3. PROBABILITY

Jointly with **statistics**, **probability** is a branch of mathematics that has been developed to deal with **uncertainty**. In the study of probability there are basically three kinds of questions:

1) What do we mean when we say that the probability of an event is, say, 0.50, 0.02, or 0.81?
2) How are the numbers we call probabilities determined, or measured in actual practice?
3) What are the mathematical rules that probabilities must obey

Sample Spaces فراغ العينة and Events الحادثة أوالحدث

In **probability** theory, an **experiment** or trial is any procedure or process that can be infinitely repeated and has a well-defined set of possible outcomes (results), known as the **sample space**. An **experiment** is said to be random if it has more than one possible outcome, and deterministic (مجددة) if it has only one.

sample space

The **sample space** denoted by ***(S)*** of an experiment is a set consisting of all of the possible experimental outcomes.

The following examples help illustrate the concept of a sample space.

Example 1

Machine Breakdowns: An engineer in charge of the maintenance of a particular machine notices that its breakdowns can be characterized as due to an **electrical failure** within the machine, **a mechanical failure** of some component of the machine, or **operator misuse**. When the machine is running, the engineer is uncertain what will be the cause of the next breakdown. The problem can be thought of as an experiment with the sample space ***S*** **= {electrical, mechanical, misuse}**

Example 2

Defective Computer Chips: A company sells computer chips in boxes of 500, and each chip can be classified as either satisfactory or defective. The number of defective chips in a particular box is uncertain, and the sample space is:

S **= {0 defectives, 1 defective, 2 defectives, . . . , 499 defectives, 500 defectives}**

Example 3

Power Plant Operation: A manager supervises the operation of three power plants, plant **X**, plant **Y**, and plant **Z**. At any given time, each of the three plants can be classified as either generating electricity (1) or being idle (0). With the notation (0, 1, 0) used to represent the situation where plant Y is generating electricity but plants X and Z are both idle, the sample space for the status of the three plants at a particular point in time is

S **= {(0, 0, 0) (0, 0, 1) (0, 1, 0) (0, 1, 1) (1, 0, 0) (1, 0, 1) (1, 1, 0) (1, 1, 1)}**

Events

An **event** A is a subset of the sample space S. It collects outcomes of particular interest.

Example 4

The toss of a single coin has a sample space

$$S = \{\text{head, tail}\}$$

In this case, the sample space contains simple event which is either head or tail.

O the other hand, suppose toss of two coins, the corresponding of the sample space is:

$$S = \{(\text{head, head})\ (\text{head, tail})\ (\text{tail, head})\ (\text{tail, tail})\}$$

where **(head, tail),** represents the **event** (multiple elements or several outcomes) that the first coin resulted in a head and the second coin resulted in a tail.

Combining events by union, intersection, and complement

Unions of Events: "The event $A \cup B$ is the union of events A and B and consists of the outcomes that are contained within at least one of the events A and B"

Suppose an experiment with sample space S. Let A and B be events in S and let E be the event "either A occurs or B occurs". Then E occurs if the outcome of the experiment is either in A, or in B, or in both A and B. Therefore we conclude that:

$$E = A \cup B\text{, } E \text{ is union of } A \text{ and } B$$

Intersections of Events: "The event $A \cap B$ is the intersection of the events A and B and consists of the outcomes that are contained within both events A and B"

If we let F be the event "both A and B occur", then

$$F = A \cap B\text{, } F \text{ is intersection of } A \text{ and } B$$

Example 5:

Roll a fair die. The sample space of equally likely simple events is:

$$S = \{1, 2, 3, 4, 5, 6\}.$$

Let A be the event "an odd number turns up" and let B be the event "the number that turns up is divisible by 3".

(*a*) Find the event E = "the number that turns up is odd or is divisible by 3".

(*b*) Find the event F = "the number that turns up is odd and is divisible by 3".

Solutions:

(*a*) $A = \{1, 3, 5\}$, $B = \{3, 6\}$, $E = A \cup B = \{1, 3, 5, 6\}$

Since the simple events are equally likely,

(*b*) $F = A \cap B = \{3\}$

Examples 6:

Roll a pair of fair dice.

(*a*) What is the events that the sum of the numbers is 7 or 11?

(*b*) What is the event that both dice either turn up the same number or that numbers is less than 5?

Solutions:

(*a*) Let A = "sum is 7" and B = 'sum is 11". Then ,

$A = \{(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)\}$, $B = \{(5,6), (6,5)\}$, and $A \cap B = \emptyset$.

