Chapter 7

Work and Kinetic Energy

Units of Chapter 7: Work and Kinetic Energy

- Work Done by a Constant Force
- Work Done by a Variable Force
- Kinetic Energy and the Work-Energy Theorem
- Power

Learning goals of this chapter

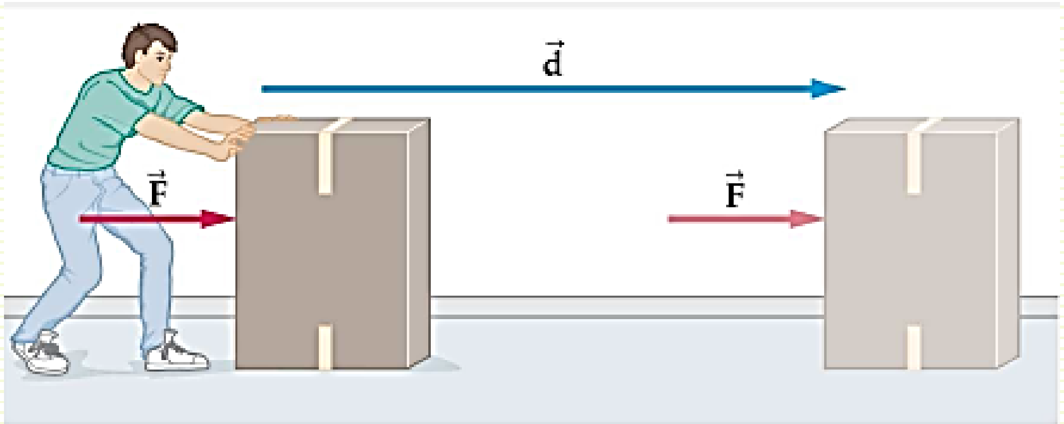
- **On completing this chapter, the student will be able to :**
- Define the concept of the work and its unit.
- Recognize how the force can do the work.
- Express the work done by variable force.
- Proof the Work-Energy theorem.
- Define the power and its unit.

Work Done by a Constant Force

- Definition:
- Work done is the scalar product of the force acting on a body by the displacement, i.e.:

$$W = F.d = Fd\cos\theta$$

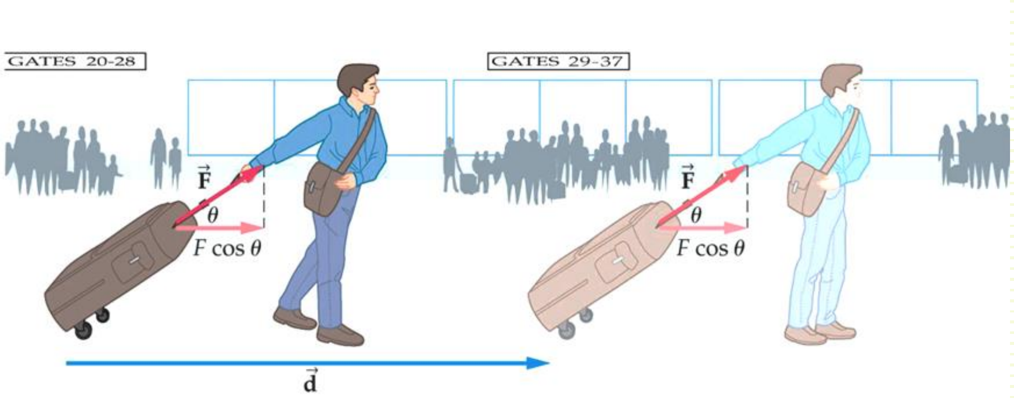
- The work done is a scalar quantity.
- SI unit: = Newton . meter (N.m)= Joule, J



