Chapter 17

Fluid Statics

Units of Chapter 17: Fluid Statics

- FLUIDS AND SOLIDS
- PRESSURE AND DENSITY
- VARIATIOIN OF PRESSURE IN A FLUID AT REST
- PASCAL’S PRINCIPLE AND ARCHIMEDES’ PRINCIPLE
- MEASUREMENT OF PRESSURE

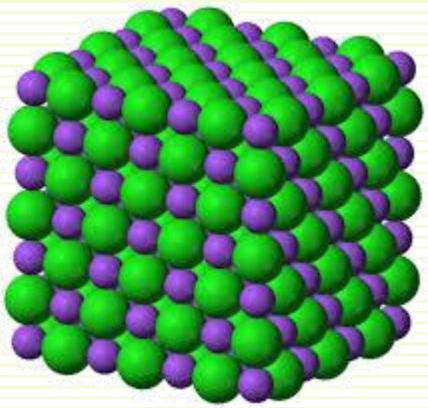
Learning goals of this chapter

- **On completing this chapter, the student will be able to :**
- Differentiate between Solid and liquid
- Define the density, the pressure and their units
- Define Pascal’s Principle.
- Define Archimedes’ Principle
- Proof Archimedes’ Principle
- Apply both Pascal’s and Archimedes’ principles to solve problems
- Recognize the principle of measuring the pressure.
- Calculate the pressure at any depth below the surface of the liquid.
- Convert the unit of the pressure from system to another.

Fluids and Solids

- A solid is a three dimensional object. It is completely bounded by a surface and take its own fixed shape. The bonding force between the molecules of solids are high enough to make the material to be rigid and possess a definite shape.
- A fluid is a substance that can flow from place to place and take the shape of the container rather than retain a shape of their own. The bonding force between the molecules is not high enough, so they can flow.
- Fluids could be:

  1- Liquids. 2- Gases.

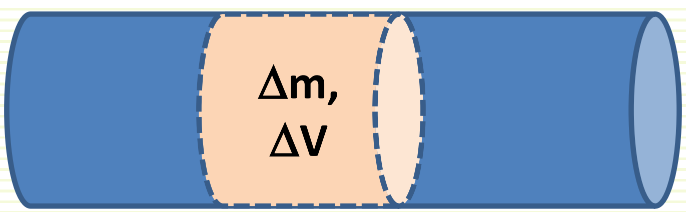


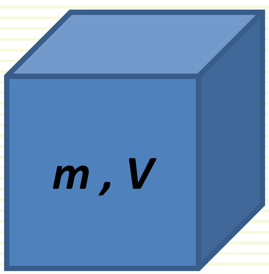
Density and Pressure

- Density: ρ
- The density of the material, ρ, is its mass per unit volume.

$$\rho = \frac{m}{V} = \frac{\Delta m}{\Delta V}$$

- Where m or Δm is the mass and V or ΔV is the volume of the material.
- its SI unit is the kilogram per cubic meter (kg/m^3)
- In cgs system of unit (centimeter, gram, second system) the unit of density is g/cm^3.





Densities of Common Substances

Substance	Density (kg/m^3)
Gold	19,300
Mercury	13,600
Lead	11,300
Silver	10,500
Iron	7860
Aluminum	2700
Ebony (wood)	1220
Ethylene glycol (antifreeze)	1114
Whole blood (37 °C)	1060
Seawater	1025
Freshwater	1000
Olive oil	920
Ice	917
Ethyl alcohol	806
Cherry (wood)	800
Balsa (wood)	120
Styrofoam	100
Oxygen	1.43
Air	1.29
Helium	0.179

