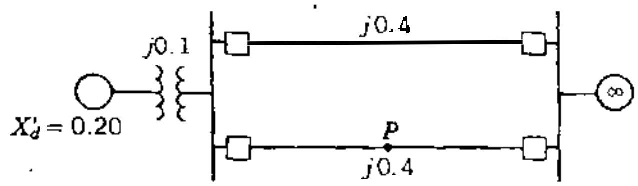


FIGURE 16.4
One-line diagram for Examples 16.3 and 16.4. Point P is at the center of the line.

indicate the values of the reactances on a common system base. The transient reactance of the generator is 0.20 per unit as indicated. Determine the power-angle equation for the given system operating conditions.

Solution. The reactance diagram for the system is shown in Fig. 16.5(a). The series reactance between the terminal voltage and the infinite bus is

$$X = 0.10 + \frac{0.4}{2} = 0.3 \text{ per unit}$$

and therefore the 1.0 per-unit power output of the generator is determined by

$$\frac{|V_t||V|}{X}\sin\alpha = \frac{(1.0)(1.0)}{0.3}\sin\alpha = 1.0$$

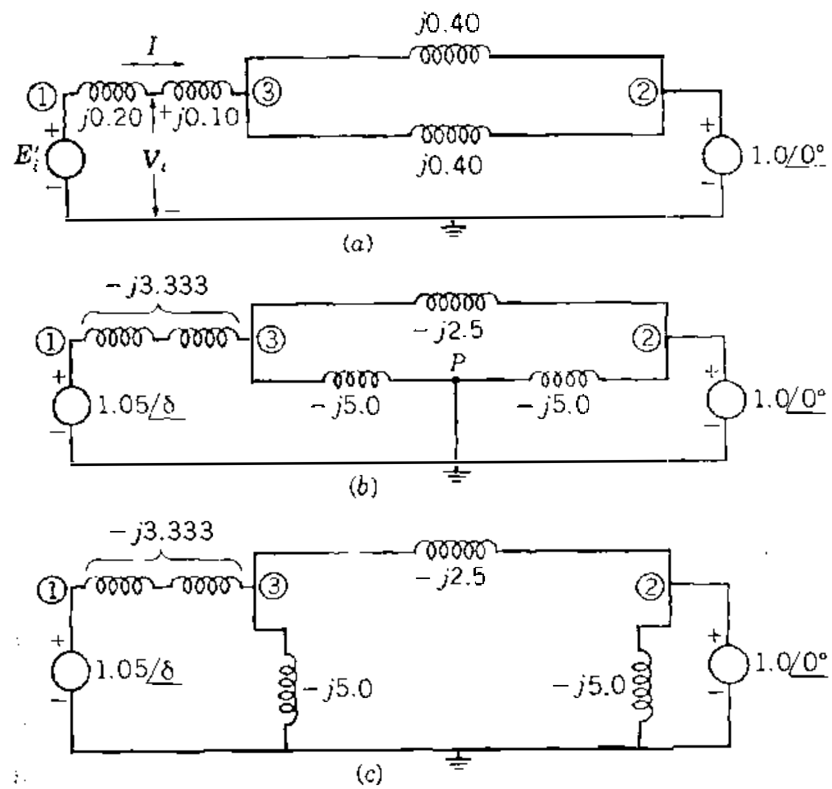


FIGURE 16.5
Reactance diagram for: (a) the prefault network for Example 16.3 with impedances in per unit; (b) and (c); the faulted network for Example 16.4 with the same impedances converted to admittances and marked in per unit.

where V is the voltage of the infinite bus and α is the angle of the terminal voltage relative to the infinite bus. Solving for α, we obtain

$$\alpha = \sin^{-1} 0.3 = 17.458°$$

so that the terminal voltage is

$$V_t = 1.0\angle 17.458° = 0.954 + j0.300 \text{ per unit}$$

The output current from the generator is now calculated as

$$I = \frac{1.0\angle\delta_2 - 1.0\angle 0°}{j0.3}$$

$$= 1.0 + j0.1535 = 1.012\angle 8.729° \text{ per unit}$$

and the transient internal voltage is then found to be

$$E'_i = (0.954 + j0.30) + j(0.2)(1.0 + j0.1535)$$

$$= 0.923 - j0.5 = 1.050\angle 28.44° \text{ per unit}$$

The power-angle equation relating the transient internal voltage E'_i and the infinite-bus voltage V is determined by the *total* series reactance

$$X = 0.2 + 0.1 + \frac{0.4}{2} = 0.5 \text{ per unit}$$

Hence, the desired equation is

$$P_e = \frac{(1.050)(1.0)}{0.5} \sin\delta = 2.10 \sin\delta \text{ per unit}$$

where δ is the machine rotor angle with respect to the infinite bus.

The power-angle equation of the preceding example is plotted in Fig. 16.6. Note that the mechanical input power P_m is constant and intersects the

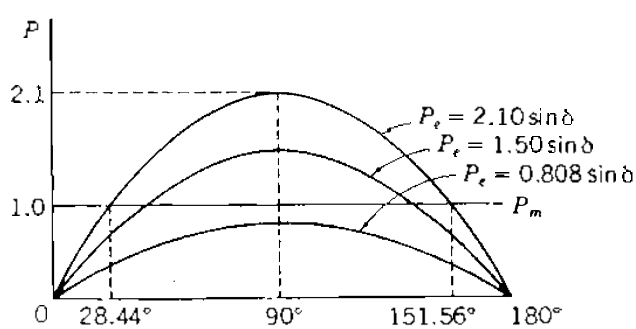


FIGURE 16.6
Plot of power-angle curves found in Examples 16.3 to 16.5.

sinusoidal power-angle curve at the operating angle $\delta_0 = 24.44°$. This is the initial angular position of the generator rotor corresponding to the given operating conditions. The swing equation for the machine may be written

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 2.10 \sin\delta \text{ per unit} \qquad (16.38)$$

where H is in megajoules per megavoltampere, f is the electrical frequency of the system, and δ is in electrical degrees. We can easily check the results of Example 16.3 since under the given operating conditions, $P_e = 2.10 \sin 28.44° = 1.0$ per unit, which corresponds exactly to the mechanical power input P_m and the acceleration is zero.

In the next example we determine the power-angle equation for the same system with a three-phase fault at P, the midpoint of one of the transmission lines. Positive acceleration is shown to exist due to the fault.

Example 16.4. The system of Example 16.3 is operating under the indicated conditions when a three-phase fault occurs at point P of Fig. 16.4. Determine the power-angle equation for the system with the fault on and the corresponding swing equation. Take $H = 5$ MJ/MVA.

Solution. The reactance diagram is shown in Fig. 16.5(*b*) with the fault on the system at point P. Values shown are admittances in per unit. The effect of the short circuit caused by the fault is clearly shown by redrawing the reactance diagram, as in Fig. 16.5(*c*). As calculated in Example 16.3, transient internal voltage of the generator remains at $E_i' = 1.05\angle 28.44°$ based on the assumption of constant flux linkage in the machine. The net transfer admittance connecting the voltage sources remains to be determined. The buses are numbered as shown and the $\mathbf{Y}_{bus}$ is formed by inspection of Fig. 16.5(*c*) as follows:

$$\mathbf{Y}_{bus} = \begin{matrix} & ① & ② & ③ \\ ① & -j3.333 & 0.00 & j3.333 \\ ② & 0.000 & -j7.50 & j2.500 \\ ③ & j3.333 & j2.50 & -j10.833 \end{matrix}$$

Bus ③ has no external source connection, and it may be removed by the node elimination procedure of Sec. 7.4 to yield the reduced bus admittance matrix

$$\begin{matrix} & ① & ② \\ ① & Y_{11} & Y_{12} \\ ② & Y_{21} & Y_{22} \end{matrix} = \begin{matrix} & ① & ② \\ ① & -j2.308 & j0.769 \\ ② & j0.769 & -j6.923 \end{matrix}$$

The magnitude of the transfer admittance is 0.769, and thus

$$P_{\max} = |E_1'||E_2'||Y_{12}| = (1.05)(1.0)(0.769) = 0.808 \text{ per unit}$$

The power-angle equation with the fault on the system is therefore

$$P_e = 0.808 \sin \delta \text{ per unit}$$

and the corresponding swing equation is

$$\frac{5}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin \delta \text{ per unit} \qquad (16.39)$$

Due to its inertia, the rotor cannot change position instantly upon occurrence of the fault. Therefore, the rotor angle δ is initially 28.44°, the same as in Example 16.3, and the electrical power output is $P_e = 0.808 \sin 28.44° = 0.385$. The initial accelerating power is

$$P_a = 1.0 - 0.385 = 0.615 \text{ per unit}$$

and the initial acceleration is positive with the value given by

$$\frac{d^2\delta}{dt^2} = \frac{180f}{5}(0.615) = 22.14f \text{ elec deg/s}^2$$

where f is the system frequency.

Relaying schemes sensing the fault on the line will act to clear the fault by simultaneous opening of the line-end breakers. When this occurs, another power-angle equation applies because of the network change.

Example 16.5. The fault on the system of Example 16.4 is cleared by simultaneous opening of the circuit breakers at each end of the affected line. Determine the power-angle equation and the swing equation for the postfault period.

Solution. Inspection of Fig. 16.5(a) shows that upon removal of the faulted line, the net transfer admittance across the system is

$$\frac{1}{j(0.2 + 0.1 + 0.4)} = -j1.429 \text{ per unit}$$

so that in the bus admittance matrix

$$Y_{12} = j1.429$$

Therefore, the postfault power-angle equation is

$$P_e = (1.05)(1.0)(1.429)\sin \delta = 1.500 \sin \delta$$

and the corresponding swing equation is

$$\frac{5}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 1.500 \sin \delta$$

The acceleration at the instant of clearing the fault depends on the angular position of the rotor at that time. The power-angle curves for Examples 16.3 through 16.5 are compared in Fig. 16.6.

16.5 SYNCHRONIZING POWER COEFFICIENTS

In Example 16.3 the operating point on the sinusoidal P_e curve of Fig. 16.6 is found to be at $\delta_0 = 28.44°$, where the mechanical power input P_m equals the electrical power output P_e. In the same figure it is also seen that P_e equals P_m at $\delta = 151.56°$, and this might appear to be an equally acceptable operating point. However, this is not the case as now shown.

