Chapter 44

Spherical Mirrors and Lenses

Units of Chapter 44

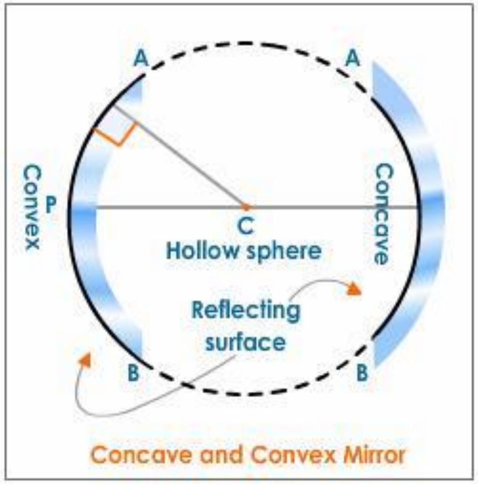
- SPHERICAL MIRROS
- SPHERICAL REFRACTING SURFACES *(optional)*
- THIN LENSES

Learning goals of this chapter

- **On completing this chapter, the student will be able to :**
- Define the following terms:
  - The focal length of a spherical mirror.
  - The focal length of a lens.
  - The equation of mirrors.
  - The equation of lenses.
  - Concave and convex mirrors.
  - Concave and convex lenses.
  - The image formed by mirrors and lenses.

SPHERICAL MIRROS

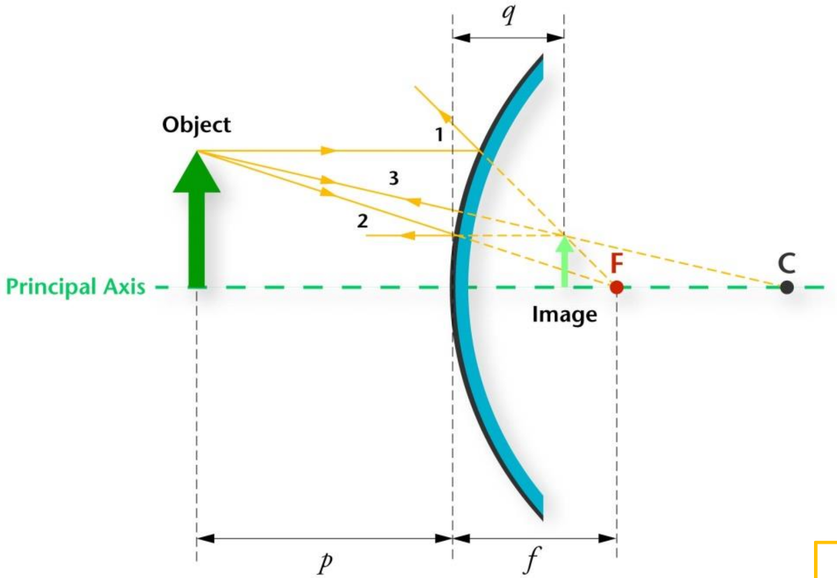
- Spherical Mirrors are either **Concave or Convex** mirrors
- Both mirrors have
  - principal axis,
  - Radius of curvature *R*,
  - focal length $f = R/2$



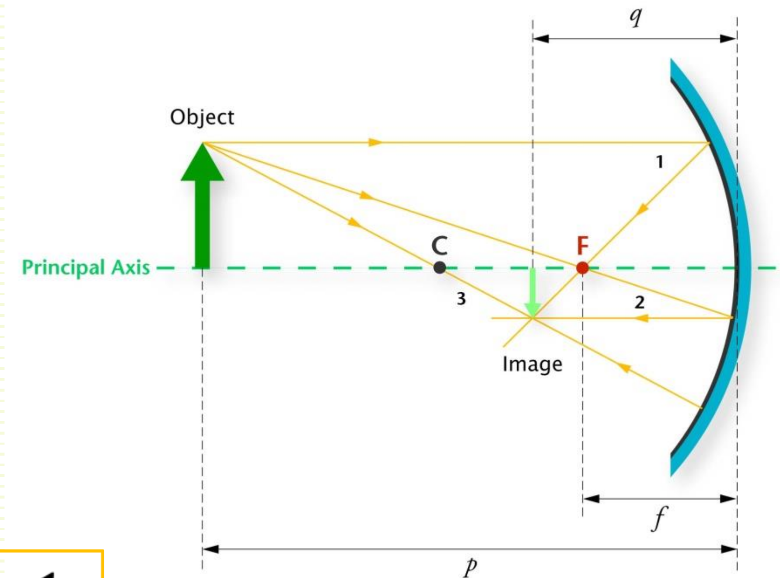
- Image is real if it is formed in front of the mirror, and virtual if it is formed behind the mirror.
- Concave mirrors produce ***real*** images
- Convex mirrors produce ***virtual*** images