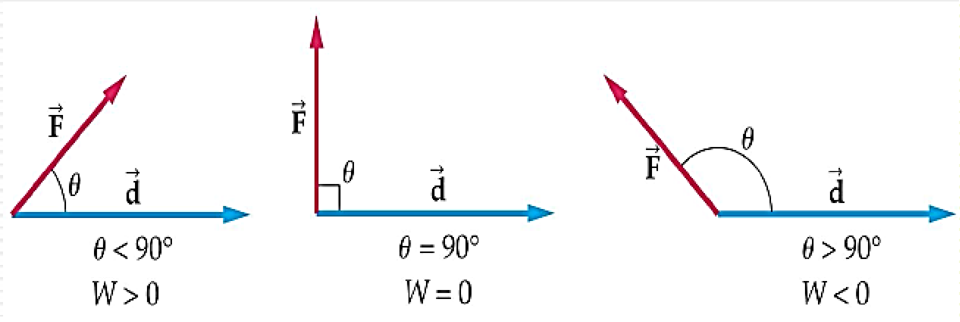
Work Done by a Constant Force

- If the force is at an angle θ to the displacement d, then

$$W = F.d = Fd\cos\theta$$

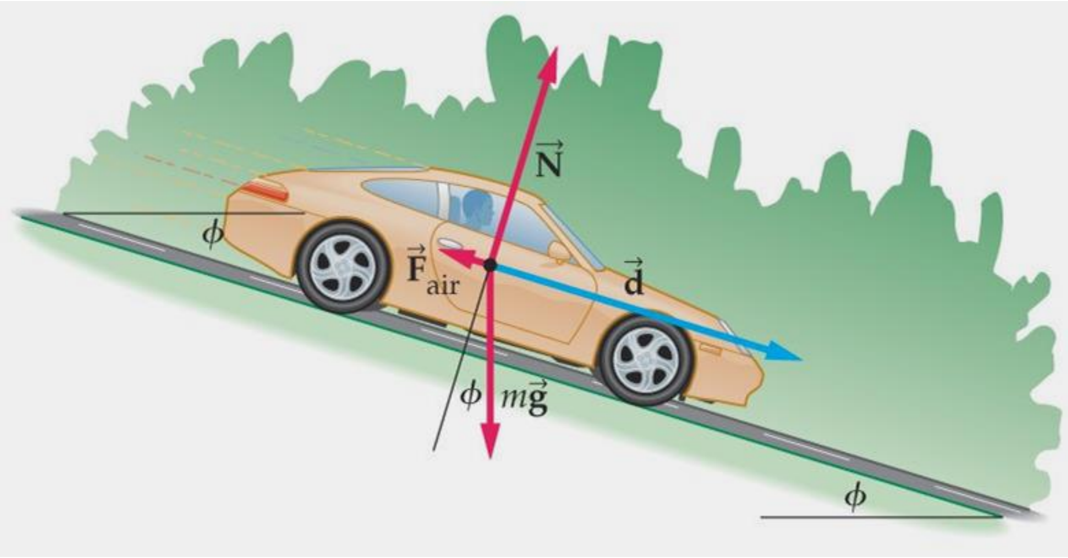


- The work done may be positive , zero, negative value, depending on the angle between the force and the displacement.