A commonsense requirement for an acceptable operating point is that the generator should not lose synchronism when small temporary changes occur in the electrical power output from the machine. To examine this requirement for fixed mechanical input power P_m, consider small incremental changes in the operating point parameters; that is, consider

$$\delta = \delta_0 + \delta_\Delta \qquad P_e = P_{e0} + P_{e\Delta} \tag{16.40}$$

where the subscript zero denotes the steady-state operating point values and the subscript delta (Δ) identifies the incremental variations from those values. Substituting Eqs. (16.40) in Eq. (16.37), we obtain the power-angle equation for the general two-machine system in the form

$$\begin{aligned} P_{e0} + P_{e\Delta} &= P_{\max} \sin(\delta_0 + \delta_\Delta) \\ &= P_{\max}(\sin \delta_0 \cos \delta_\Delta + \cos \delta_0 \sin \delta_\Delta) \end{aligned}$$

Since δ_Δ is a small incremental displacement from δ_0,

$$\sin \delta_\Delta \cong \delta_\Delta \qquad \text{and} \qquad \cos \delta_\Delta \cong 1 \tag{16.41}$$

and when strict equality is assumed, the previous equation becomes

$$P_{e0} + P_{e\Delta} = P_{\max} \sin \delta_0 + (P_{\max} \cos \delta_0)\, \delta_\Delta \tag{16.42}$$

At the initial operating point δ_0

$$P_m = P_{e0} = P_{\max} \sin \delta_0 \tag{16.43}$$

and it then follows from Eq. (16.42) that

$$P_m - (P_{e0} + P_{e\Delta}) = -(P_{\max} \cos \delta_0)\, \delta_\Delta \tag{16.44}$$

Substituting the incremental variables of Eq. (16.40) in the basic swing equation, Eq. (16.12), we obtain

$$\frac{2H}{\omega_s} \frac{d^2(\delta_0 + \delta_\Delta)}{dt^2} = P_m - (P_{e0} + P_{e\Delta}) \tag{16.45}$$

Replacing the right-hand side of this equation by Eq. (16.44) and transposing terms, we obtain

$$\frac{2H}{\omega_s} \frac{d^2\delta_\Delta}{dt^2} + (P_{\max} \cos \delta_0)\, \delta_\Delta = 0 \tag{16.46}$$

since δ_0 is a constant value. Noting that $P_{\max} \cos \delta_0$ is the slope of the power-angle curve at the angle δ_0, we denote this slope as S_p and define it as

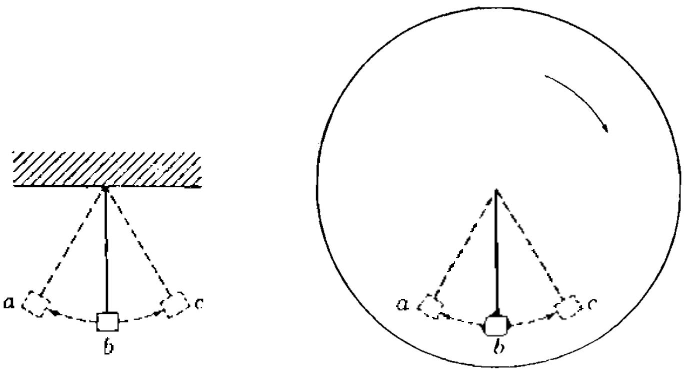
$$S_p = \left. \frac{dP_e}{d\delta} \right|_{\delta=\delta_0} = P_{\max} \cos \delta_0 \tag{16.47}$$

where S_p is called the *synchronizing power coefficient*. When S_p is used in Eq. (16.46), the swing equation governing the incremental rotor-angle variations may be rewritten in the form

$$\frac{d^2\delta_\Delta}{dt^2} + \frac{\omega_s S_p}{2H} \delta_\Delta = 0 \tag{16.48}$$

This is a linear, second-order differential equation, the solution to which depends on the algebraic sign of S_p. When S_p is positive, the solution $\delta_\Delta(t)$ corresponds to that of simple harmonic motion; such motion is represented by the oscillations of an undamped swinging pendulum.[3] When S_p is negative, the solution $\delta_\Delta(t)$ increases exponentially without limit. Therefore, in Fig. 16.6 the operating point $\delta_0 = 28.44°$ is a point of stable equilibrium, in the sense that the rotor-angle swing is bounded following a small perturbation. In the physical situation damping will restore the rotor angle to δ_0 following the temporary electrical perturbation. On the other hand, the point $\delta = 151.56°$ is a point of

[3]The equation of simple harmonic motion is $d^2x/dt^2 + \omega_n^2 x = 0$, which has the general solution $A \cos \omega_n t + B \sin \omega_n t$ with constants A and B determined by the initial conditions. The solution when plotted is an undamped sinusoid of angular frequency ω_n.



(a) Pendulum (b) Pendulum on disk

FIGURE 16.7
Pendulum and rotating disk to illustrate a rotor swinging with respect to an infinite bus.

unstable equilibrium since S_p is negative there. So, this point is not a valid operating point.

The changing position of the generator rotor swinging with respect to the infinite bus may be visualized by an analogy. Consider a pendulum swinging from a pivot on a stationary frame, as shown in Fig. 16.7(*a*). Points *a* and *c* are the maximum points of the oscillation of the pendulum about the equilibrium point *b*. Damping will eventually bring the pendulum to rest at *b*. Now imagine a disk rotating in a clockwise direction about the pivot of the pendulum, as shown in Fig. 16.7(*b*), and superimpose the motion of the pendulum on the motion of the disk. When the pendulum is moving from *a* to *c*, the combined angular velocity is slower than that of the disk. When the pendulum is moving from *c* to *a*, the combined angular velocity is faster than that of the disk. At points *a* and *c* the angular velocity of the pendulum alone is zero and the combined angular velocity equals that of the disk. If the angular velocity of the disk corresponds to the synchronous speed of the rotor, and if the motion of the pendulum alone represents the swinging of the rotor with respect to an infinite bus, the superimposed motion of the pendulum on that of the disk represents the actual angular motion of the rotor.

From the above discussion we conclude that the solution of Eq. (16.48) represents sinusoidal oscillations, provided the synchronizing power coefficient S_p is positive. The angular frequency of the undamped oscillations is given by

$$\omega_n = \sqrt{\frac{\omega_s S_p}{2H}} \text{ elec rad/s} \tag{16.49}$$

which corresponds to a frequency of oscillation given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{\omega_s S_p}{2H}} \text{ Hz} \tag{16.50}$$

Example 16.6. The machine of Example 16.3 is operating at $\delta = 28.44°$ when it is subjected to a slight temporary electrical system disturbance. Determine the frequency and period of oscillation of the machine rotor if the disturbance is removed before the prime mover responds. Take $H = 5$ MJ/MVA.

Solution. The applicable swing equation is Eq. (16.48) and the synchronizing power coefficient at the operating point is

$$S_p = 2.10 \cos 28.44° = 1.8466$$

The angular frequency of oscillation is therefore

$$\omega_n = \sqrt{\frac{\omega_s S_p}{2H}} = \sqrt{\frac{377 \times 1.8466}{2 \times 5}} = 8.343 \text{ elec rad/s}$$

The corresponding frequency of oscillation is

$$f_n = \frac{8.343}{2\pi} = 1.33 \text{ Hz}$$

and the period of oscillation is

$$T = \frac{1}{f_n} = 0.753 \text{ s}$$

The above example is an important one from the practical viewpoint since it indicates the order of magnitude of the frequencies which can be superimposed upon the nominal 60-Hz frequency in a large system having many interconnected machines. As load on the system changes randomly throughout the day, intermachine oscillations involving frequencies of the order of 1 Hz tend to arise, but these are quickly damped out by the various damping influences caused by the prime mover, the system loads, and the machine itself. It is worthwhile to note that even if the transmission system in our example contains resistance, nonetheless, the swinging of the rotor is harmonic and undamped. Problem 16.8 examines the effect of resistance on the synchronizing power coefficient and the frequency of oscillations. In a later section we again discuss the concept of synchronizing coefficients. In the next section we examine a method of determining stability under transient conditions caused by large disturbances.

16.6 EQUAL-AREA CRITERION OF STABILITY

In Sec. 16.4 we developed swing equations which are nonlinear in nature. Formal solutions of such equations cannot be explicitly found. Even in the case of a single machine swinging with respect to an infinite bus, it is very difficult to obtain literal-form solutions, and computer methods are therefore normally

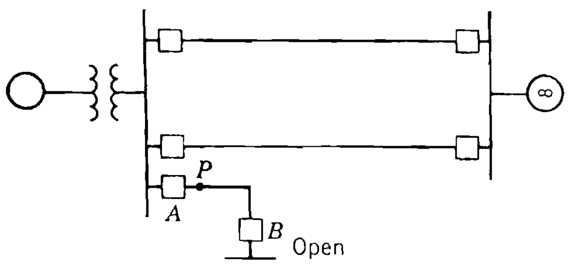


FIGURE 16.8
One-line diagram of the system of Fig. 16.4 with the addition of a short transmission line.

used. To examine the stability of a two-machine system without solving the swing equation, a direct approach is possible as now discussed.

The system shown in Fig. 16.8 is the same as that considered previously except for the addition of a short transmission line. Initially, circuit breaker A is considered to be closed while circuit breaker B at the opposite end of the short line is open. Therefore, the initial operating conditions of Example 16.3 may be considered unaltered. At point P close to the bus a three-phase fault occurs and is cleared by circuit breaker A after a short period of time. Thus, the effective transmission system is unaltered except while the fault is on. The short circuit caused by the fault is effectively at the bus, and so the electrical power output from the generator is zero until the fault is cleared. The physical conditions before, during, and after the fault can be understood by analyzing the power-angle curves of Fig. 16.9.

The generator is operating initially at synchronous speed with a rotor angle of δ_0, and the input mechanical power P_m equals the output electrical power P_e, as shown at point a in Fig. 16.9(a). When the fault occurs at $t = 0$, the electrical power output is suddenly zero while the input mechanical power is unaltered, as shown in Fig. 16.9(b). The difference in power must be accounted for by a rate of change of stored kinetic energy in the rotor masses. This can be accomplished only by an increase in speed which results from the constant accelerating power P_m. If we denote the time to clear the fault by t_c, then the acceleration is constant for time t less than t_c and is given by

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H}P_m \tag{16.51}$$

While the fault is on, the velocity increase above synchronous speed is found by integrating this equation to obtain

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H}P_m\,dt = \frac{\omega_s}{2H}P_m t \tag{16.52}$$

A further integration with respect to time yields for the rotor angle

$$\delta = \frac{\omega_s P_m}{4H}t^2 + \delta_0 \tag{16.53}$$

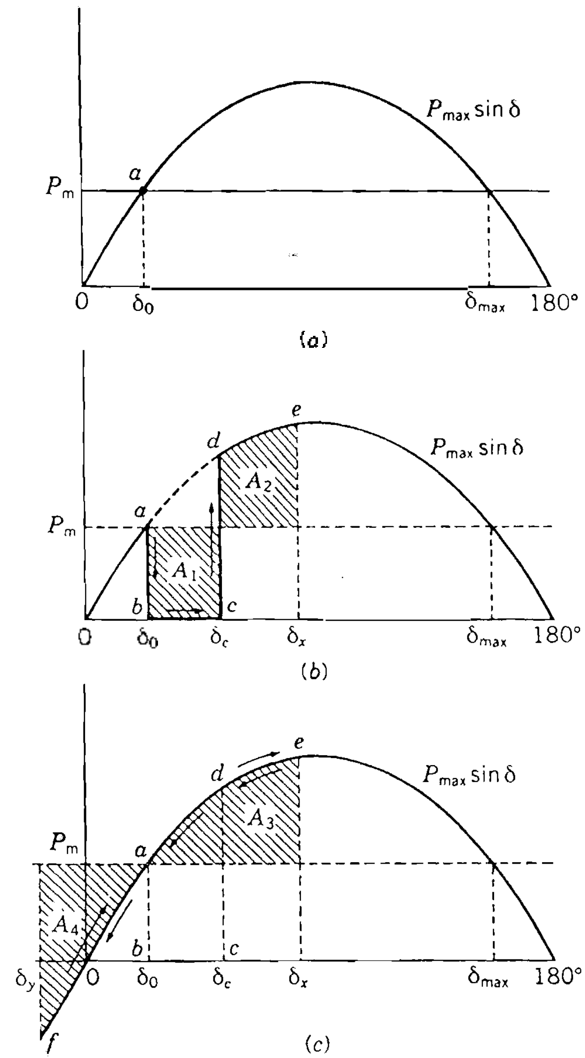


FIGURE 16.9
Power-angle curves for the generator shown in Fig. 16.8. Areas A_1 and A_2 are equal as are areas A_3 and A_4.