SPHERICAL MIRROS

- **Convex Mirrors**



- **Concave Mirrors**



- **Mirrors Equation is:**

$$\boxed{\frac{1}{o} + \frac{1}{i} = \frac{1}{f}}$$

- ***o*** is the distance from the object to the mirror
- ***i*** is the distance from the image to the mirror
- ***f*** is the focal length.

SPHERICAL MIRROS

- **Magnification** $$M = \frac{image\ height}{object\ height} = -\frac{i}{o}$$

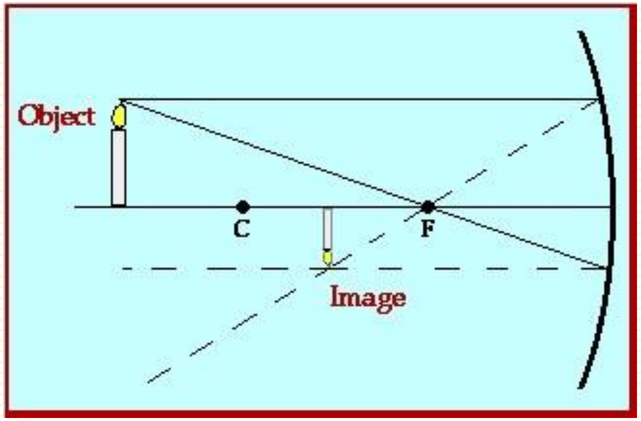
  - $M > 0$, if image is upright with respect to the object
  - $M < 0$, if image is inverted with respect to the object

- **Sign Convention**
  - o is always positive (always in front of the mirror)
  - i is positive if the image is real, and negative if the image is virtual.
  - f is positive for concave mirrors and negative for convex mirrors.

SPHERICAL MIRROS

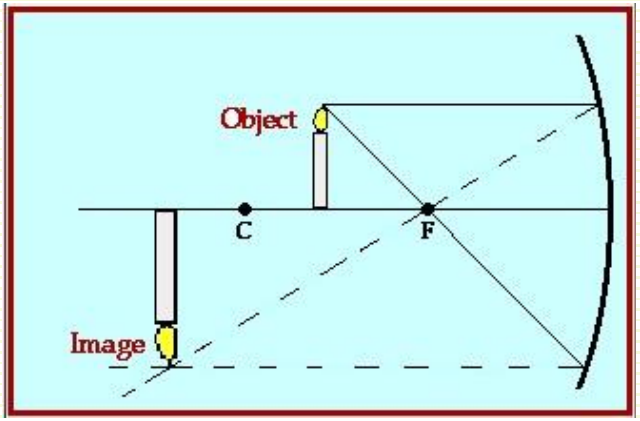
Ray Diagrams

To predict the image formed, draw two rays from the tip of the object as shown. The crossing of the reflected rays is the image.



1- Object beyond C

The image is located between C and F. It is a real, inverted image that is smaller in size than the object

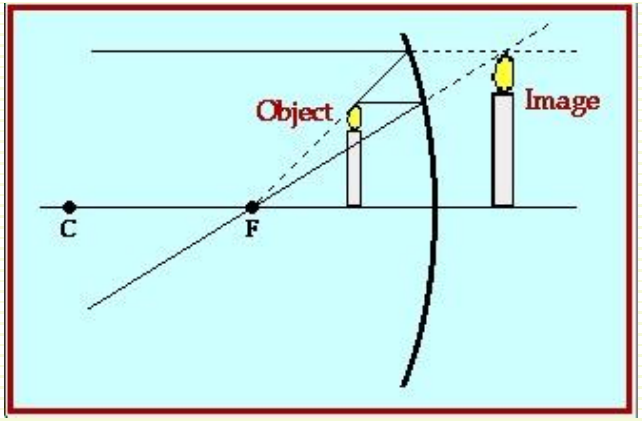


2- Object between C and F

The image is located beyond C. It is a real, inverted image that is larger in size than the object.