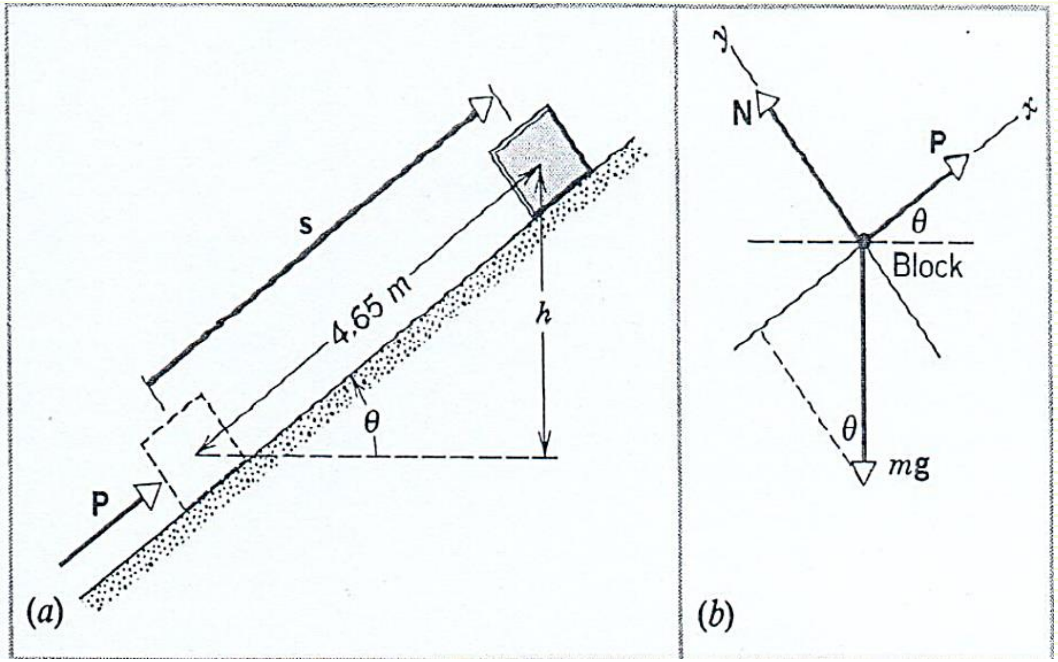
Work Done by a Constant Force

- If there is more than one force acting on an object (engine, gravity, ground friction, air drag, etc…) we can find the work done by each force, and also the work done by the net force:



Problem 1

- Block of mass m=11.7 kg is pushed a distance s=4.65 m along a frictionless incline to raise it a distance h = 2.86 m. Calculate work you would do if you apply a force parallel to the incline to push the block up at constant speed (assume no friction force in the process)



Problem 1

Solution: Let P = the force pushing the block up. Because the motion is not accelerated (speed is constant), the net force parallel to the plane must be zero. If we choose our x axis parallel to the plane, with positive direction up the plane, we have, from Newton's second law:

$$xcomponent: \quad P - mg\sin\theta = 0,$$

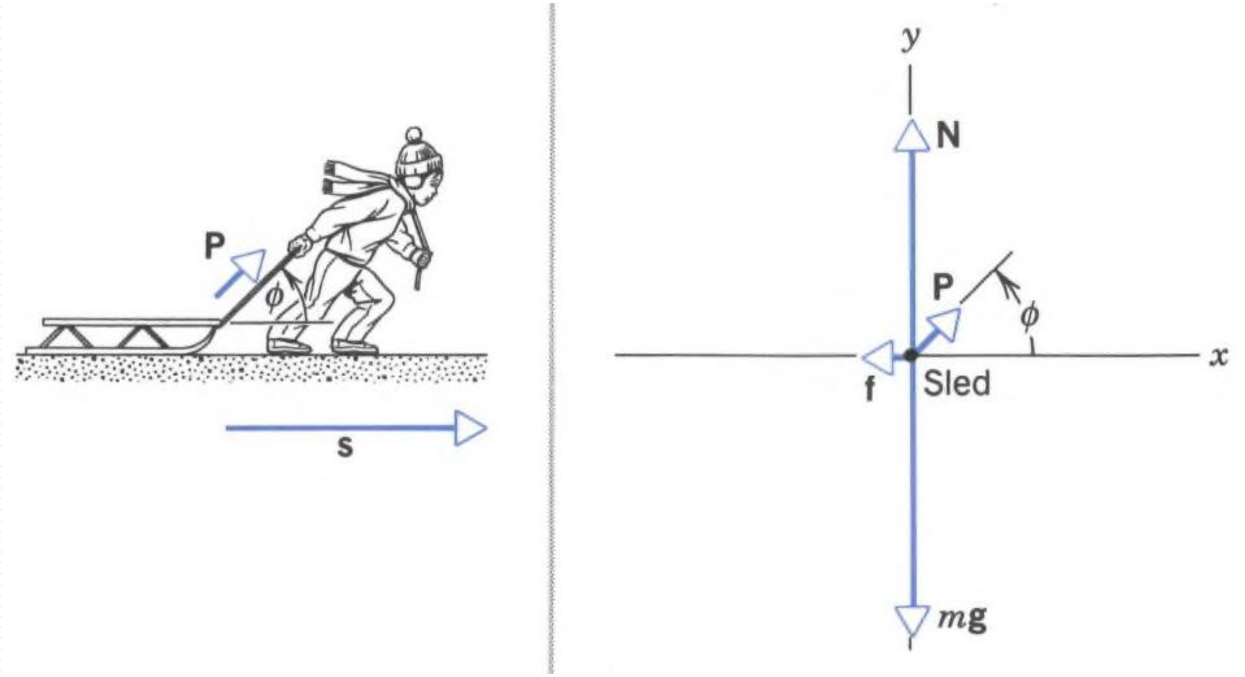
$$P = mg\sin\theta = (11.7\,\text{kg})(9.80\,\text{m/s}^2)\left(\frac{2.86\,\text{m}}{4.65\,\text{m}}\right) = 70.5\,\text{N}.$$