Equations (16.52) and (16.53) show that the velocity of the rotor relative to synchronous speed increases linearly with time while the rotor angle advances from δ_0 to the angle δ_c at clearing; that is, the angle δ goes from b to c in Fig. 16.9(b). At the instant of fault clearing the increase in rotor speed and the angle separation between the generator and the infinite bus are given, respectively, by

$$\left.\frac{d\delta}{dt}\right|_{t=t_c} = \frac{\omega_s P_m}{2H} t_c \tag{16.54}$$

and

$$\delta(t)|_{t=t_c} = \frac{\omega_s P_m}{4H} t_c^2 + \delta_0 \tag{16.55}$$

When the fault is cleared at the angle δ_c, the electrical power output abruptly increases to a value corresponding to point d on the power-angle curve. At d the electrical power output exceeds the mechanical power input, and thus the accelerating power is negative. As a consequence, the rotor slows down as P_e goes from d to e in Fig. 16.9(c). At e the rotor speed is again synchronous although the rotor angle has advanced to δ_x. The angle δ_x is determined by the fact that areas A_1 and A_2 must be equal, as explained later. The accelerating power at e is still negative (retarding), and so the rotor cannot remain at synchronous speed but must continue to slow down. The relative velocity is negative and the rotor angle moves back from δ_x at e along the power-angle curve of Fig. 16.9(c) to point a at which the rotor speed is less than synchronous. From a to f the mechanical power exceeds the electrical power and the rotor increases speed again until it reaches synchronism at f. Point f is located so that areas A_3 and A_4 are equal. In the absence of damping the rotor would continue to oscillate in the sequence f-a-e, e-a-f, and so on, with synchronous speed occurring at e and f.

We shall soon show that the shaded areas A_1 and A_2 in Fig. 16.9(b) must be equal, and similarly, areas A_3 and A_4 in Fig. 16.9(c) must be equal. In a system where one machine is swinging with respect to an infinite bus we may use this principle of equality of areas, called the *equal-area criterion*, to determine the stability of the system under transient conditions without solving the swing equation. Although the criterion is not applicable to multimachine systems, it helps in understanding how certain factors influence the transient stability of any system.

The derivation of the equal-area criterion is made for one machine and an infinite bus although the considerations in Sec. 16.3 show that the method can be readily adapted to general two-machine systems. The swing equation for the machine connected to the bus is

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{16.56}$$

Define the angular velocity of the rotor relative to synchronous speed by

$$\omega_r = \frac{d\delta}{dt} = \omega - \omega_s \tag{16.57}$$

Differentiating Eq. (16.57) with respect to t and substituting in Eq. (16.56), we obtain

$$\frac{2H}{\omega_s}\frac{d\omega_r}{dt} = P_m - P_e \tag{16.58}$$

When the rotor speed is synchronous, it is clear that ω equals ω_s and ω_r is zero. Multiplying both sides of Eq. (16.58) by $\omega_r = d\delta/dt$, we have

$$\frac{H}{\omega_s} 2\omega_r \frac{d\omega_r}{dt} = (P_m - P_e)\frac{d\delta}{dt} \tag{16.59}$$

The left-hand side of this equation can be rewritten to give

$$\frac{H}{\omega_s}\frac{d(\omega_r^2)}{dt} = (P_m - P_e)\frac{d\delta}{dt} \tag{16.60}$$

Multiplying by dt and integrating, we obtain

$$\frac{H}{\omega_s}(\omega_{r2}^2 - \omega_{r1}^2) = \int_{\delta_1}^{\delta_2}(P_m - P_e)\,d\delta \tag{16.61}$$

The subscripts for the ω_r terms correspond to those for the δ limits. That is, the rotor speed ω_{r1} corresponds to that at the angle δ_1 and ω_{r2} corresponds to δ_2. Since ω_r represents the *departure* of the rotor speed from synchronous speed, we readily see that if the rotor speed is synchronous at δ_1 *and* δ_2, then, correspondingly, $\omega_{r1} = \omega_{r2} = 0$. Under this condition, Eq. (16.61) becomes

$$\int_{\delta_1}^{\delta_2}(P_m - P_e)\,d\delta = 0 \tag{16.62}$$

This equation applies to any two points δ_1 and δ_2 on the power-angle diagram, provided they are points at which the rotor speed is synchronous. In Fig. 16.9(*b*) two such points are a and e corresponding to δ_0 and δ_x, respectively. If we perform the integration of Eq. (16.62) in two steps, we can write

$$\int_{\delta_0}^{\delta_c}(P_m - P_e)\,d\delta + \int_{\delta_c}^{\delta_x}(P_m - P_e)\,d\delta = 0 \tag{16.63}$$

or

$$\int_{\delta_0}^{\delta_c}(P_m - P_e)\,d\delta = \int_{\delta_c}^{\delta_x}(P_e - P_m)\,d\delta \tag{16.64}$$

The left integral applies to the fault period, whereas the right integral corresponds to the immediate postfault period up to the point of maximum swing δ_x. In Fig. 16.9(*b*) P_e is zero during the fault. The shaded area A_1 is given by the left-hand side of Eq. (16.64) and the shaded area A_2 is given by the right-hand side. So, the two areas A_1 and A_2 are equal.

Since the rotor speed is also synchronous at δ_x and at δ_y in Fig. 16.9(*c*), the same reasoning as above shows that A_3 equals A_4. The areas A_1 and A_4 are directly proportional to the increase in kinetic energy of the rotor while it is

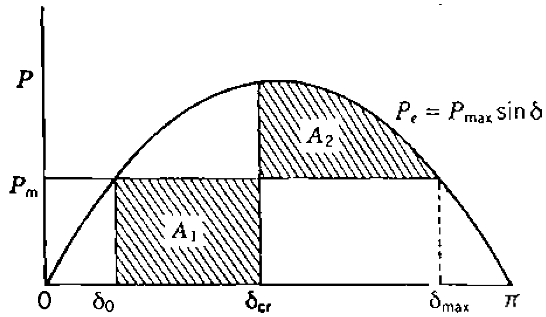


FIGURE 16.10
Power-angle curve showing the critical clearing angle δ_{cr}. Areas A_1 and A_2 are equal.

accelerating, whereas areas A_2 and A_3 are proportional to the decrease in kinetic energy of the rotor while it is decelerating. This can be seen by inspection of both sides of Eq. (16.61). Therefore, the equal-area criterion states that the kinetic energy added to the rotor following a fault must be removed after the fault in order to restore the rotor to synchronous speed.

The shaded area A_1 is dependent on the time taken to clear the fault. If there is delay in clearing, the angle δ_c is increased; likewise, the area A_1 increases and the equal-area criterion requires that area A_2 also increase to restore the rotor to synchronous speed at a larger angle of maximum swing δ_x. If the delay in clearing is prolonged so that the rotor angle δ swings beyond the angle δ_{max} in Fig. 16.9, then the rotor speed at that point on the power-angle curve is above synchronous speed when positive accelerating power is again encountered. Under the influence of this positive accelerating power, the angle δ will increase without limit and instability results. Therefore, there is a critical angle for clearing the fault in order to satisfy the requirements of the equal-area criterion for stability. This angle, called the *critical clearing angle* δ_{cr}, is shown in Fig. 16.10. The corresponding critical time for removing the fault is called the *critical clearing time* t_{cr}. Thus, the critical clearing time is the maximum elapsed time from the initiation of the fault until its isolation such that the power system is transiently stable.

In the particular case of Fig. 16.10 *both* the critical clearing angle *and* the critical clearing time can be calculated as follows. The rectangular area A_1 is

$$A_1 = \int_{\delta_0}^{\delta_{cr}} P_m \, d\delta = P_m(\delta_{cr} - \delta_0) \tag{16.65}$$

while the area A_2 is

$$\begin{aligned} A_2 &= \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m) \, d\delta \\ &= P_{max}(\cos \delta_{cr} - \cos \delta_{max}) - P_m(\delta_{max} - \delta_{cr}) \end{aligned} \tag{16.66}$$

Equating the expressions for A_1 and A_2 and transposing terms, yield

$$\cos \delta_{cr} = (P_m/P_{max})(\delta_{max} - \delta_0) + \cos \delta_{max} \tag{16.67}$$

We see from the sinusoidal power-angle curve that

$$\delta_{max} = \pi - \delta_0 \text{ elec rad} \tag{16.68}$$

and

$$P_m = P_{max} \sin \delta_0 \tag{16.69}$$

Substituting for δ_{max} and P_m in Eq. (16.67), simplifying the result, and solving for the critical clearing angle δ_{cr}, we obtain

$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0)\sin \delta_0 - \cos \delta_0] \tag{16.70}$$

Substituting this value of δ_{cr} in the left-hand side of Eq. (16.55) yields

$$\delta_{cr} = \frac{\omega_s P_m}{4H} t_{cr}^2 + \delta_0 \tag{16.71}$$

from which we find the critical clearing time

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}} \tag{16.72}$$

Example 16.7. Calculate the critical clearing angle and the critical clearing time for the system of Fig. 16.8 when the system is subjected to a three-phase fault at point P on the short transmission line. The initial conditions are the same as those in Example 16.3, and $H = 5$ MJ/MVA.

Solution. In Example 16.3 the power-angle equation and initial rotor angle are

$$P_e = P_{max} \sin \delta = 2.10 \sin \delta$$

$$\delta_0 = 28.44° = 0.496 \text{ elec rad}$$

Mechanical input power P_m is 1.0 per unit, and Eq. (16.70) then gives

$$\delta_{cr} = \cos^{-1}[(\pi - 2 \times 0.496)\sin 28.44° - \cos 28.44°]$$

$$= 81.697° = 1.426 \text{ elec rad}$$

Using this value with the other known quantities in Eq. (16.72) yields

$$t_{cr} = \sqrt{\frac{4 \times 5(1.426 - 0.496)}{377 \times 1}} = 0.222 \text{ s}$$

which is equivalent to a critical clearing time of 13.3 cycles on a frequency base of 60 Hz.

This example serves to establish the concept of critical clearing time which is essential to the design of proper relaying schemes for fault clearing. In more general cases the critical clearing time cannot be explicitly found without solving the swing equations by computer simulation.