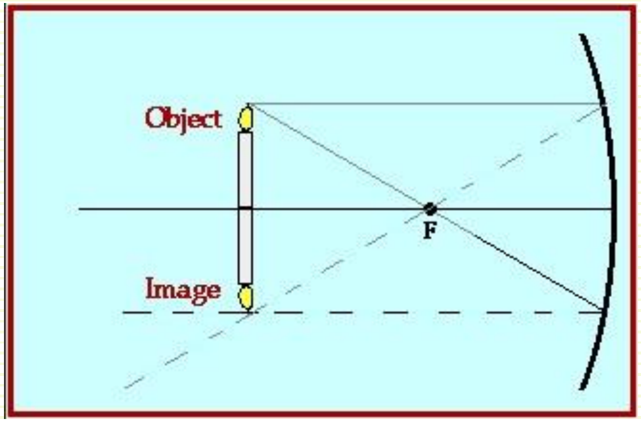
SPHERICAL MIRROS

Ray Diagrams



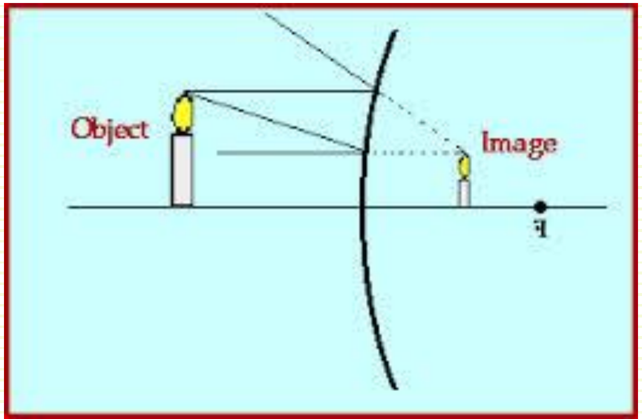
3- Object in front of F

The image is located behind the mirror. It is a virtual, upright image that is larger in size than the object.



4- Object at C

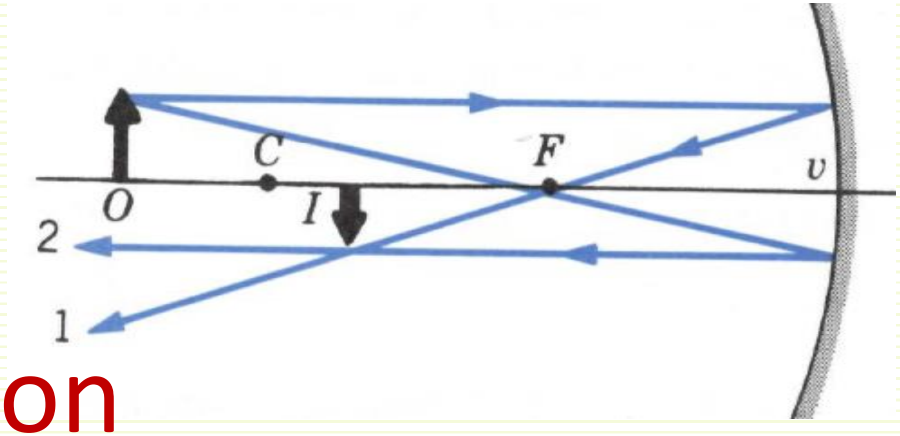
The image will be formed at C also, but it will be inverted. It will be real and the same size as the object.



In <u>convex mirrors</u>, the image of an object is always virtual, upright, and smaller in size.

Problem 1

In the figure, suppose f =12 cm and o = 30 cm. Find the position of the image and the magnification.



Solution

Solving Eq. 1 for $1/i$, we obtain

$$\frac{1}{i} = \frac{1}{f} - \frac{1}{o} = \frac{1}{12\text{ cm}} - \frac{1}{30\text{ cm}},$$

$$i = 20\text{ cm}.$$

$$m = -\frac{i}{o} = -\frac{20\text{ cm}}{30\text{ cm}} = -0.67.$$

Problem 2

A convex mirror has a radius of curvature of 22 cm. An object is placed 14 cm from the mirror. Locate and describe the image using (a) graphical and (b) algebraic methods.