Then the work done by **P**, from Eq. (6.1.3) with $\phi = 0°$, is

$$W = \mathbf{P}\cdot\mathbf{s} = Ps\cos 0° = Ps = (70.5\,\text{N})(4.65\,\text{m}) = 328\,\text{J}.$$

Problem 2

- A child pulls a 5.6 kg sled distance of S=12m on a horizontal surface at a constant speed. What work does the child do on the sled if the coefficient of kinetic friction μ_k is 0.20 and the cord makes an angle of φ = 45° with the horizontal?



Problem 2

The work done by the child on the sled is: $W \ = \ P\,S\,\cos\phi$.

The sled is moving at a constant speed i.e. net force = 0:

X component $\quad P\cos\phi - \ f \ = \ 0$, and

Y component $\quad P\cos\phi - \ +N - mg \ = \ 0$,

Where f is the static friction, N is the normal force and

$$f \ = \mu_k N$$

Solving these equations, we can find *P*

$$P = \frac{\mu_k mg}{\cos\phi + \mu_k \sin\phi} \qquad \frac{(0.20)(55\ \mathrm{N})}{\cos 45° + (0.20)(\sin 45°)} = 13\ \mathrm{N}.$$

Then the work done is

$$W = Ps\cos\phi = (13\ \mathrm{N})(12\ \mathrm{m})(\cos 45°) = 110\ \mathrm{J}.$$

Work Done by a Variable Force

- If the force is not constant on each displacement, the work done on each segment