(*b*) Let A = "both dice turn up the same number" and B = "the sum is less than 5". Then,

$A = \{\mathbf{(1,1)}, \mathbf{(2,2)}, (3,3), (4,4), (5,5), (6,6)\}$, $B = \{\mathbf{(1,1)},(1,2),(1,3),(2,1),\mathbf{(2,2)},(3,1)\}$

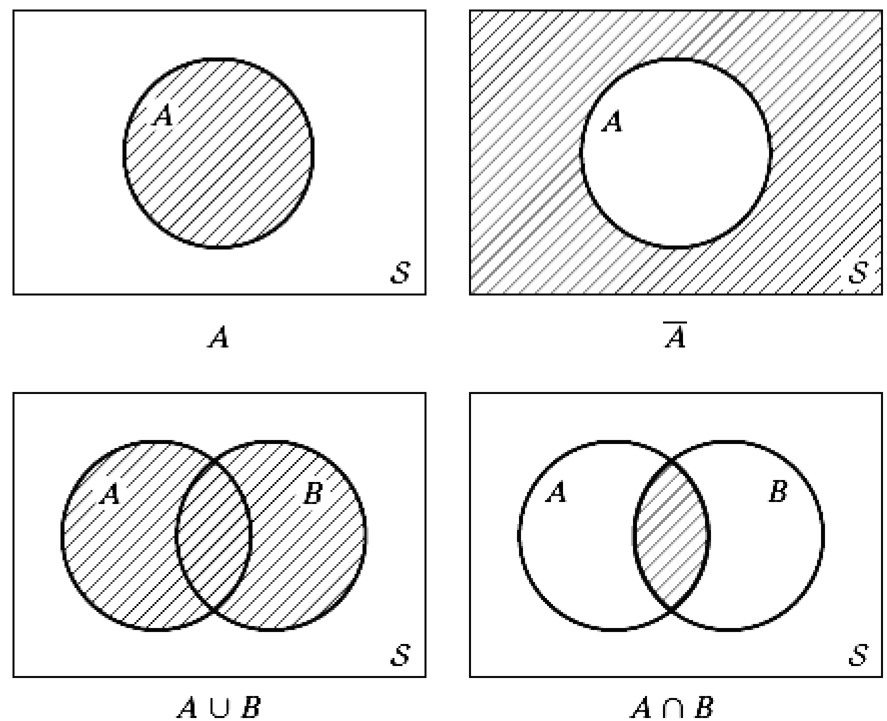
$A \cup B = \{\mathbf{(1,1)},(1,2),(1,3),(2,1),\mathbf{(2,2)},(3,1)\}$ = **event that both dice either turn up the same number and that numbers is less than 5**

<u>Complements of Events</u>: "The event *A*, the complement of an event *A*, is the event consisting of everything in the sample space *S* that is not contained within the event *A*"

Suppose that $\boldsymbol{A}$ is an event in a sample space $\boldsymbol{S}$. Since $\overline{A}$, the *complement of* $\boldsymbol{A}$ which is $\overline{A}$, together with $\boldsymbol{A}$ satisfies $\boldsymbol{A} \cup \overline{A} = \boldsymbol{S}$ and $\boldsymbol{A} \cap \overline{A} = \emptyset$ ($\emptyset$ denotes to empty "nothing")

<u>Venn diagrams</u>

Sample spaces and events, particularly relationships among events, are often depicted by means of **Venn diagrams.** In Venn diagram, the sample space is represented by a rectangle, whereas events are represented by regions within the rectangle, usually by circles or parts of circles. As shown in the given figure, the shaded regions of the four Venn diagrams of the figure represent event *A*, the **complement** of event *A* "which is $\overline{A}$", the **union** of events *A* and *B* "$A \cup B$ ", and the **intersection** of events *A* and *B* "$A \cap B$ ".



Examples 7: Relating events to regions of the Venn diagram

A manufacturer of small motors is concerned with three major types of defects as the following:

A is the **event** that ***the shaft size is too large***;

B is the **event** that ***the windings are improper***; and

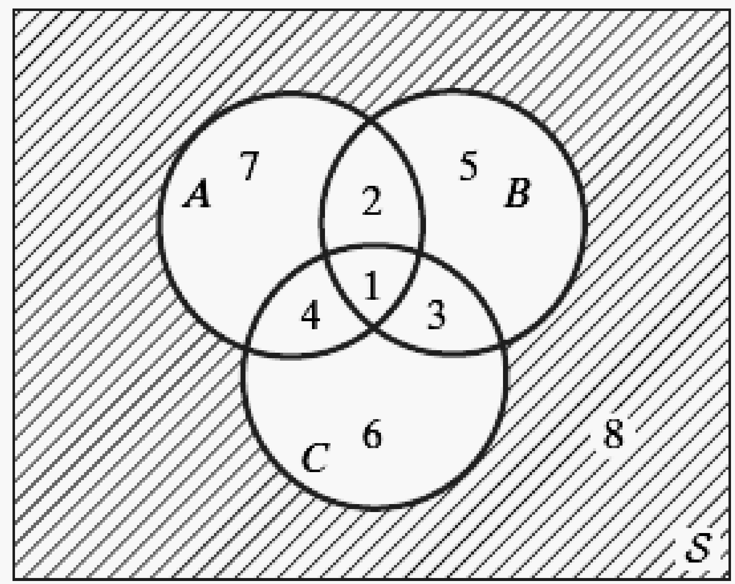
C is the **event** that ***the electrical connections are unsatisfactory***.