Solution

(a) The image is virtual, upright and smaller.

(b)

$$\frac{1}{o}+\frac{1}{i}=\frac{2}{r}$$

$$\frac{1}{+14\text{ cm}}+\frac{1}{i}=\frac{2}{-22\text{ cm}},$$

$$i=-6.2\text{ cm.}$$

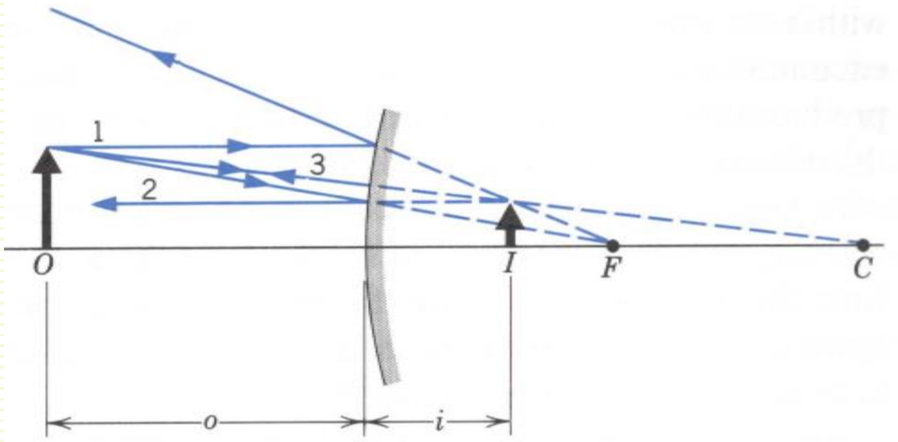


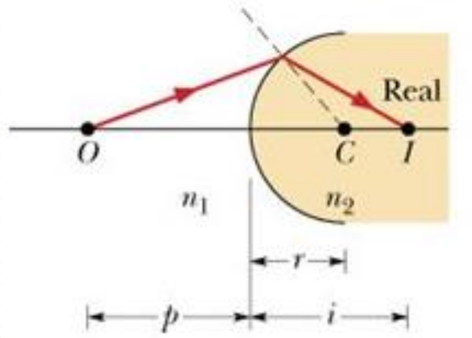
Image is virtual (negative) and smaller

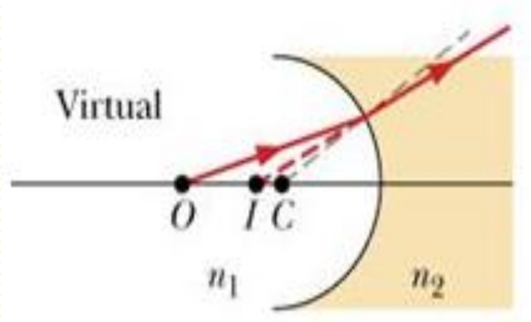
$$m=-\frac{i}{o}=-\frac{-6.2\text{ cm}}{+14\text{ cm}}=+0.44,$$

Image is upright (m>0)

SPHERICAL REFRACTING SURFACES

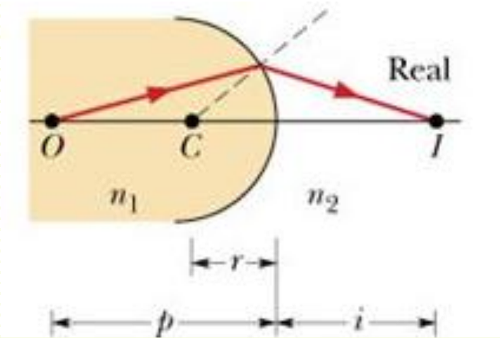
If light from O in a medium whose refractive index n_1, (e.g. air) falls on a spherical refracting surface of radius of curvature r, and refractive index n_2 (e.g. glass) such that $n_1 < n_2$





If the surface is convex, real image I will be formed inside the glass

If the surface is concave ,virtual image I will be formed outside the glass



If the surface is concave with respect to the light, but in this case $n_1 > n_2$, image I will be formed outside the glass

SPHERICAL REFRACTING SURFACES

In these cases, the image distance i, object distance o, radius of curvature r, and the two refractive indices n_1, n_2 are related by:

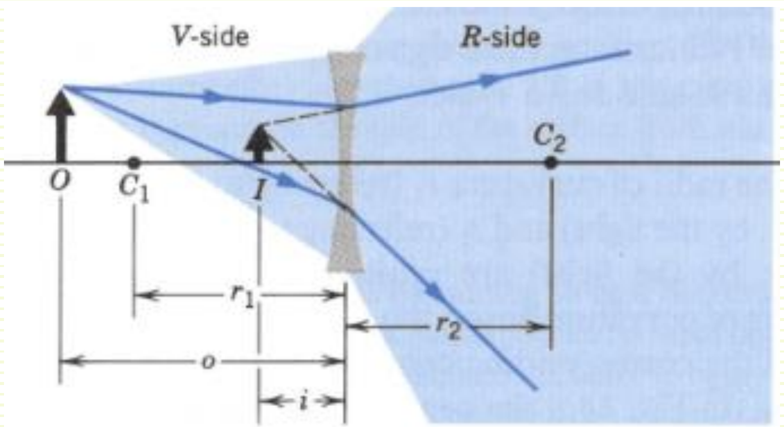
$$\frac{n_1}{o} + \frac{n_2}{i} = \frac{n_2 - n_1}{r}$$

Sign Convention

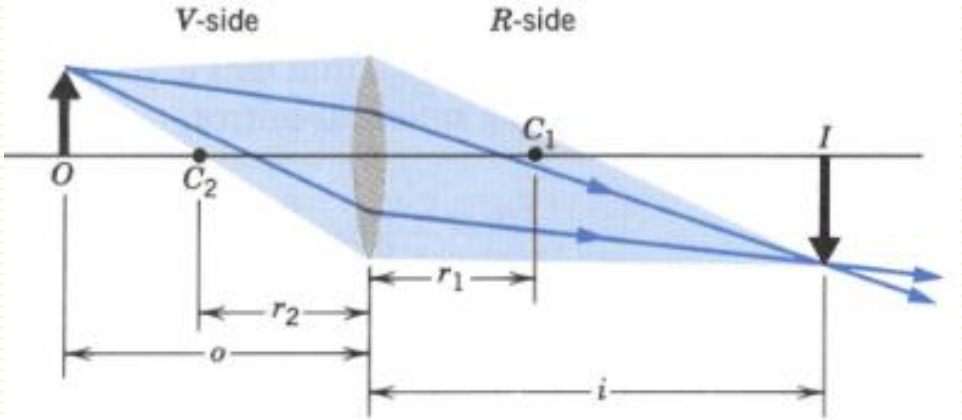
- Real images appear on the opposite side of the incident light
- Virtual images appear on the same side as the incident light
- r is positive if the curvature is on the object's side
- r is negative if the curvature is on the image's side or the glass side.

THIN LENSES

Consider a lens having an index of refraction n and two spherical surfaces with radii of curvature r_1 and r_2