16.7 FURTHER APPLICATIONS OF THE EQUAL-AREA CRITERION

The equal-area criterion is a very useful means for analyzing stability of a system of two machines, or of a single machine supplied from an infinite bus. However, the computer is the only practical way to determine the stability of a large system. Because the equal-area criterion is so helpful in understanding transient stability, we continue to examine it briefly before discussing the determination of swing curves by the computer approach.

When a generator is supplying power to an infinite bus over parallel transmission lines, opening one of the lines may cause the generator to lose synchronism even though the load could be supplied over the remaining line under steady-state conditions. If a three-phase short circuit occurs on the bus to which two parallel lines are connected, no power can be transmitted over either line. This is essentially the case in Example 16.7. However, if the fault is at the end of one of the lines, opening breakers at both ends of the line will isolate the fault from the system and allow power to flow through the other parallel line. When a three-phase fault occurs at some point on a double-circuit line other than on the paralleling buses or at the extreme ends of the line, there is some impedance between the paralleling buses and the fault. Therefore, some power is transmitted while the fault is still on the system. The power-angle equation in Example 16.4 demonstrates this fact.

When power is transmitted during a fault, the equal-area criterion is applied, as shown in Fig. 16.11, which is similar to the power-angle diagram of Fig. 16.6. Before the fault $P_{max} \sin \delta$ is the power which can be transmitted; during the fault $r_1 P_{max} \sin \delta$ is the power which can be transmitted; and $r_2 P_{max} \sin \delta$ is the power which can be transmitted after the fault is cleared by switching at the instant when $\delta = \delta_{cr}$. Examination of Fig. 16.11 shows that δ_{cr} is the critical clearing angle in this case. By evaluating the areas A_1 and A_2

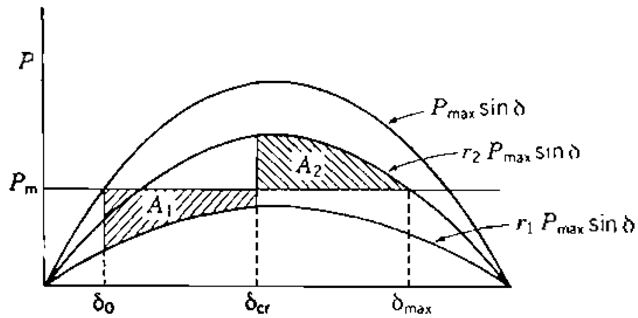


FIGURE 16.11
Equal-area criterion applied to fault clearing when power is transmitted during the fault. Areas A_1 and A_2 are equal.

using the procedural steps of the previous section, we find that

$$\cos \delta_{cr} = \frac{(P_m/P_{max})(\delta_{max} - \delta_0) + r_2 \cos \delta_{max} - r_1 \cos \delta_0}{r_2 - r_1} \qquad (16.73)$$

A literal-form solution for the critical clearing time t_{cr} is not possible in this case. For the particular system and fault location shown in Fig. 16.8 the applicable values are $r_1 = 0$, $r_2 = 1$, and Eq. (16.73) then reduces to Eq. (16.67).

Short-circuit faults which do not involve all three phases allow some power to be transmitted over the unaffected phases. Such faults are represented by connecting an impedance (rather than a short circuit) between the fault point and the reference node in the positive-sequence impedance diagram. The larger the impedance shunted across the positive-sequence network to represent the fault, the larger the power transmitted during the fault. The amount of power transmitted during the fault affects the value of A_1 for any given clearing angle. Thus, smaller values of r_1 result in greater disturbances to the system, as smaller amounts of power are transmitted during the fault. Consequently, the area A_1 of acceleration is larger. In order of increasing severity (that is, decreasing $r_1 P_{max}$) the various faults are:

1. A single line-to-ground fault
2. A line-to-line fault
3. A double line-to-ground fault
4. A three-phase fault

The single line-to-ground fault occurs most frequently, and the three-phase fault is the least frequent. For complete reliability a system should be designed for transient stability for three-phase faults at the worst locations, and this is virtually the universal practice.

Example 16.8. Determine the critical clearing angle for the three-phase fault described in Examples 16.4 and 16.5 when the initial system configuration and prefault operating conditions are as described in Example 16.3.

Solution. The power-angle equations obtained in the previous examples are

$$\text{Before the fault:} \qquad P_{max} \sin \delta = 2.100 \sin \delta$$

$$\text{During the fault:} \quad r_1 P_{max} \sin \delta = 0.808 \sin \delta$$

$$\text{After the fault:} \quad r_2 P_{max} \sin \delta = 1.500 \sin \delta$$

Hence,

$$r_1 = \frac{0.808}{2.100} = 0.385 \qquad r_2 = \frac{1.500}{2.100} = 0.714$$

From Example 16.3 we have

$$\delta_0 = 28.44° = 0.496 \text{ rad}$$

and from Fig. 16.11 we calculate

$$\delta_{max} = 180° - \sin^{-1}\left[\frac{1.000}{1.500}\right] = 138.190° = 2.412 \text{ rad}$$

Therefore, inserting numerical values in Eq. (16.73), we obtain

$$\cos \delta_{cr} = \frac{\left(\dfrac{1.0}{2.10}\right)(2.412 - 0.496) + 0.714\cos(138.19°) - 0.385\cos(28.44°)}{0.714 - 0.385}$$

$$= 0.127$$

Hence,

$$\delta_{cr} = 82.726°$$

To determine the critical clearing time for this example, we must obtain the swing curve of δ versus t. In Sec. 16.9 we discuss one method of computing such swing curves.

16.8 MULTIMACHINE STABILITY STUDIES: CLASSICAL REPRESENTATION

The equal-area criterion cannot be used directly in systems where three or more machines are represented. Although the physical phenomena observed in the two-machine problems are basically the same as in the multimachine case, nonetheless, the complexity of the numerical computations increases with the number of machines considered in a transient stability study. When a multimachine system operates under electromechanical transient conditions, intermachine oscillations occur through the medium of the transmission system connecting the machines. If any one machine could be considered to act alone as the single oscillating source, it would send into the interconnected system an electromechanical oscillation determined by its inertia and synchronizing power. A typical frequency of such an oscillation is of the order of 1–2 Hz, and this is superimposed upon the nominal 60-Hz frequency of the system. When many machine rotors are simultaneously undergoing transient oscillation, the swing curves reflect the combined presence of many such oscillations. Therefore, the transmission system frequency is not unduly perturbed from nominal frequency, and the assumption is made that the 60-Hz network parameters are still applicable. To ease the complexity of system modeling, and thereby the computational burden, the following additional assumptions are commonly made in transient stability studies:

1. The mechanical power input to each machine remains constant during the entire period of the swing curve computation.
2. Damping power is negligible.
3. Each machine may be represented by a constant transient reactance in series with a constant transient internal voltage.
4. The mechanical rotor angle of each machine coincides with δ, the electrical phase angle of the transient internal voltage.
5. All loads may be considered as shunt impedances to ground with values determined by conditions prevailing immediately prior to the transient conditions.

The system stability model based on these assumptions is called the *classical stability model*, and studies which use this model are called *classical stability studies*. These assumptions, which we shall adopt, are in addition to the fundamental assumptions set forth in Sec. 16.1 for *all* stability studies. Of course, detailed computer programs with more sophisticated machine and load models are available to modify one or more of assumptions 1 to 5. Throughout this chapter, however, the classical model is used to study system disturbances originating from three-phase faults.

The system conditions before the fault occurs, and the network configuration both during and after its occurrence, must be known in any transient stability study, as we have seen. Consequently, in the multimachine case two preliminary steps are required:

1. The steady-state prefault conditions for the system are calculated using a production-type power-flow program.
2. The prefault network representation is determined and then modified to account for the fault and for the postfault conditions.

From the first preliminary step we know the values of power, reactive power, and voltage at each generator terminal and load bus, with all angles measured with respect to the slack bus. The transient internal voltage of each generator is then calculated using the equation

$$E = V_t + jX'_d I \tag{16.74}$$

where V_t is the corresponding terminal voltage and I is the output current. Each load is converted into a constant admittance to ground at its bus using the equation

$$Y_L = \frac{P_L - jQ_L}{|V_L|^2} \tag{16.75}$$

where $P_L + jQ_L$ is the load and $|V_L|$ is the magnitude of the corresponding bus voltage. The bus admittance matrix which is used for the prefault power-flow calculation is now augmented to include the transient reactance of each generator and the shunt admittance of each load, as suggested in Fig. 16.12. Note that the injected current is zero at all buses except the internal buses of the generators.

In the second preliminary step the bus admittance matrix is modified to correspond to the faulted and postfault conditions. Since only the generator internal buses have injections, all other buses can be eliminated by Kron

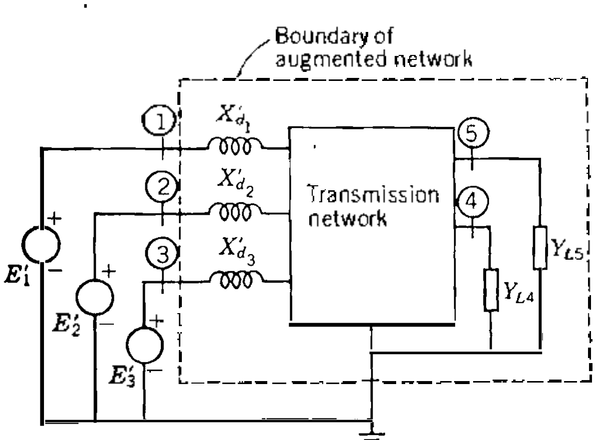


FIGURE 16.12
Augmented network of a power system.

reduction. The dimensions of the modified matrices then correspond to the number of generators. During and after the fault the power flow into the network from each generator is calculated by the corresponding power-angle equation. For example, in Fig. 16.12 the power out of generator 1 is given by

$$P_{e1} = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}|\cos(\delta_{12} - \theta_{12}) + |E_1'||E_3'||Y_{13}|\cos(\delta_{13} - \theta_{13}) \tag{16.76}$$

where δ_{12} equals $\delta_1 - \delta_2$. Similar equations are written for P_{e2} and P_{e3} using the Y_{ij} elements of the 3×3 bus admittance matrices appropriate to the fault or postfault condition. The P_{ei} expressions form part of the swing equations

$$\frac{2H_i}{\omega_s}\frac{d^2\delta_i}{dt^2} = P_{mi} - P_{ei} \qquad i = 1, 2, 3 \tag{16.77}$$

which represent the motion of each rotor during the fault and postfault periods. The solutions depend on the location and duration of the fault, and the $\mathbf{Y}_{\text{bus}}$ resulting when the faulted line is removed. The basic procedures used in computer programs for classical stability studies are revealed in the following examples.