It I required to express in words what events are represented by the following regions of the Venn diagram of the given figure:

(a) region 2

(b) regions 1 and 3 together

(c) regions 3, 5, 6, and 8 together



Solutions:

(a) Since region **2** is contained in *A* and *B* but not in *C*, it represents **the event that the shaft is too large and the windings improper, but the electrical connections are satisfactory**.

(b) Since region **1** and **3** together, it represents **only the event *B* of the improper windings and the unsatisfactory of electrical connections as well as all concerns**

(c) Since this is the entire region outside *A*, it represents **the event that the shaft size is not too large**.

Counting

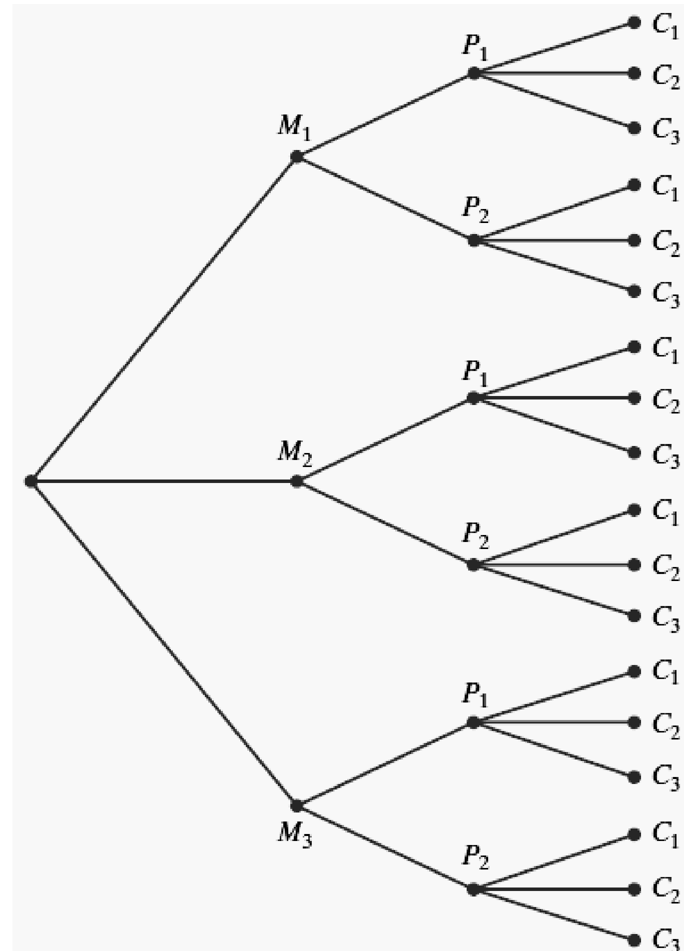
Sometimes it can be difficult to determine the number of elements in a finite sample space by direct enumeration.

Example 8: Suppose all newer used cars in a large city can be classified as **low**, **medium**, or **high** current mileage; **moderate** or **high** priced; and be **inexpensive**, **average**, or **expensive** to operate.

In how many ways can a used car be categorized?

To handle this kind of problem systematically, it helps to draw a **tree diagram** , where the three alternatives for current mileage are denoted by $M1$, $M2$, and $M3$, where $M1$ is low mileage. The price is either $P1$ or $P2$, where $P1$ is moderate; and the three alternatives for operating costs are denoted by $C1$, $C2$, and $C3$, where $C1$ is inexpensive. Following a given path from left to right along the branches of the tree, we obtain a particular categorization, namely a particular element of the sample space. It can be seen that all together there are **18 possibilities.**

This result could also have been obtained by observing that each M-branch forks into two P-branches, and each P-branch forks into three C-branches. Thus, there are 3x2x 3 = 18 combinations of branches, or paths.



tree diagram

Previous result is a special case of the following theorem often called the ***fundamental theorem of counting***

Theorem: **Multiplication of choices**

If sets $A_1, A_2, \ldots, A_k$ contain, respectively, $n_1, n_2, \ldots, n_k$ elements, there are $(= n_1 \times n_2 \times \ldots \times n_k)$ ways of choosing first an element of A_1, then an element of $A_2, \ldots,$ and finally an element of A_k.

In our example we had $n_1 = 3$, $n_2 = 2$, and $n_3 = 3$, and hence, **3 x 2 x 3 = 18** possibilities.

Example 9: Determining the size of an experiment

A manufacturer is experiencing difficulty getting consistent readings of tensile strength between three machines located on the production floor, research lab, and quality control lab, respectively. There are also four possible technicians

(a) How many operator-machine pairs must be included in a designed experiment where every operator tries every machine?

(b) If each operator-machine pair is required to test eight specimens, how many test specimens are required for the entire procedure? Note: A specimen is destroyed when its tensile strength is measured.

Solution : (a) There are $n_1 = 4$ operators and $n_2 = 3$ machines, so **4 x 3 = 12 pairs** are required.

(b) There are $n_3 = 8$ test specimens required for each operator-machine pair, so **8 x 12 = 96 test specimens** are required for the designed experiment.