Converging lens



Diverging lens

The lens formula is $$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$$

where $$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

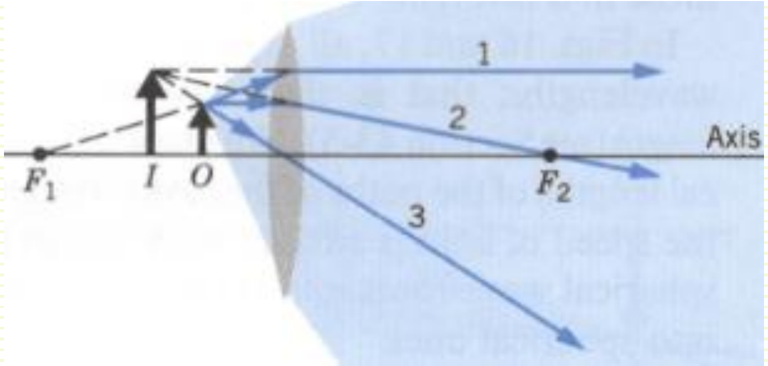
THIN LENSES

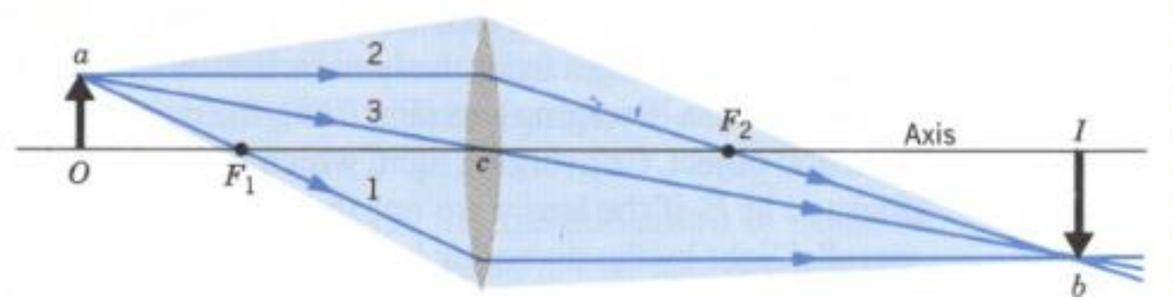
- Sign Convention
- For each surface of the lens
  - r is positive if the center of curvature is on the R-side
  - r is negative if the center of curvature is on the V-side
- The object distance *o* is positive if the object is real and lies on the V-side of the lens
- The image distance i is positive if the image is on the R-side
- The image distance i is negative if the image is on the V-side
- Magnification is negative when both i and o are positive.

THIN LENSES

Ray Diagrams

To predict the image formed, draw three rays from the tip of the object as shown.
The crossing of the three rays after passing through the lens is the image.