Example 16.9. A 60-Hz, 230-kV transmission system shown in Fig. 16.13 has two generators of finite inertia and an infinite bus. The transformer and line data are given in Table 16.2. A three-phase fault occurs on line ④–⑤ near bus ④. Using the prefault power-flow solution given in Table 16.3, determine the swing equation for each machine during the fault period. The generators have reactances

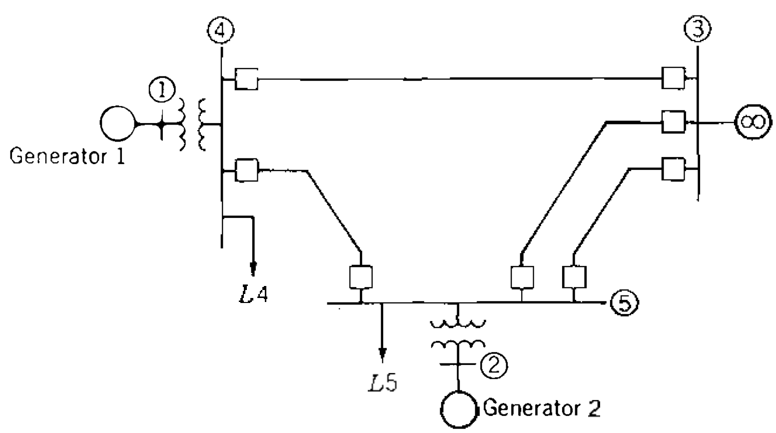


FIGURE 16.13
One-line diagram for Example 16.9.

TABLE 16.2
Line and transformer data for Example 16.9†

Bus to bus	Series Z R	Series Z X	Shunt Y B
Transformer ①-④	—	0.022	
Transformer ②-⑤	—	0.040	
Line ③-④	0.007	0.040	0.082
Line ③-⑤(1)	0.008	0.047	0.098
Line ③-⑤(2)	0.008	0.047	0.098
Line ④-⑤	0.018	0.110	0.226

†All values in per unit on 230-kV, 100-MVA base.

TABLE 16.3
Bus data and prefault load-flow values†

Bus	Voltage	Generation P	Generation Q	Load P	Load Q
①	$1.030\angle 8.88^\circ$	3.500	0.712		
②	$1.020\angle 6.38^\circ$	1.850	0.298		
③	$1.000\angle 0^\circ$	—	—		
④	$1.018\angle 4.68^\circ$	—	—	1.00	0.44
⑤	$1.011\angle 2.27^\circ$	—	—	0.50	0.16

†Values are in per unit on 230-kV, 100-MVA base.

and H values expressed on a 100-MVA base as follows:

Generator 1: 400 MVA, 20 kV, $X'_d = 0.067$ per unit, $H = 11.2$ MJ/MVA

Generator 2: 250 MVA, 18 kV, $X'_d = 0.10$ per unit, $H = 8.0$ MJ/MVA

Solution. In order to formulate the swing equations, we must first determine the transient internal voltages. Based on the data of Table 16.3, the current into the network at bus ① is

$$I_1 = \frac{(P_1 + jQ_1)^*}{V_1^*} = \frac{3.50 - j0.712}{1.030\angle -8.88^\circ} = 3.468\angle -2.619^\circ$$

Similarly, the current into the network at bus ② is

$$I_2 = \frac{(P_2 + jQ_2)^*}{V_2^*} = \frac{1.850 - j0.298}{1.020\angle -6.38^\circ} = 1.837\angle -2.771^\circ$$

From Eq. (16.74) we then calculate

$$E_1' = 1.030\underline{/8.88^\circ} + j0.067 \times 3.468\underline{/-2.619^\circ} = 1.100\underline{/20.82^\circ}$$

$$E_2' = 1.020\underline{/6.38^\circ} + j0.10 \times 1.837\underline{/-2.771^\circ} = 1.065\underline{/16.19^\circ}$$

At the infinite bus we have

$$E_3' = E_3 = 1.000\underline{/0.0^\circ}$$

and so $\qquad \delta_{13} = \delta_1 \qquad \delta_{23} = \delta_2$

The P-Q loads at buses ④ and ⑤ are converted into equivalent shunt admittances using Eq. (16.75), which yields

$$Y_{L4} = \frac{1.00 - j0.44}{(1.018)^2} = 0.9649 - j0.4246 \text{ per unit}$$

$$Y_{L5} = \frac{0.50 - j0.16}{(1.011)^2} = 0.4892 - j0.1565 \text{ per unit}$$

The prefault bus admittance matrix is now modified to include the load admittances and the transient reactances of the machines. Buses ① and ② designate the fictitious internal nodes behind the transient reactances of the machines. So, in the prefault bus admittance matrix, for example,

$$Y_{11} = \frac{1}{j0.067 + j0.022} = -j11.236 \text{ per unit}$$

$$Y_{34} = -\frac{1}{0.007 + j0.040} = -4.2450 + j24.2571 \text{ per unit}$$

The sum of the admittances connected to buses ③, ④, and ⑤ must include the shunt capacitances of the transmission lines. So, at bus ④ we have

$$Y_{44} = -j11.236 + \frac{j0.082}{2} + \frac{j0.226}{2} + 4.2450 - j24.2571$$

$$+ \frac{1}{0.018 + j0.110} + 0.9649 - j0.4246$$

$$= 6.6587 - j44.6175 \text{ per unit}$$

The new prefault bus admittance matrix is displayed as Table 16.4 on page 733.

Bus ④ must be short-circuited to the reference to represent the fault. Row 4 and column 4 of Table 16.4 thereby disappear because node ④ is now merged with the reference node. Next, the row and column representing bus ⑤ are eliminated by Kron reduction, and we obtain the bus admittance matrix shown in the upper half of Table 16.5. The faulted-system $\mathbf{Y}_{bus}$ shows that bus ① decouples from the other buses during the fault and that bus ② is connected directly to bus ③. This reflects the physical fact that the short circuit at bus ④ reduces to zero the power injected into the system from generator 1 and causes generator 2 to deliver its power radially to bus ③. Under fault conditions, the power-angle equations based on values from Table 16.5 are

$$P_{e1} = 0$$

$$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_3||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2(0.1362) + (1.065)(1.0)(5.1665)\cos(\delta_2 - 90.755°)$$

$$= 0.1545 + 5.5023\sin(\delta_2 - 0.755°) \text{ per unit}$$

Therefore, while the fault is on the system, the desired swing equations (values of P_{m1} and P_{m2} from Table 16.3) are

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{H_1}(P_{m1} - P_{e1}) = \frac{180f}{H_1}P_{a1}$$

$$= \frac{180f}{11.2}(3.5) \text{ elec deg/s}^2$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H_2}(P_{m2} - P_{e2}) = \frac{180f}{H_2}P_{a2}$$

$$= \frac{180f}{8.0}\left\{\overbrace{1.85}^{P_m} - \left[\overbrace{0.1545}^{P_c} + \overbrace{5.5023}^{P_{max}}\sin\left(\delta_2 - \overbrace{0.755°}^{\gamma}\right)\right]\right\}$$

$$= \frac{180f}{8.0}\left[\underbrace{1.6955}_{P_m - P_c} - \underbrace{5.5023}_{P_{max}}\sin\left(\delta_2 - \underbrace{0.755°}_{\gamma}\right)\right] \text{ elec deg/s}^2$$

Example 16.10. The three-phase fault in Example 16.9 is cleared by simultaneously opening the circuit breakers at the ends of the faulted line. Determine the swing equations for the postfault period.

Solution. Since the fault is cleared by removing line ④–⑤, the prefault $\mathbf{Y}_{bus}$ of Table 16.4 must be modified again. This is accomplished by substituting zero for Y_{45} and Y_{54} and by subtracting the series admittance of line ④–⑤ and the capacitive susceptance of one-half the line from elements Y_{44} and Y_{55} of Table 16.4. The reduced bus admittance matrix applicable to the postfault network is shown in the lower half of Table 16.5. The zero elements in the first and second rows reflect the fact that generators 1 and 2 are not interconnected when line

TABLE 16.4
Elements of prefault bus admittance matrix for Example 16.9†

Bus	①	②	③	④	⑤
①	$-j11.2360$	0.0	0.0	$j11.2360$	0.0
②	0.0	$-j7.1429$	0.0	0.0	$j7.1429$
③	0.0	0.0	11.2841 $-j65.4731$	-4.2450 $+j24.2571$	-7.0392 $+j41.3550$
④	$j11.2360$	0.0	-4.2450 $+j24.2571$	6.6588 $-j44.6175$	-1.4488 $+j8.8538$
⑤	0.0	$j7.1429$	-7.0392 $+j41.3550$	-1.4488 $+j8.8538$	8.9772 $-j57.2972$

†Admittances in per unit.

TABLE 16.5
Elements of faulted and postfault bus admittance matrices for Example 16.9†

Bus	①	②	③
		Faulted network	
①	$0.0000 - j11.2360$ $(11.2360\angle -90°)$	$0.0 + j0.0$	$0.0 + j0.0$
②	$0.0 + j0.0$	$0.1362 - j6.2737$ $(6.2752\angle -88.7563°)$	$-0.0681 + j5.1661$ $(5.1665\angle 90.7552°)$
③	$0.0 + j0.0$	$-0.681 + j5.1661$ $(5.1665\angle 90.7552°)$	$5.7986 - j35.6299$ $(36.0987\angle -80.7564°)$
		Postfault network	
①	$0.5005 - j7.7897$ $(7.8058\angle -86.3237°)$	$0.0 + j0.0$	$-0.2216 + j7.6291$ $(7.6323\angle 91.6638°)$
②	$0.0 + j0.0$	$0.1591 - j6.1168$ $(6.1189\angle -88.5101°)$	$-0.0901 + j6.0975$ $(6.0982\angle 90.8466°)$
③	$-0.2216 + j7.6291$ $(7.6323\angle 91.6638°)$	$-0.0901 + j6.0975$ $(6.0982\angle 90.8466°)$	$1.3927 - j13.8728$ $(13.9426\angle -84.2672°)$

†Admittances in per unit.

④–⑤ is removed. Accordingly, each generator is connected radially to the infinite bus, and we can write power-angle equations for the postfault conditions as follows:

$$P_{e1} = |E_1'|^2 G_{11} + |E_1'||E_3||Y_{13}|\cos(\delta_{13} - \theta_{13})$$

$$= (1.100)^2(0.5005) + (1.100)(1.0)(7.6323)\cos(\delta_1 - 91.664°)$$

$$= 0.6056 + 8.3955\sin(\delta_1 - 1.664°) \text{ per unit}$$

and $$P_{e2} = |E_2'|^2 G_{22} + |E_2'||E_3||Y_{23}|\cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2(0.1591) + (1.065)(1.0)(6.0982)\cos(\delta_2 - 90.847°)$$

$$= 0.1804 + 6.4934\sin(\delta_2 - 0.847°) \text{ per unit}$$

For the postfault period the applicable swing equations are given by

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{11.2}\{3.5 - [0.6056 + 8.3955\sin(\delta_1 - 1.664°)]\}$$

$$= \frac{180f}{11.2}[2.8944 - 8.3955\sin(\delta_1 - 1.664°)] \text{ elec deg/s}^2$$

and $$\frac{d^2\delta_2}{dt^2} = \frac{180f}{8.0}\{1.85 - [0.1804 + 6.4934\sin(\delta_2 - 0.847°)]\}$$

$$= \frac{180f}{8.0}[1.6696 - 6.4934\sin(\delta_2 - 0.847°)] \text{ elec deg/s}^2$$

The power-angle equations obtained in Examples 16.9 and 16.10 are of the form of Eq. (16.35), and the corresponding swing equations assume the form

$$\frac{d^2\delta}{dt^2} = \frac{180f}{H}[P_m - P_c - P_{\max}\sin(\delta - \gamma)] \tag{16.78}$$

The bracketed right-hand term represents the accelerating power of the rotor. Accordingly, we may write Eq. (16.78) as

$$\frac{d^2\delta}{dt^2} = \frac{180f}{H}P_a \text{ elec deg/s}^2 \tag{16.79}$$

where $$P_a = P_m - P_c - P_{\max}\sin(\delta - \gamma) \tag{16.80}$$

In the next section we discuss how to solve equations of the form of Eq. (16.79) in order to obtain δ as a function of time for specified clearing times.