Factorial, Permutations, Combinations

Factorial Notation:

For any integer n, n **factorial** is defined as

$$n! = n \times (n-1) \times (n-2) \times \dots \times 2 \times 1 \qquad \textbf{Note } 0! = 1$$

Permutations Notation:

The number of permutations of r objects selected from a set of n distinct objects is

$$nPr = n \times (n-1) \times (n-2) \times \dots \times (n-r+1)$$

or, in factorial notation,

$$nPr = \frac{n!}{(n-r)!}$$

Combinations Notation:

The number of ways in which r objects can be selected from a set of n distinct objects is

$$\binom{n}{r} = \frac{nPr}{r!} = \frac{n \times (n-1) \times (n-1) \times \dots \times (n-r+1)}{r!}$$

or, in factorial notation,

$$\binom{n}{r} = \frac{n!}{r! \times (n-r)!}$$

Example 10: The number of ways to assemble chips in a controller

An electronic controlling mechanism requires 5 distinct, but interchangeable, memory chips. In how many ways can this mechanism be assembled

(a) by placing the 5 chips in the 5 positions within the controller?

(b) by placing 3 chips in the odd numbered positions within the controller?

Solution:

(a) When all **5 chips** must be placed, the answer is **5!**. Alternatively, in the permutation notation with ***n* = 5** and ***r* = 5**, the first formula yields

$$5P5 = n \times (n-1) \times (n-2) \times \ldots\ldots \times (n-r+1) = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

and the second formula yields

$$nPr = \frac{n!}{(n-r)!} = 5P5 = \frac{5!}{(5-5)!} = \frac{5!}{0!} = 5! = 120$$

(b) For ***n* = 5** chips placed in ***r* = 3** positions, the permutation is

$$5P3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = \frac{5\text{x}4\text{x}3\text{x}2\text{x}1}{2\text{x}1} = 60$$

Example 11: **Selection of machines for an experiment**

A calibration study needs to be conducted to see if the readings on **15 test machines** are giving similar results. In how many **ways can 3 of the 15** be selected for the initial investigation?

Solution:

$$\binom{n}{r} = \frac{nPr}{r!} = \binom{15}{3} = \frac{15P\,3}{3!} = \frac{15 \text{ x } 14 \text{ x } 13}{3\text{x}2\text{x}1} = 455$$

Note that selecting which 3 machines to use is the same as selecting which 12 not to include. That is, according to the second formula,

$$\binom{n}{r} = \frac{n!}{r! \text{ x } (n-r)!} = \binom{15}{3} = \frac{15!}{3! \text{ x } (15-3)!} = \frac{15!}{3! \text{ x } 12!} = \frac{15\text{x}14\text{x}13\text{x}12!}{3\text{x}2\text{x}1\text{x}12!} = 455$$

Probability

The classical probability concept:

If there are $\boldsymbol{m}$ equally likely possibilities, of which one must occur and s are regarded as favorable, or as a "success," then the probability of a "success" is given by $\frac{s}{m}$

Example 12: What is the probability of drawing an ace from a well-shuffled deck of 52 playing cards?

Solution : There are s **= 4 aces** among the m **= 52** cards, so we get

$$\frac{s}{m} = \frac{4}{52} = \frac{1}{13}$$

Example 13:

If records show that 294 of 300 ceramic insulators tested were able to withstand a certain thermal shock, what is the probability that any one untested insulator will be able to withstand the thermal shock?

Solution Among the insulators tested, $\frac{294}{300} = 0.98$ were able to withstand the thermal shock.

The Axioms of Probability

In **The axioms of probability for a finite sample space are:**

Axiom 1: The first axiom of probability is that the probability of any event is a nonnegative real number: $\mathbf{0 \le P(A) \le 1}$ for each event A in S.

Axiom 2: The second axiom of probability is that the probability of the entire sample space is one: $\mathbf{P(S) = 1}$.

Axiom 3: If A and B are mutually exclusive events in S, meaning that they have an empty intersection then $\mathbf{P(A \cup B) = P(A) + P(B)}$

Example 14: If an experiment has the three possible and mutually exclusive outcomes A, B, and C, check in each case whether the assignment of probabilities is permissible:

(a) $P(A) = 1/3$, $P(B) = 1/3$, and $P(C) = 1/33$

(b) $P(A) = 0.64$, $P(B) = 0.38$, and $P(C) = -0.02$

(c) $P(A) = 0.35$, $P(B) = 0.52$, and $P(C) = 0.26$

(d) $P(A) = 0.57$, $P(B) = 0.24$, and $P(C) = 0.19$

Solution **(a)** The assignment of probabilities is permissible because the values are all on the interval from 0 to 1, and their sum is $1/3+1/3+1/3=1$

(b) The assignment is not permissible because $P(C)$ is negative.

(c) The assignment is not permissible because $0.35 + 0.52 + 0.26 = 1.13$, which exceeds 1.