$$\delta W_1 = F_1\, \delta x.$$

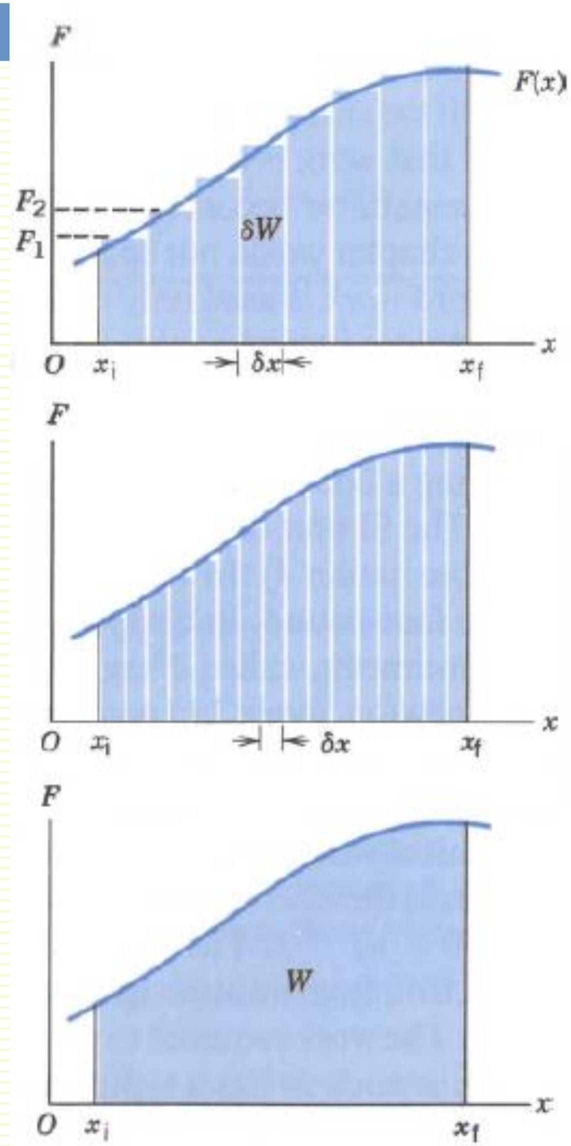
- the total work done is

$$\begin{aligned} W &= \delta W_1 + \delta W_2 + \delta W_3 + \cdots \\ &= F_1\, \delta x + F_2\, \delta x + F_3\, \delta x + \cdots \end{aligned}$$

$$W = \sum_{n=1}^{N} F_n\, \delta x,$$

- If the displacement is very small, then

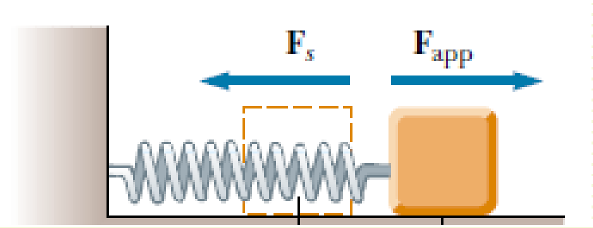
$$W = \int_{x_i}^{x_f} F(x)\, dx.$$



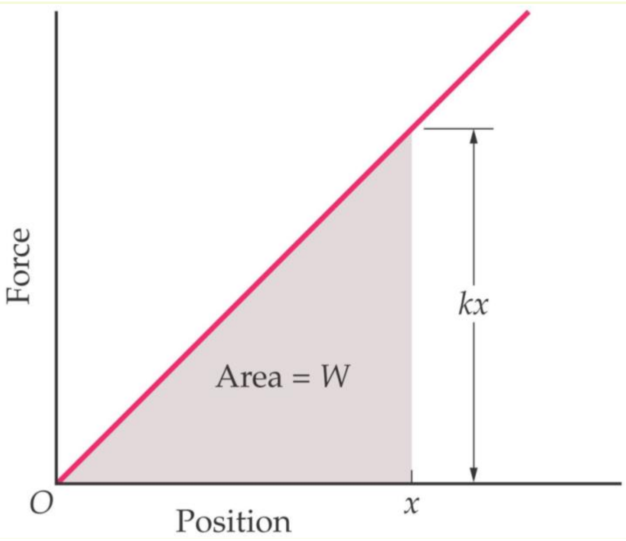
Work Done by a Variable Force

- The force needed to stretch a spring an amount x is

$$F = -k.x.$$



Where k is the spring constant.