16.9 STEP-BY-STEP SOLUTION OF THE SWING CURVE

For large systems we depend on the computer, which determines δ versus t for all machines in which we are interested; and δ may be plotted versus t for a machine to obtain the swing curve of that machine. The angle δ is calculated as a function of time over a period long enough to determine whether δ will increase without limit or reach a maximum and start to decrease. Although the latter result usually indicates stability, on an actual system where a number of variables are taken into account it may be necessary to plot δ versus t over a long enough interval to be sure δ will not increase again without returning to a low value.

By determining swing curves for various clearing times, we can find the length of time permitted before clearing a fault. Standard interrupting times for circuit breakers and their associated relays are commonly 8, 5, 3, or 2 cycles after a fault occurs, and thus breaker speeds may be specified. Calculations should be made for a fault in the position which will allow the least transfer of power from the machine and for the most severe type of fault for which protection against loss of stability is justified.

A number of different methods is available for the numerical evaluation of second-order differential equations in step-by-step computations for small increments of the independent variable. The more elaborate methods are practical only when the computations are performed on a computer. The step-by-step method used for hand calculations is necessarily simpler than some of the methods recommended for computers. In the methods for hand calculation the change in the angular position of the rotor during a short interval of time is computed by making the following assumptions:

1. The accelerating power P_a computed at the beginning of an interval is constant from the middle of the preceding interval to the middle of the interval considered.
2. Throughout any interval the angular velocity is constant at the value computed for the middle of the interval.

Of course, neither of the above assumptions is accurate since δ is changing continuously and both P_a and ω are functions of δ. As the time interval is decreased, the computed swing curve becomes more accurate. Figure 16.14(a) will help in visualizing the assumptions. The accelerating power is computed for the points enclosed in circles at the ends of the $n-2$, $n-1$, and n intervals, which are the beginnings of the $n-1$, n, and $n+1$ intervals. The step curve of P_a in Fig. 16.14(a) results from the assumptions that P_a is constant between the midpoints of the intervals. Similarly, ω_r, the *excess* of the angular velocity ω over the synchronous angular velocity ω_s, is shown in Fig. 16.14(b) as a step curve that is constant throughout the interval at the value computed for the midpoint. Between the ordinates $n-\frac{3}{2}$ and $n-\frac{1}{2}$ there is a change of speed caused by the constant accelerating power. The change in speed is the product of the acceleration and the time interval, and so

$$\omega_{r,n-1/2}-\omega_{r,n-3/2}=\frac{d^2\delta}{dt^2}\Delta t=\frac{180f}{H}P_{a,n-1}\Delta t \qquad (16.81)$$

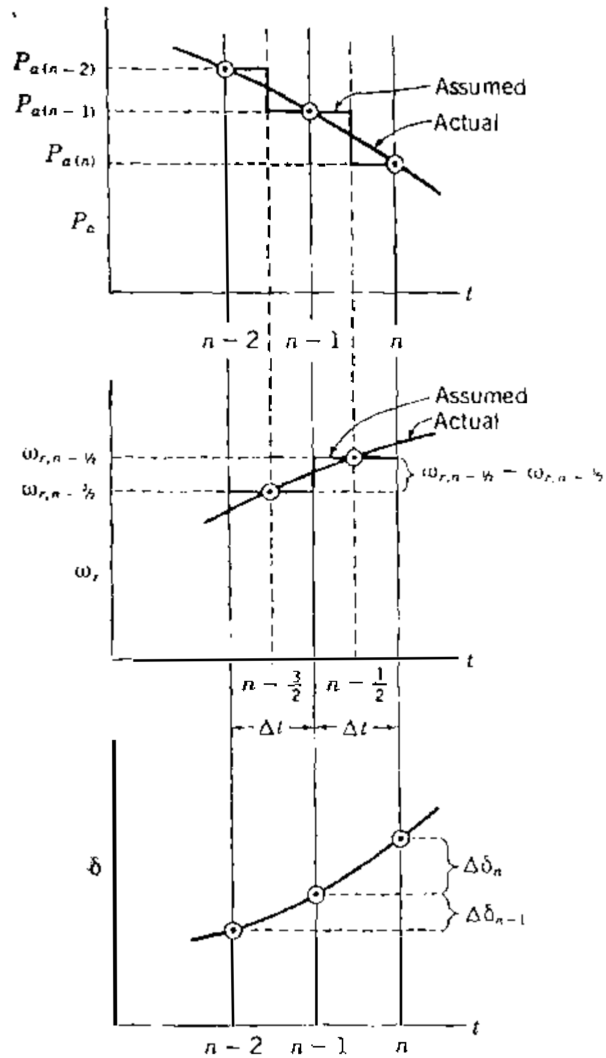


FIGURE 16.14
Actual and assumed values of P_a, ω_r, and δ as functions of time.

The change in δ over an interval is the product of ω_r for the interval and the time of the interval. Thus, the change in δ during the $n-1$ interval is

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t \times \omega_{r,n-3/2} \tag{16.82}$$

and during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t \times \omega_{r,n-1/2} \tag{16.83}$$

Subtracting Eq. (16.82) from Eq. (16.83) and substituting Eq. (16.81) in the resulting equation eliminate all values of ω_r, and we find that

$$\Delta\delta_n = \delta_{n-1} + kP_{a,n-1} \tag{16.84}$$

where
$$k = \frac{180f}{H}(\Delta t)^2 \tag{16.85}$$

Equation (16.84) is important for the step-by-step solution of the swing equation, for it shows how to calculate the change in δ during an interval based on the accelerating power for that interval and the change in δ for the preceding interval. The accelerating power is calculated at the beginning of each new interval and the solution progresses until enough points are obtained for plotting the swing curve. Greater accuracy is obtained when Δt is small. A value of $\Delta t = 0.05$ s is usually satisfactory.

The occurrence of a fault causes a discontinuity in the accelerating power P_a, which has a zero value before the fault and a nonzero value immediately following the fault. The discontinuity occurs at the beginning of the interval when $t = 0$. Reference to Fig. 16.14 shows that our method of calculation assumes that the accelerating power computed at the beginning of an interval is constant from the middle of the preceding interval to the middle of the interval being considered. When the fault occurs, we have two values of P_a at the beginning of an interval, and we must take the average of these two values as the constant accelerating power. The procedure is illustrated in the following example.

Example 16.11. Prepare a table showing the steps taken to plot the swing curve of machine 2 for the fault on the 60-Hz system of Examples 16.9 and 16.10. The fault is cleared at 0.225 s by simultaneously opening the circuit breakers at the ends of the faulted line.

Solution. Without loss of generality, we consider the detailed computations for machine 2. Computations to plot the swing curve for machine 1 are left to the student. Accordingly, we drop the subscript 2 as the indication of the machine number from all symbols in what follows. All calculations are made in per unit on a 100-MVA base. For the time interval $\Delta t = 0.05$ s the parameter k applicable to machine 2 is

$$k = \frac{180f}{H}(\Delta t)^2 = \frac{180 \times 60}{8.0} \times (0.05)^2 = 3.375 \text{ elec deg}$$

When the fault occurs at $t = 0$, the rotor angle of machine 2 cannot change instantly. Hence, from Example 16.9

$$\delta_0 = 16.19°$$

and during the fault

$$P_e = 0.1545 + 5.5023 \sin(\delta - 0.755°) \text{ per unit}$$

Therefore, as already seen in Example 16.9,

$$P_a = P_m - P_e = 1.6955 - 5.5023\sin(\delta - 0.755°) \text{ per unit}$$

At the beginning of the first interval there is a discontinuity in the accelerating power of each machine. Immediately before the fault occurs $P_a = 0$. Immediately after the fault occurs

$$P_a = 1.6955 - 5.5023\sin(16.19° - 0.755°) = 0.231 \text{ per unit}$$

The average value of P_a at $t = 0$ is $\frac{1}{2} \times 0.2310 = 0.1155$ per unit. We then find that

$$kP_a = 3.375 \times 0.1155 = 0.3898°$$

Identifying the time intervals by numerical subscripts, we find that the change in the rotor angle of machine 2 as time advances over the *first* interval from 0 to Δt is given by

$$\Delta\delta_1 = 0 + 0.3898 = 0.3898°$$

At the end of the first time interval we then have

$$\delta_1 = \delta_0 + \Delta\delta_1 = 16.19° + 0.3898° = 16.5798°$$

and
$$\delta_1 - \gamma = 16.5798° - 0.755° = 15.8248°$$

At $t = \Delta t = 0.05$ s we find that

$$kP_{a,1} = 3.375[(P_m - P_c) - P_{max}\sin(\delta_1 - \gamma)]$$

$$= 3.375[1.6955 - 5.5023\sin(15.8248°)] = 0.6583°$$

and it follows that the increase in the rotor angle over the *second* time interval is

$$\Delta\delta_2 = \Delta\delta_1 + kP_{a,1} = 0.3898° + 0.6583° = 1.0481°$$

At the end of the second time interval

$$\delta_2 = \delta_1 + \Delta\delta_2 = 16.5798° + 1.0481° = 17.6279°$$

The subsequent steps in the computations are shown in Table 16.6. Note that the postfault equation found in Example 16.10 is needed.