(d) The assignment is permissible because the values are all on the interval from 0 to 1 and their sum is $0.57 + 0.24 + 0.19 = 1$

Some Elementary Theorems

Generalization of the third axiom of probability:

If $A1, A2, \ldots, An$ are mutually exclusive events in a sample space S, then:

$$P(A1 \cup A2 \cup \ldots\ldots\ldots \cup An) = P(A1) + P(A2) + \ldots\ldots\ldots + P(An)$$

Example 15: The probability that a consumer testing service will rate a new antipollution device for cars very poor, poor, fair, good, very good, or excellent are 0.07, 0.12, 0.17, 0.32, 0.21, and 0.11. What are the probabilities that it will rate the device:

(a) very poor, poor, fair, or good;

(b) good, very good, or excellent?

Solution: Probability of **very poor = 0.07**; **poor = 0.21**; **fair = 0.17**; **good=0.32**; **v.good=0.21** and **excellent =0.11**

(a) Since the probabilities are all mutually exclusive, direct substitution into the formula of previous theorem yields:

Probability of **very poor**, poor, **fair**, or **good** =**0.07** + **0.12** + **0.17** + **0.32** = **0.68**

(b) Probability of **good, v.good, excellent** = **0.32** + **0.21** + **0.11** = **0.64**

Note:

As it can be shown that a sample space of n points (outcomes) has 2^n subsets, it would seem that the problem of specifying a probability function (namely, a probability for each subset or event) can easily become very tedious. Indeed, for $n = 20$ **there are already more than 1 million possible events.**

Fortunately, this task can be simplified considerably by the use of the following theorem:

If A is an event in the finite sample space S, then $P(A)$ equals the sum of the probabilities of the individual outcomes comprising A.

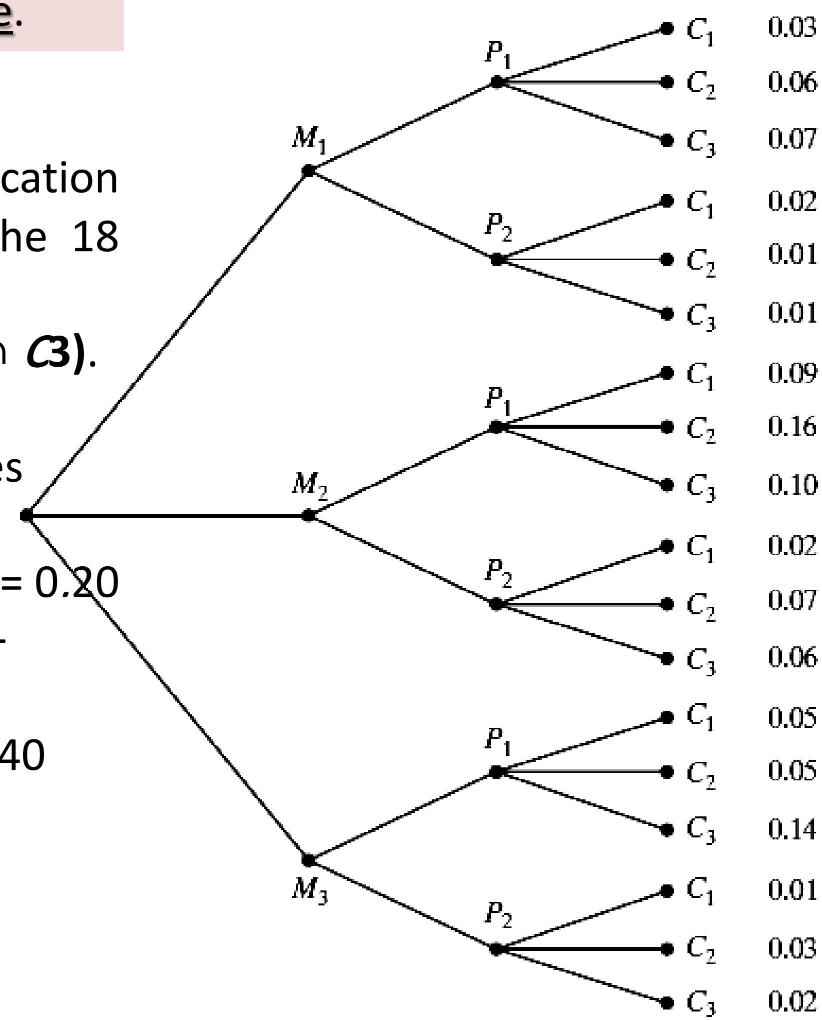
To prove, let $E1, E2, \ldots, En$ be the n outcomes comprising event A, so that:

$A = E1 \cup E2 \cup \cdot \cdot \cdot \cup En$. Since the E's are individual outcomes, they are mutually exclusive, then:

$$P(A) = P(E1 \cup E2 \cup \ldots\ldots \cup En) = P(E1) + P(E2) + \ldots\ldots\ldots + P(En)$$

Example 8: Suppose all newer used cars in a large city can be classified as **low**, **medium**, or **high** current mileage; **moderate** or **high** priced; and be **inexpensive**, **average**, or **expensive** to operate.

Example 16: Refer to the used car classification **example 8**, Suppose that the probabilities of the 18 outcomes are as shown in the figure.
Find $\boldsymbol{P(M1), P(P1), P(C3), P(M1 \cap P1)}$, **and** $\boldsymbol{P(M1 \cap C3)}$.