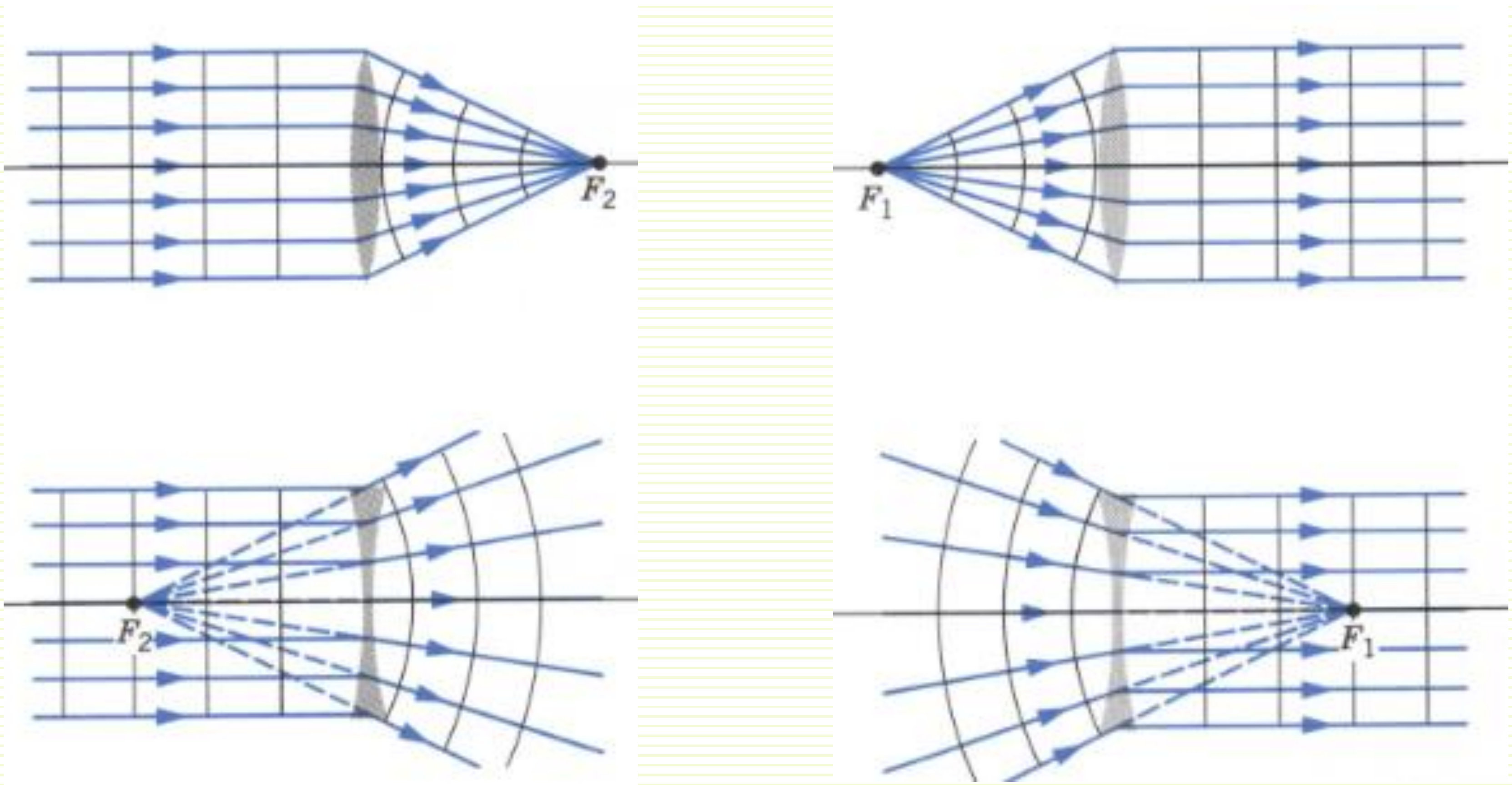




THIN LENSES

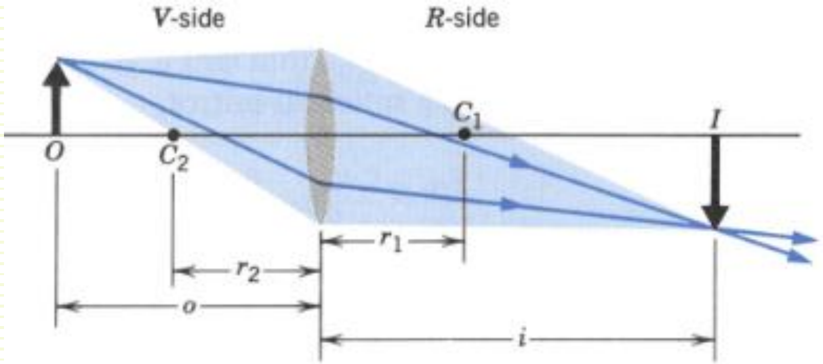
Ray Diagrams

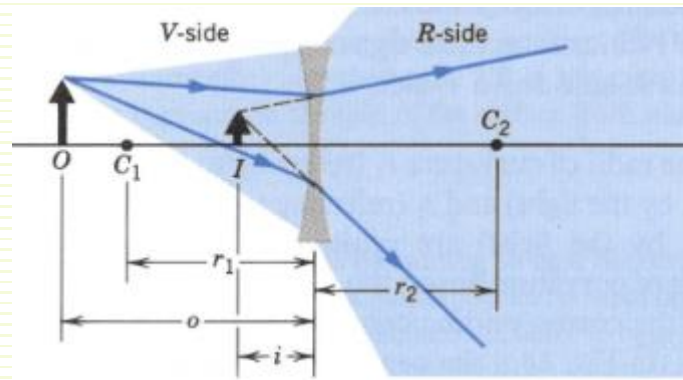
Special cases will occur if parallel light rays are incident on the lenses, or goes of the focus of the lens as shown.



Problem 5

The lenses in the figures have radii of curvature of 42 cm and are made of glass with n= 1.65. Compute their focal lengths.





Solution

r_1 is positive, r_2 is negative

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= (1.65 - 1)\left(\frac{1}{+42\text{ cm}} - \frac{1}{-42\text{ cm}}\right)$$

$$f = +32\text{ cm}.$$

r_1 is negative, r_2 is positive

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= (1.65 - 1)\left(\frac{1}{-42\text{ cm}} - \frac{1}{+42\text{ cm}}\right)$$

$$f = -32\text{ cm}.$$

Problem 6

An object is 38 cm in front of a diverging lens of focal length -24 cm. Find the location and magnification of the image using (a) graphical and (b) algebraic techniques.

Solution

$$\frac{1}{o}+\frac{1}{i}=\frac{1}{f}$$

$$\frac{1}{+38\text{ cm}}+\frac{1}{i}=\frac{1}{-24\text{ cm}}$$

$$i=-15\text{ cm},$$

$$m=-\frac{i}{o}=-\frac{-15\text{ cm}}{+38\text{ cm}}=+0.39,$$