In Table 16.6 the terms $P_{max}\sin(\delta - \gamma)$, P_a, and δ_n have values computed at the time t shown in the first column but $\Delta\delta_n$ is the *change* in the rotor angle *during* the interval that begins at the time indicated. For example, in the row for $t = 0.10$ s the angle 17.6279° is the first value calculated and is found by adding the change in angle during the preceding time interval (0.05 to 0.10 s) to the angle at $t = 0.05$ s. Next, $P_{max}\sin(\delta - \gamma)$ is calculated for $\delta = 17.6279°$. Then, $P_a = (P_m - P_c) - P_{max}\sin(\delta - \gamma)$ and kP_a are calculated. The value of kP_a is 0.3323°, which is added to the angular change of 1.0481° during the preceding interval to find the

TABLE 16.6
Computation of swing curve for machine 2 of Example 16.11 for clearing at 0.225 s

$k = (180f/H)(\Delta t)^2 = 3.375$ elec deg. Before clearing $P_m - P_c = 1.6955$ p.u., $P_{\max} = 5.5023$ p.u., and $\gamma = 0.755°$. After clearing these values become 1.6696, 6.4934, and 0.847, respectively.

t, s	$(\delta_n - \gamma)$, elec deg	$P_{\max}\sin(\delta_n - \gamma)$, per unit	P_a, per unit	$kP_{a,n-1}$, elec deg	$\Delta\delta_n$, elec deg	δ_n, elec deg
0 −	—	—	0.00	—		16.19
0 +	15.435	1.4644	0.2310	—		16.19
0 av	—	—	0.1155	0.3898	—	16.19
					0.3898	
0.05	15.8248	1.5005	0.1950	0.6583	—	16.5798
					1.0481	
0.10	16.8729	1.5970	0.0985	0.3323	—	17.6279
					1.3804	
0.15	18.2533	1.7234	−0.0279	−0.0942	—	19.0083
					1.2862	
0.20	19.5395	1.8403	−0.1448	−0.4886	—	20.2945
					0.7976	
0.25	20.2451	2.2470	−0.5774	−1.9487	—	21.0921
					−1.1511	
0.30	19.0940	2.1241	−0.4545	−1.534	—	19.9410
					−2.6852	
0.35	16.4088	1.8343	−0.1647	−0.5559	—	17.2558
					−3.2410	
0.40	13.1678	1.4792	0.1904	0.6425	—	14.0148
					−2.5985	
0.45	10.5693	1.1911	0.4785	1.6151	—	11.4163
					−0.9833	
0.50	9.5860	1.0813	0.5883	1.9854	—	10.4330
					1.0020	
0.55	10.5880	1.1931	0.4765	1.6081	—	11.4350
					2.6101	
0.60	13.1981	1.4826	0.1870	0.6312	—	14.0451
					3.2414	
0.65	16.4395	1.8376	−0.1680	−0.5672	—	17.2865
					2.6742	
0.70	19.1137	2.1262	−0.4566	−1.5411	—	19.9607
					1.1331	
0.75	20.2468	2.2471	−0.5775	−1.9492	—	21.0938
					−0.8161	
0.80	19.4307	2.1601	−0.4905	−1.6556	—	20.2777
					−2.4716	
0.85	—	—	—	—	—	17.8061

change of 1.3804° during the interval beginning at $t = 0.10$ s. This value added to 17.6279° gives the value $\delta = 19.0083°$ at $t = 0.15$ s. Note that the value of $P_m - P_c$ changes at 0.25 s because the fault is cleared at 0.225 s. The angle γ hasalso changed from 0.755° to 0.847°.

Whenever a fault is cleared, a discontinuity occurs in the accelerating power P_a. When clearing is at 0.225 s, as in Table 16.6, no special approach is required since our procedure assumes a discontinuity at the middle of an interval. At the beginning of the interval following clearing the assumed constant value of P_a is that determined for δ at the beginning of the interval following clearing.

When clearing is at the *beginning* of an interval such as at 3 cycles (0.05 s), two values of accelerating power result from the two expressions for the power output of the generator. One applies during and one after clearing the fault. For the system of Example 16.11 if the discontinuity occurs at 0.05 s, the average of the two values is assumed as the constant value of P_a from 0.025 to 0.075 s. The procedure is the same as that followed upon occurrence of the fault at $t = 0$, as demonstrated in Table 16.6.

Following the same procedures as in Table 16.6, we can determine δ versus t for machine 1 for clearing at 0.025 s and for both machines for clearing at 0.05 s. In the next section we see computer printouts of δ versus t for both machines calculated for clearing at 0.05 and 0.225 s.

Swing curves plotted for the two machines in Fig. 16.15 show that machine 1 is unstable for clearing at 0.225 s. For clearing at 0.20 s, however, it can be shown that the system is stable. The equal-area criterion confirms that the actual critical clearing time is between 0.20 and 0.225 s (see Prob. 16.16).

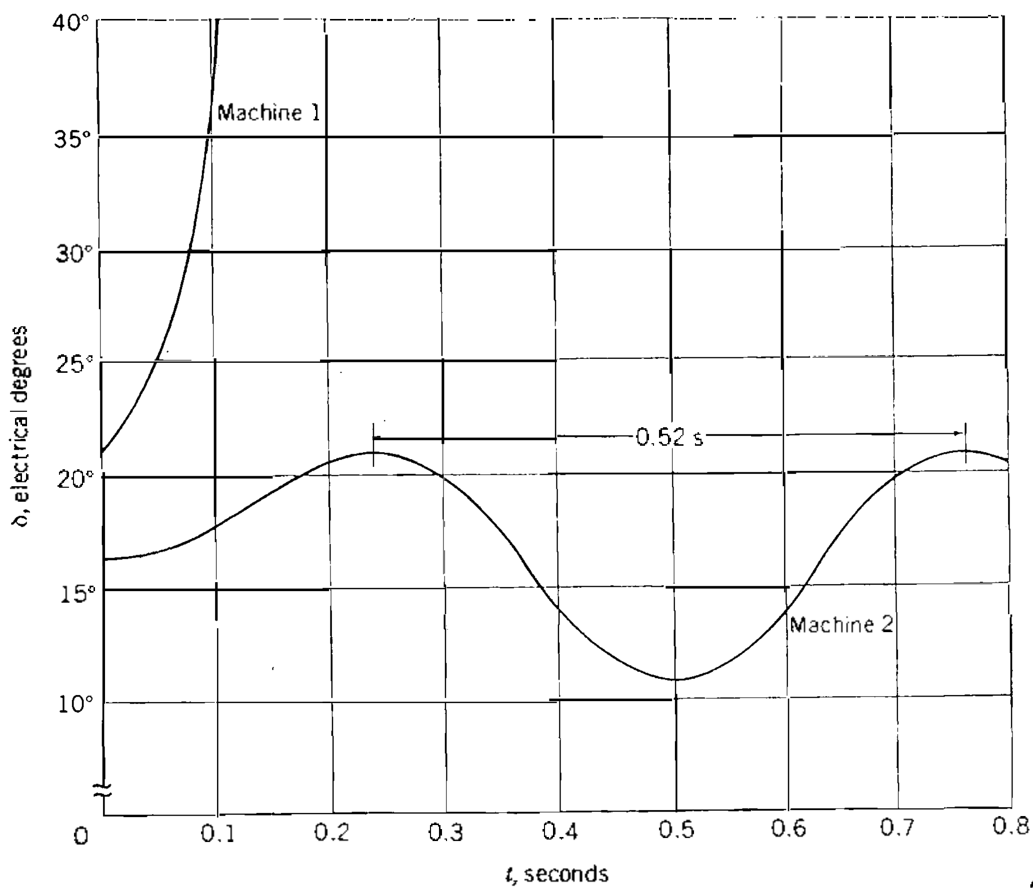


FIGURE 16.15
Swing curves for machines 1 and 2 of Examples 16.9 to 16.11 for clearing at 0.225 s.

Figure 16.15 shows that the change in the rotor angle of machine 2 is quite small, even though the fault is not cleared until 13.5 cycles after it occurs. It is interesting, therefore, to calculate the approximate frequency of oscillation of the rotor using the linearization procedure of Sec. 16.5. The synchronizing power coefficient calculated from the postfault power-angle equation for machine 2 is given by

$$S_p = \frac{dP_e}{d\delta} = \frac{d}{d\delta}[0.1804 + 6.4934\sin(\delta - 0.847°)]$$

$$= 6.4934\cos(\delta - 0.847°)$$

We note from Table 16.6 that the angle of machine 2 varies between 10.43° and 21.09°. Using either angle makes little difference in the value found for S_p. If we use the average value of 15.76°, we find that

$$S_p = 6.274 \text{ per-unit power/elec rad}$$

and by Eq. (16.50) the frequency of oscillation is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{377 \times 6.274}{2 \times 8}} = 1.935 \text{ Hz}$$

from which the period of oscillation is calculated to be

$$T = \frac{1}{f_n} = \frac{1}{1.935} = 0.517 \text{ s}$$

Figure 16.15 and Table 16.6 confirm this value of T for machine 2. When faults are of shorter duration than 0.225 s, even more accurate results can be expected since the swing of the rotor is correspondingly smaller.

In the above examples it is possible to calculate the swing curves for each machine separately because of the fault location considered. When other fault locations are chosen, intermachine oscillations occur because the two generators do not decouple. The swing-curve computations are then more unwieldy. For such cases manual calculations are time-consuming and should be avoided. Computer programs of great versatility are generally available and should be used.

16.10 COMPUTER PROGRAMS FOR TRANSIENT STABILITY STUDIES

Present-day computer programs for transient stability studies have evolved from two basic needs (1) the requirement to study very large interconnected systems with numerous machines and (2) the need to represent machines and their associated control systems by more detailed models. The classical machine representation is suitable for many studies. However, more elaborate models may be required to represent modern turboalternators with dynamic characteristics determined by the many technological advances in the design of machine and control systems.

The simplest possible synchronous machine model is that used in classical stability studies. The more complicated two-axis machine models of Chap. 3 provide for direct- and quadrature-axis flux conditions during the subtransient and transient periods following a system disturbance. By providing for varying flux linkages of the field winding in the direct axis, these models allow representation of the action of the continuously acting automatic voltage regulator and excitation system with which all modern machines are equipped. Turbine control systems, which automatically govern the mechanical power input to the generating unit, also have dynamic response characteristics which can influence rotor dynamics. If these control schemes are to be represented, the model of the generating unit must be further extended.

The more complex generator models give rise to a larger number of differential and algebraic equations for each machine. In large system studies many generators supply widely dispersed load centers through extensive transmission systems whose performance also must be represented by a very large number of algebraic equations. Therefore, two sets of equations need to be solved simultaneously for each interval of time following the occurrence of a system disturbance. One set consists of the *algebraic* equations for the *steady-state* behavior of the network and its loads, and the algebraic equations relating V_t and E' of the synchronous machines. The other set consists of the *differential* equations which describe the *dynamic* electromechanical performance of the machines and their associated control systems.

The Newton-Raphson power-flow procedure described in Chap. 9 is the most commonly used solution technique for the network equations. Any one of several well-known step-by-step procedures may be chosen for numerical integration of the differential equations. The fourth-order Runge-Kutta method is very often used in production-type transient stability programs. Other methods known as the Euler method, the modified Euler method, the trapezoidal method, and predictor-corrector methods similar to the step-by-step method developed in Sec. 16.9 are alternatives. Each of these methods has advantages and disadvantages associated with numerical stability, time-step size, computational effort per integration step, and accuracy of solutions obtained.[4]

Table 16.7 shows the computer printout for swing curves of machines 1 and 2 of Example 16.11 for clearing at 0.225 s and at 0.05 s. These results were obtained by use of a production-type stability program, which couples a Newton-Raphson power-flow program with a fourth-order Runge-Kutta proce-

[4]For further information, see G. W. Stagg and A. H. El-Abiad, *Computer Methods in Power System Analysis*, Chaps. 9 and 10, McGraw-Hill, Inc. New York, 1968.