Solution : Adding the probabilities of the outcomes comprising the respective events, we get

$P(M1) = (0.03 + 0.06 + 0.07) + (0.02 + 0.01 + 0.01) = 0.20$

$P(P1) = (0.03 + 0.06 + 0.07) +(0.09 + 0.16 + 0.10) + (0.05+0.05 + 0.14) = 0.75$

$P(C3) = 0.07 + 0.01 + 0.10 + 0.06 + 0.14 + 0.02 = 0.40$

$P(M1 \cap P1) = 0.03 + 0.06 + 0.07 = 0.16$

and

$P(M1 \cap C3) = 0.07 + 0.01 = 0.08$

General addition rule for probability

If A and B are any events in S, then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Example 17: With reference to the used car **example 16**, find the probability that a car will have **low mileage** or be **expensive to operate**, namely $P(M1 \cup C3)$.

Solution: Making use of the previous obtained results , $P(M1) = 0.20$, $P(C3) = 0.40$, and $P(M1 \cap C3) = 0.08$, we substitute into the general addition rule to get

$$\begin{aligned} P(M1 \cup C3) &= P(M1) + P(C3) - P(M1 \cap C3) \\ &= 0.20 + 0.40 - 0.08 \\ &= 0.52 \end{aligned}$$

Probability rule of the complement

If A is any event in S, then $P(\overline{A}) = 1 - P(A)$

Example 18: Referring to the used car example of **Example 16**, find:

(a) the probability that **a used car will not have low mileage**

(b) the probability that **a used car will either not have low mileage or not be expensive to operate**

Solution By the rule of the complement

(a) $P(\overline{M}_1) = 1 - P(M_1) = 1 - 0.20 = 0.80$

(b) Since $\overline{M}_1 \cup \overline{C3} = \overline{M_1 \cap C_3}$ by the rule of the complement we get

$$P(\overline{M}_1 \cup \overline{C}_3) = 1 - P(M1 \cap C3) = 1 - 0.08 = 0.92$$

Conditional Probability

The probability that event A occurs conditional on event B having occurred is written $P(A/B)$. Its interpretation is that if the outcome occurring is known to be contained within the event B, then this **conditional probability** measures the probability that the outcome is also contained within the event A. Conditional probabilities can easily be obtained using the following formula:

If A and B are any events in S and $P(B) > 0$, the **conditional probability** of A given B is

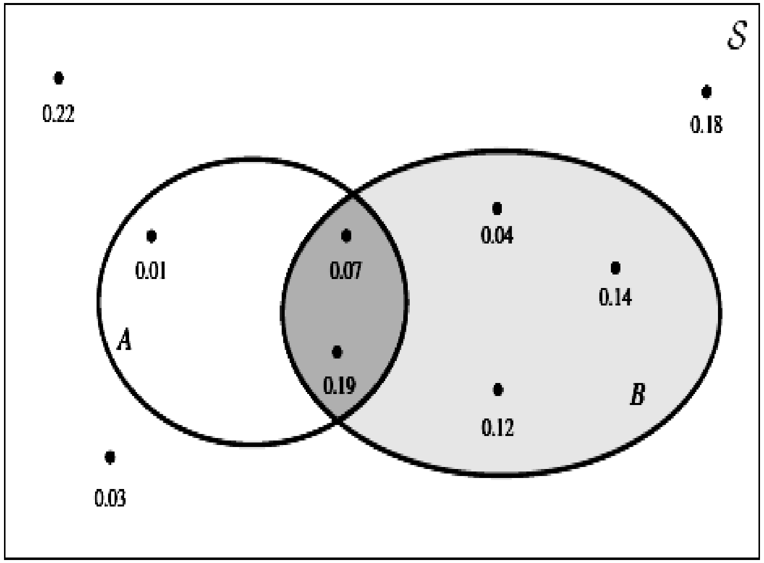
$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

It measures the probability that event A occurs when it is known that event B occurs.

One simple example of **conditional probability** concerns the situation in which two events A and B **are mutually exclusive**. Since in this case events A and B have no outcomes in common, it is clear that the occurrence of event B prevents the possibility of event A occurring, so that automatically, the probability of event A conditional on event B must be zero. Accordingly, since $A \cap B = \emptyset$ for mutually exclusive events, this natural reasoning is in agreement with the formula $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{P(B)} = 0$

Another simple example of conditional probability concerns the situation in which an event B is contained within an event A, that is $B \subset A$. Then if event B occurs, it is clear that event A must also occur, so that intuitively, the probability of event A conditional on event B must be **one**. Again, since $A \cap B = B$, here, this intuitive reasoning is in agreement with the formula $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B)}{P(B)} = 1$

Example 19: consider the given figure and the events ***A*** and ***B*** shown there. Suppose that event ***B*** is known is known to occur. In other words, suppose that it is known that the outcome occurring is one of the **five outcomes** contained within the event ***B***.
What then is the conditional probability of event ***A*** occurring?