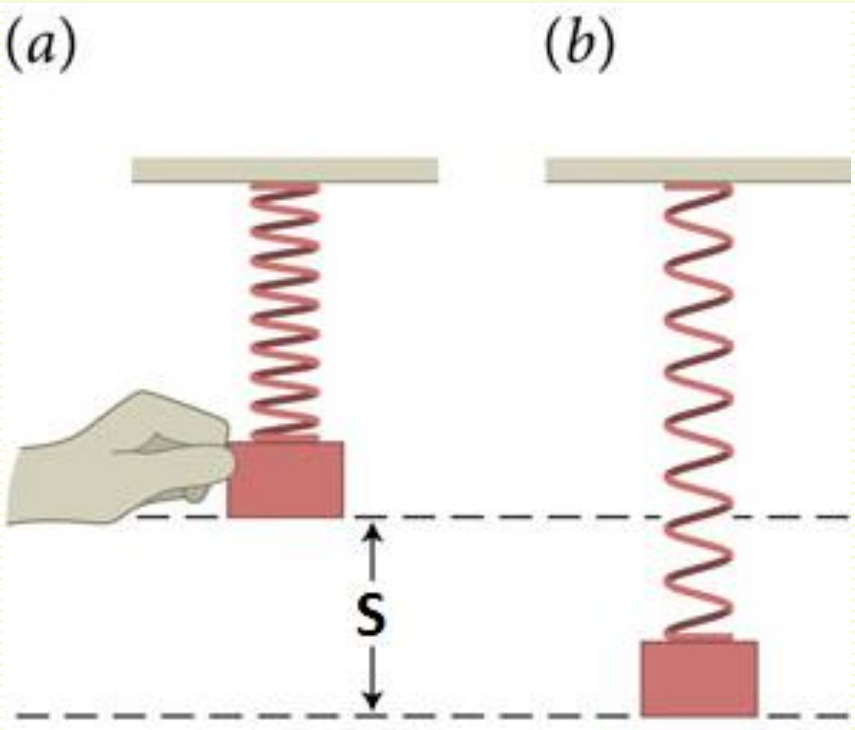
- As x increases, the force needed F also increase.
- Therefore, the work done in stretching the spring is

$$W = \int_{x_1}^{x_2} F(x)dx = \int_{x_i}^{x_f} kxdx = \frac{-1}{2}kx_f^{\ 2} - \frac{1}{2}kx_i^{\ 2} = -\frac{1}{2}kx^2$$

Problem 3

A spring hangs vertically in equilibrium. A mass m = 6.40 kg is attached to the spring, and is held in place so that at first the spring does not stretch. Now the hand holding the block is slowly lowered, allowing the block to descend at constant speed until equilibrium is reached, The hand is then removed. The spring has been stretched by a distance S= 0.124m. Find the work done on the block in this process by

(a) gravity, and (b) the spring.



Problem 3

The net force acting on the spring is

$$\sum F = mg - ks = 0.$$

The spring constant is

$$k = mg/s = (6.40 \text{ kg})(9.80 \text{ m/s}^2)/(0.124 \text{ m}) = 506 \text{ N/m}.$$

(a) Work done by gravity is:

$$W_g = Fs = mgs = (6.40 \text{ kg})(9.80 \text{ m/s}^2)(0.124 \text{ m}) = +7.78 \text{ J}$$

It is positive because the force and displacement are in the same direction.

(b) Work done by spring is:

$$W_s = -\tfrac{1}{2}ks^2 = -\tfrac{1}{2}(506 \text{ N/m})(0.124 \text{ m})^2 = -3.89 \text{ J}$$

It is negative because the force and displacement are in opposite directions.

Kinetic Energy and the Work-Energy Theorem

- Using the equation $$v_f^2 = v_i^2 + 2a(x_f - x_i)$$
- Then $$a(x_f - x_i) = \frac{1}{2}v_f^2 - \frac{1}{2}v_i^2$$
- Multiplying by m $$ma(x_f - x_i) = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$
- Since the force acting on the object *F = ma*, then the work done on the object to move from $x_{i=}$ to x_f is given by W = m a (x_f-x_i), Therefore, the last equation can be written as:

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

- We define the kinetic energy (K) : $$K = \frac{1}{2}mv^2$$
- <u>Work-Energy Theorem</u> state that: The total work done on an object is equal to its change in kinetic energy.

$$W_{total} = \Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

Problem 5

One method of determining the kinetic energy of neutrons in a beam, such as from a nuclear reactor, is to measure how long it takes a particle in the beam to pass two fixed points a known distance apart. This technique is known as the *time-of-flight* method. Suppose a neutron travels a distance of d= 6.2 m in a time of t= 160 μs.

What is its kinetic energy? The mass of a neutron is 1.67 X 10^{-27} kg.