TABLE 16.7
Computer printout of swing curves for machines 1 and 2 of Examples 16.9 to 16.11 for clearing at 0.225 and 0.05 s

Clearing at 0.225 s			Clearing at 0.05 s		
Time	Mach. 1 angle	Mach. 2 angle	Time	Mach. 1 angle	Mach. 2 angle
0.00	20.8	16.2	0.00	20.8	16.2
0.05	25.1	16.6	0.05	25.1	16.6
0.10	37.7	17.6	0.10	32.9	17.2
0.15	58.7	19.0	0.15	37.3	17.2
0.20	88.1	20.3	0.20	36.8	16.7
0.25	123.1	20.9	0.25	31.7	15.9
0.30	151.1	19.9	0.30	23.4	15.0
0.35	175.5	17.4	0.35	14.6	14.4
0.40	205.1	14.3	0.40	8.6	14.3
0.45	249.9	11.8	0.45	6.5	14.7
0.50	319.3	10.7	0.50	10.1	15.6
0.55	407.0	11.4	0.55	17.7	16.4
0.60	489.9	13.7	0.60	26.6	17.1
0.65	566.0	16.8	0.65	34.0	17.2
0.70	656.4	19.4	0.70	37.6	16.8
0.75	767.7	20.8	0.75	36.2	16.0

dure. It is interesting to compare the closeness of the hand-calculated values of Table 16.6 with those for machine 2 in Table 16.7 for the case where the fault is cleared at 0.225 s.

The assumption of constant admittances for the loads allows us to absorb these admittances into $\mathbf{Y}_{bus}$ and thereby to avoid power-flow calculations, which are required when more accurate solutions using Runge-Kutta calculations are desired. The latter, being of the fourth order, require four iterative power-flow computations per time step.

16.11 FACTORS AFFECTING TRANSIENT STABILITY

Two factors which indicate the relative stability of a generating unit are (1) the angular swing of the machine during and following fault conditions and (2) the critical clearing time. It is apparent from this chapter that both the H constant and the transient reactance X_d' of the generating unit have a direct effect on both of these factors.

Equations (16.84) and (16.85) show that the smaller the H constant, the larger the angular swing during any time interval. On the other hand, Eq. (16.36) shows that P_{max} decreases as the transient reactance of the machine increases. This is so because the transient reactance forms part of the overall series reactance which is the reciprocal of the transfer admittance of the system.

Examination of Fig. 16.11 shows that all three power curves are lowered when P_{max} is decreased. Accordingly, for a given shaft power P_m, the initial rotor angle δ_0 is increased, δ_{max} is decreased, and a smaller difference between δ_0 and δ_{cr} exists for a smaller P_{max}. The net result is that a decreased P_{max} constrains a machine to swing through a smaller angle from its original position before it reaches the critical clearing angle. Thus, any developments which lower the H constant and increase transient reactance X'_d of the machine cause the critical clearing time to decrease and lessen the probability of maintaining stability under transient conditions. As power systems continually increase in size, there may be a corresponding need for higher-rated generating units. These larger units have advanced cooling systems which allow higher-rated capacities without comparable increase in rotor size. As a result, H constants continue to decrease with potential adverse impact on generating unit stability. At the same time this uprating process tends to result in higher transient and synchronous reactances, which makes the task of designing a reliable and stable system even more challenging.

Fortunately, stability control techniques and transmission system designs have also been evolving to increase overall system stability. The control schemes include:

- Excitation systems
- Turbine valve control
- Single-pole operation of circuit breakers
- Faster fault clearing times

System design strategies aimed at lowering system reactance include:

- Minimum transformer reactance
- Series capacitor compensation of lines
- Additional transmission lines

When a fault occurs, the voltages at all buses of the system are reduced. At generator terminals the reduced voltages are sensed by the automatic voltage regulators which act within the excitation system to restore generator terminal voltages. The general effect of the excitation system is to reduce the initial rotor angle swing following the fault. This is accomplished by boosting the voltage applied to the field winding of the generator through action of the amplifiers in the forward path of the voltage regulators. The increased air-gap flux exerts a restraining torque on the rotor, which tends to slow down its motion. Modern excitation systems employing thyristor controls can respond rapidly to bus-voltage reduction and can effect from 0.5 to 1.5 cycles gain in critical clearing times for three-phase faults on the high-side bus of the generator step-up transformer.

Modern electrohydraulic turbine-governing systems have the ability to close turbine valves to reduce unit acceleration during severe system faults near the unit. Immediately upon detecting differences between mechanical input and electrical output, control action initiates the valve closing, which reduces the power input. A gain of 1 to 2 cycles in critical clearing time can be achieved.

Reducing the reactance of the system during fault conditions increases $r_1 P_{max}$ and decreases the acceleration area of Fig. 16.11. The possibility of maintaining stability is thereby enhanced. Since single-phase faults occur more often than three-phase faults, relaying schemes which allow independent or selective circuit-breaker pole operation can be used to clear the faulted phase while keeping the unfaulted phases intact. Separate relay systems, trip coils, and operating mechanisms can be provided for each pole so as to mitigate *stuck-breaker* contingencies following three-phase faults. Independent-pole operation of critical circuit breakers can extend the critical clearing time by 2 to 5 cycles depending on whether one or two poles fail to open under fault conditions. Such gain in critical clearing time can be important especially if backup clearing times are a problem for system stability.

Reducing the reactance of a transmission line is another way of raising P_{max}. Compensating for line reactance by series capacitors is often an economical means of increasing stability. Increasing the number of parallel lines between two points is a common means of reducing reactance. When parallel transmission lines are used instead of a single line, some power is transferred over the remaining line even during a three-phase fault on one of the lines—unless the fault occurs at a paralleling bus. For other types of faults on one line more power is transferred during the fault if there are two lines in parallel than is transferred over a single faulted line. For more than two lines in parallel the power transferred during the fault is even greater. Power transferred into the system is subtracted from power input to the generator to obtain accelerating power. Thus, the more power is transferred into the system during a fault, the lower the acceleration of the machine rotor and the greater the degree of stability.

16.12 SUMMARY

This chapter presents the basics of power system stability analysis. Starting with elementary principles of rotational motion, the swing equation governing the electromechanical dynamic behavior of each generating unit is developed. The swing equation is shown to be nonlinear because the electrical power output from the generating unit is a nonlinear function of the rotor angle. Because of this nonlinearity, iterative step-by-step methods of solution of the swing equation are generally required. In the special case of two finite machines (or one machine operating into an infinite bus) the equal-area criterion of stability can be used to calculate the critical clearing angle. It is shown, however, that finding the critical clearing time (which is the maximum elapsed time from the initiation of a fault until its isolation such that the system is transiently stable) generally requires a numerical solution of the swing equation.

Classical stability studies and their underlying assumptions are explained for the multimachine case and a simple step-by-step procedure for solving the swing equations of the system is illustrated numerically. A basis is thereby provided for further study of the more powerful numerical techniques employed in industry-based production-type computer programs.

Transient stability of the power system is affected by many other factors related to the design of the system network, its protection system, and the control schemes associated with each of the generating units. These factors are discussed in summary form.

PROBLEMS

16.1. A 60-Hz four-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of $H = 7.5$ MJ/MVA. Find (*a*) the kinetic energy stored in the rotor at synchronous speed and (*b*) the angular acceleration if the electrical power developed is 400 MW when the input less the rotational losses is 740,000 hp.

16.2. If the acceleration computed for the generator described in Prob. 16.1 is constant for a period of 15 cycles, find the change in δ in electrical degrees in that period and the speed in revolutions per minute at the end of 15 cycles. Assume that the generator is synchronized with a large system and has no accelerating torque before the 15-cycle period begins.

16.3. The generator of Prob. 16.1 is delivering rated megavoltamperes at 0.8 power-factor lag when a fault reduces the electric power output by 40%. Determine the accelerating torque in newton-meters at the time the fault occurs. Neglect losses and assume constant power input to the shaft.

16.4. Determine the WR^2 of the generator of Prob. 16.1.

16.5. A generator having $H = 6$ MJ/MVA is connected to a synchronous motor having $H = 4$ MJ/MVA through a network of reactances. The generator is delivering power of 1.0 per unit to the motor when a fault occurs which reduces the delivered power. At the time when the reduced power delivered is 0.6 per unit, determine the angular acceleration of the generator with respect to the motor.

16.6. A power system is identical to that of Example 16.3, except that the impedance of each of the parallel transmission lines is $j0.5$ and the delivered power is 0.8 per unit when both the terminal voltage of the machine and the voltage of the infinite bus are 1.0 per unit. Determine the power-angle equation for the system during the specified operating conditions.

16.7. If a three-phase fault occurs on the power system of Prob. 16.6 at a point on one of the transmission lines at a distance of 30% of the line length away from the sending-end terminal of the line, determine (*a*) the power-angle equation during the fault and (*b*) the swing equation. Assume that the system is operating under the conditions specified in Prob. 16.6 when the fault occurs. Let $H = 5.0$ MJ/MVA, as in Example 16.4.

16.8. Series resistance in the transmission network results in positive values for P_c and γ in Eq. (16.80). For a given electrical power output, show the effects of resistance on the synchronizing coefficient S_p, the frequency of rotor oscillations, and the damping of these oscillations.

16.9. A generator having $H = 6.0$ MJ/MVA is delivering power of 1.0 per unit to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 per unit. When the fault is cleared, the original network conditions again exist. Determine the critical clearing angle and critical clearing time.

16.10. A 60-Hz generator is supplying 60% of P_{max} to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 400%. When the fault is cleared, the maximum power that can be delivered is 80% of the original maximum value. Determine the critical clearing angle for the condition described.

16.11. If the generator of Prob. 16.10 has an inertia constant of $H = 6$ MJ/MVA and P_m (equal to 0.6 P_{max}) is 1.0 per-unit power, find the critical clearing time for the condition of Prob. 16.10. Use $\Delta t = 0.05$ to plot the necessary swing curve.

16.12. For the system and fault conditions described in Probs. 16.6 and 16.7, determine the power-angle equation if the fault is cleared by the simultaneous opening of breakers at both ends of the faulted line at 4.5 cycles after the fault occurs. Then, plot the swing curve of the generator through $t = 0.25$ s.

16.13. Extend Table 16.6 to find δ at $t = 1.00$ s.

16.14. Calculate the swing curve for machine 2 of Examples 16.9 through 16.11 for fault clearing at 0.05 s by the method described in Sec. 16.9. Compare the results with the values obtained by the production-type program and listed in Table 16.7.

16.15. If the three-phase fault on the system of Example 16.9 occurs on line ④–⑤ at bus ⑤ and is cleared by the simultaneous opening of breakers at both ends of the line at 4.5 cycles after the fault occurs, prepare a table like that of Table 16.6 to plot the swing curve of machine 2 through $t = 0.30$ s.

16.16. By applying the equal-area criterion to the swing curves obtained in Examples 16.9 and 16.10 for machine 1, (*a*) derive an equation for the critical clearing angle, (*b*) solve the equation by trial and error to evaluate δ_{cr} and (*c*) use Eq. (16.72) to find the critical clearing time.