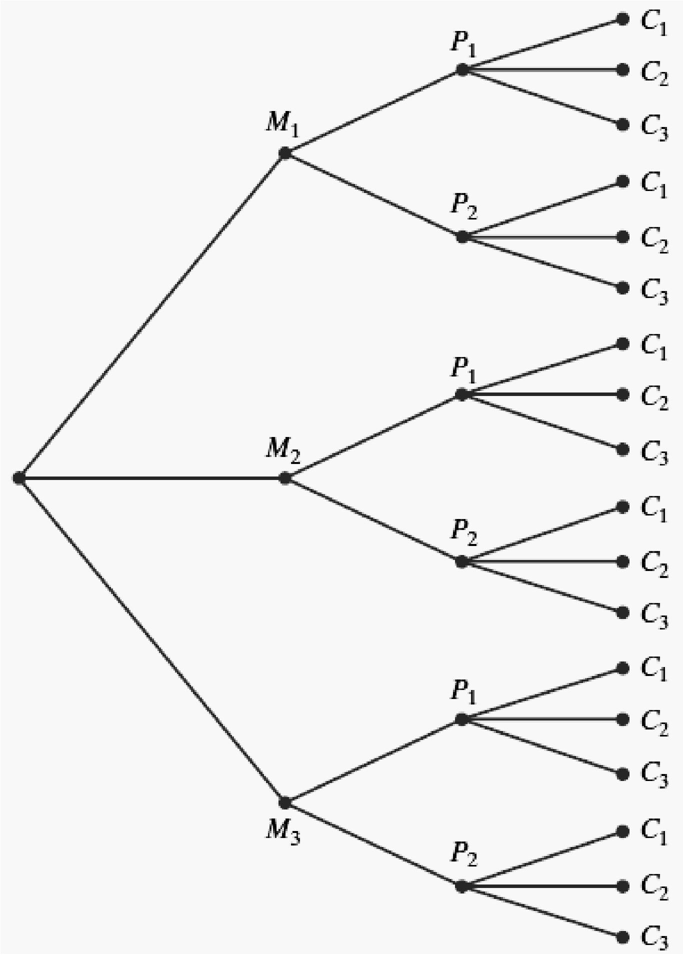
Solution: Since two of the **five outcomes** in event ***B*** are also in event ***A*** (that is, **there are two outcomes in** ***A∩B***), the conditional probability is the probability that one of these **two outcomes** occurs rather than one of the other three outcomes (which are in $\bar{A} \cap B$). The conditional probability is calculated to be:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.26}{0.56} = 0.464$$

Notice that this conditional probability is different from ***P(A)*** **= 0.27.**
If the event ***B*** is known ***not*** **to occur**, which mean ***complement of A*** then the conditional probability of event ***A*** is:

$$P(A/\bar{B}) = \frac{P(A \cap \bar{B})}{P(\bar{B})} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{0.27 - 0.26}{1 - 0.56} = 0.023$$

Example 8: Suppose all newer used cars in a large city can be classified as **low**, **medium**, or **high** current mileage; **moderate** or **high** priced; and be **inexpensive**, **average**, or **expensive** to operate.



Example 20: Refer to the used car classification **example 8**, It is known from **Example 16** that:

$$P(M1) = 0.20,\ P(C3) = 0.40 \text{ and } P(M1 \cap C3) = 0.08$$

It required to find $P(M1 \mid C3)$.

Solution : Since $P(M1 \cap C3) = 0.08$ and $P(C3) = 0.40$, the conditional probability yields:

$$P(M1 / C3) = \frac{P(M1 \cap C3)}{P(C3)} = \frac{0.08}{0.4} = 0.20$$

It is of interest to note that the value of the **conditional probability** obtained, $P(M1 \mid C3) = 0.20$, equals the value for $P(M1)$. This means that **the probability a used car has low mileage is the same whether or not it is expensive to operate**., which mean that $M1$ is <u>**independent**</u> of $C3$.

Example 21: If the **probability** that a communication system will have **high fidelity** is **0.81** and the probability that it will have high fidelity and high selectivity is **0.18**, **<u>what is probability that a system with high selectivity will also have high fidelity?</u>**

Solution If A is the event that a communication system has high selectivity and B is the event that it has high fidelity, it is clear that $P(B) = 0.81$ and $P(A \cap B) = 0.18$, and substitution into the **Conditional probability** formula yields

$$P(A / B) = \frac{P(A \cap B)}{P(B)} = \frac{0.18}{0.81} = \frac{2}{9} = 0.2\dot{2}$$

General Multiplication Law

From the definition of the conditional probability $P(A/B)$ that the probability of the intersection of two events $A \cap B$ can be calculated as:

$$P(A \cap B) = P(B)\, P(A/B)$$

That is, the probability of events A and B both occurring can be obtained by multiplying the probability of event B by the probability of event A conditional on event B. It also follows from the definition of the conditional probability $P(B/A)$ that

$$P(A \cap B) = P(A)\, P(B/A)$$

Bayes' Theorem

Theorem:

If $B1, B2, \ldots\ldots, Bn$ are mutually exclusive events of which one must occur, then the total probability of event A can be calculated as the following:

$$P(A) = \sum_{i=1}^{n} P(Bi) \,.\, P(A / Bi)$$

OR

$$P(B) = \sum_{j=1}^{n} P(Aj) \,.\, P(B / Aj)$$

Theorem Explanation:

If $B1, B2$, and $B3$ are mutually exclusive events and $A = A \cap [B1 \cup B2 \cup B3]$**.** It follows that $A \cap B1$, $A \cap B2$, and $A \cap B3$ are also mutually exclusive. By the definition of the third axiom of probability then:

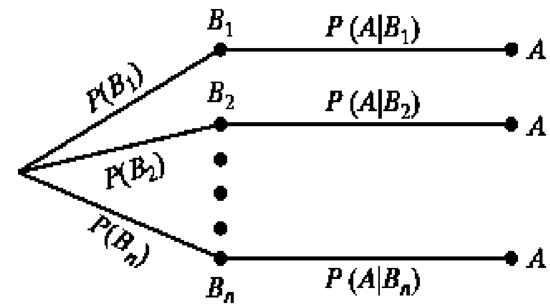
$$P(A) = P(A \cap B1) + P(A \cap B2) + P(A \cap B3)$$

If the general multiplication rules is applied to $P(A \cap B1)$, $P(A \cap B2)$, and $P(A \cap B3)$, then:

$$P(A) = P(B1) \cdot P(A / B1) + P(B2) \cdot P(A / B2) + P(B3) \cdot P(A / B3)$$

Example 22: A manufacturer of tablets receives its LED screens from three different suppliers**,** **60%** from supplier *B*1, **30%** from supplier *B*2, and **10%** from supplier *B*3. Also suppose that **95%** of the LED screens from *B*1, **80%** of those from *B*2, and **65%** of those from *B*3 perform according to specifications. It is required to know the probability that any one LED screen received by the plant will perform according to specifications.

Solution: If *A* denotes the event that a LED screen received by the plant performs according to specifications, and *B*1*, B*2*,* and *B*3 are the events come from the suppliers, then:

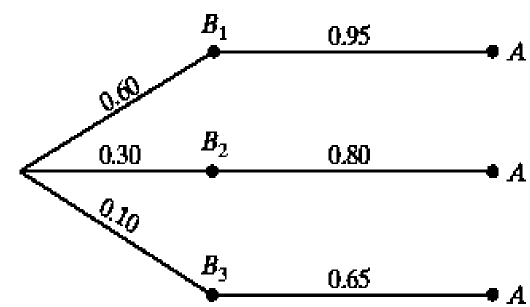


$$P(A) = \sum_{i=1}^{n} P(Bi)\ .\ P(A/Bi)$$

$$P(A) = P(B1)\ .\ P(A/B1) + P(B2)\ .\ P(A/B2) + P(B3)\ .\ P(A/B3)$$

$$P(A) = (0.6) * (0.95) + (0.3) * (0.8) + (0.1) * (0.65) = 0.875$$

If it is required to know the value of $P(B3/A)$, and to find a formula for this probability first write



$$P(B3/A) = \frac{P(A \cap B3)}{P(A)} = \frac{P(B3) \cdot P(A/B3)}{\sum_{j=1}^{3} P(Bi) . P(A/Bi)}$$

$$P(B3/A) = \frac{(0.10)(0.65)}{(0.60)(0.95) + (0.30)(0.80) + (0.10)(0.65)}$$

$$P(B3/A) = 0.074$$

the probability that an LED screen is supplied by *B*3

Theorem If *B*1*, B*2*,……, Bn* are mutually exclusive events of which one must occur, then

$$P(Br \mid A) = \frac{P(Br) \cdot P(A \mid Br)}{\sum_{i=1}^{n} P(Bi) \cdot P(A \mid Bi)} \quad \textbf{for } r = 1, 2, \ldots, n.$$

Example 23:

Four technicians regularly make repairs when breakdowns occur on an automated production line. **Technician 1**, who services **20%** of the breakdowns, makes an incomplete repair 1 time in 20; **Technician 2**, who services **60%** of the breakdowns, makes an incomplete repair 1 time in 10; **Technician 3**, who services **15%** of the breakdowns, makes an incomplete repair 1 time in 10; and **Technician 4**, who services **5%** of the breakdowns, makes an incomplete repair 1 time in 20. For the next problem with the production line diagnosed as being due to an initial repair that was incomplete, what is the probability that this initial repair was made by **Technician 1**?

Solution: Let A be the event that the initial repair was incomplete, $B1$**=0.20** that the initial repair was made by **Technician 1**, $B2$**=0.60** that it was made by **Technician 2**, $B3$**=0.15** that it was made by **Technician 3**, and $B4$**=0.05** that it was made by **Technician 4**.

Also $P(A/B1)=\frac{1}{20}=0.05$, $P(A/B2)=\frac{1}{10}=0.10$, $P(A/B3)=\frac{1}{10}=0.10$ **and** $P(A/B4)=\frac{1}{20}=0.05$

Substituting the various probabilities into the formula of previous theorem, then

$$P(B1/A)=\frac{(0.20)(0.05)}{(0.20)(0.05)+(0.60)(0.10)+(0.15)(0.10)+(0.05)(0.05)}=0.114$$