- Solution:

The velocity of the particle is:

$$v = \frac{d}{t} = \frac{6.2\ \text{m}}{160 \times 10^{-6}\ \text{s}} = 3.88 \times 10^{4}\ \text{m/s}.$$

Then, the kinetic energy is:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(3.88 \times 10^{4}\ \text{m/s})^2$$
$$= 1.26 \times 10^{-18}\ \text{J} = 7.9\ \text{eV}.$$

Power

Power: The rate at which work is done

$$P = \frac{W}{t}$$

SI unit: J/s = watt, W

- If an object is moving at a constant speed in the face of friction, gravity, air resistance, and so forth, the power exerted by the driving force can be written:

$$P = \frac{Fd}{t} = F\left(\frac{d}{t}\right) = Fv$$

Problem 8

- An elevator has an empty weight of *5160N*. It carries a maximum load of *20* passengers from the ground floor to the *25th* floor in a time of *18 seconds*. Assuming the weight of a passenger to be *710 N* and the distance between floors to be *3.5 m*, what is the power needed for the elevator motor?
- Solution

The force that must be exerted is the total weight of the elevator and passengers, $F = 5160N + 20(710)N = 19400\ N$. The work that must be done is

$$W = Fs = (19.400\ \mathrm{N})(25 \times 3.5\ \mathrm{m}) = 1.7 \times 10^6\ \mathrm{J}.$$

The minimum power is therefore

$$P = \frac{W}{t} = \frac{1.7 \times 10^6\ \mathrm{J}}{18\ \mathrm{s}} = 94\ \mathrm{kW}.$$

Some Notes

- A Horsepower (hp) is a unit of power (in British system of unit), it is equivalent to $550\ ft.lbf/s\ =\ 745.6999\ Watt$.

- Kilowatt hour (kWh) is the unit of energy, which is commonly used as a billing unit of electric energy delivered to consumers.

- $1\ kWh\ =\ 1000\ \times 1\ W \times 60 \times 60\ (s) = 3.6 \times 10^6\ \ (J)$

Summary of Ch 7

The work done by a constant force **F** acting on a particle is defined as the product of the component of the force in the direction of the particle's displacement and the magnitude of the displacement. Given a force **F** that makes an angle θ with the displacement vector **d** of a particle acted on by the force, you should be able to determine the work done by **F** using the equation

$$W \equiv Fd\cos\theta \qquad \textbf{(7.1)}$$

The **kinetic energy** of a particle of mass m moving with a speed v (where v is small compared with the speed of light) is

$$K \equiv \tfrac{1}{2}mv^2 \qquad \textbf{(7.14)}$$

The **work–kinetic energy theorem** states that the net work done on a particle by external forces equals the change in kinetic energy of the particle:

$$\sum W = K_f - K_i = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 \qquad \textbf{(7.16)}$$

The **instantaneous power** $\mathcal{P}$ is defined as the time rate of energy transfer. If an agent applies a force **F** to an object moving with a velocity **v**, the power delivered by that agent is

$$\mathcal{P} \equiv \frac{dW}{dt} = \mathbf{F}\cdot\mathbf{v} \qquad \textbf{(7.18)}$$

Homework

- 1- To push a *52 kg* crate across a floor, a worker applies a force of *190 N* with direction *22°* below the horizontal. As the crate moves *3.3 m*, how much work is done on the crate by (a) the worker, (b) the force of gravity, (c) the normal force of floor on the crate
- 2- A spring has a force constant of *15.0 N/cm*. How much work is required to extend the spring *7.60 mm* from its relaxed position?
- 3- Calculate the kinetic energies of the following objects moving at the given speeds: (a) a 110 kg football player running at 8.1 m/s; (b) a 4.2 g bullet at 950 m/s.
- 4- A *57 kg* woman runs up stairs, having a rise of *4.5 m* in *3.5 s*. What is the average power must she supply.
- 5- Force F_1 does *5 J* of work in *10* seconds, Force F_2 does *3 J* of work in *5* seconds, Force F_3 does *6 J* of work in *18* seconds, and Force F_4 does *25 J* of work in *125* seconds, Sort these forces in order of increasing power they produce.