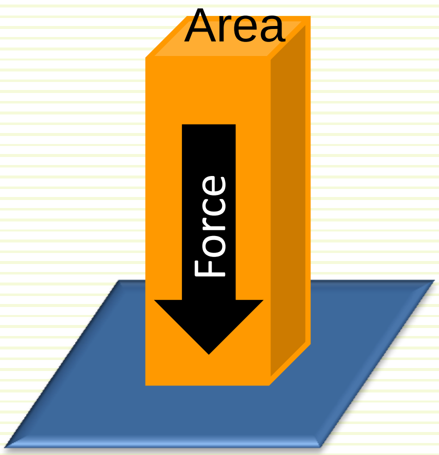
Density and Pressure

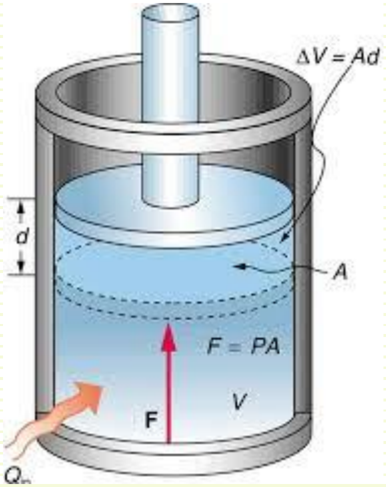
- **Pressure**: P
- It is the applied force per unit area

$$P = \frac{F}{A}$$

$$F = P\,A$$

- Where F is the magnitude of the normal force on area A.





Density and Pressure

Units of pressure

- In SI unit, the unit of Pressure is Pascal (Pa) = N/m^2 .
- 1 atmosphere (atm) is the approximate average pressure of the atmosphere at sea level. It is the weight of the atmosphere above per 1 m^2. The value of 1 atm depends on the length of mercury column of the barometer at the sea level.
- 1 atm = 1.01×10^5 Pa = 101 k Pa = 760 torr = 760 mm Hg = 14.7 (psi)
- The *torr* (named for Evangelista Torricelli, who invented the mercury barometer in 1674) was formerly called the *millimeter of mercury* (mm Hg): 1 torr = 1 mm Hg.
- One bar = 10^5 Pa ~ 1 P_{at}
- The pound-force per square inch is often abbreviated psi. 1psi = 6895 Pa

Density and Pressure

Units of pressure

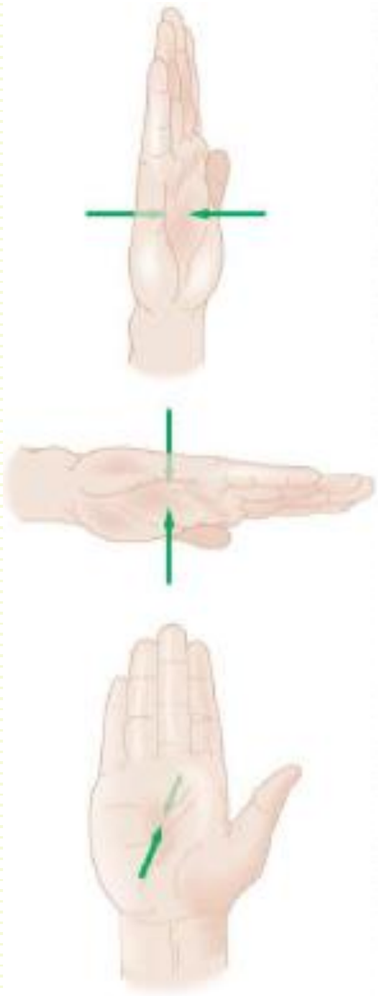
- Example: Converting of the units of the pressure from FPS (Foot-Pound-Second) system of unit to SI unit. :
- The pound-force is equal to the gravitational force exerted on a mass of one avoirdupois pound on the surface of Earth. Therefore
- $1\ lbf\ =\ 1\ lb\ (pound - mass) \times g\left(9.80665\,\frac{m}{s^2} \times 0.3048\,\frac{m}{ft}\right)$

  $= 32.174\ \frac{ft.lb}{s^2}$
- 1 slug = the mass equal 32.174 pound.
- $1\ lbf\ =\ 1\ lb\ (pound - mass\ =\ 0.45359237\ kg) \times g(9.80665\ m/s^2)$
- $1\ lbf\ =\ 4.448\ N$
- $1\ psi\ = \frac{1\ lbf}{in^2} = \frac{4.448\ N}{(0.0254)^2} = 6894.4\ Pa$
- But $1\ atm\ =\ 1.01 \times 10^5\ Pa,$ then $1\ Pa\ =\ 9.869{\times}10^{-6}\ atm.$
- Then $1\ psi\ =\ 6894.4 \times 9.869{\times}10^{-6}\ atm\ = 0.068\ atm$.
- Or $1\ atm\ = \frac{1}{0.068} = 14.6959\ psi$

Variation of pressure in a Fluid at rest

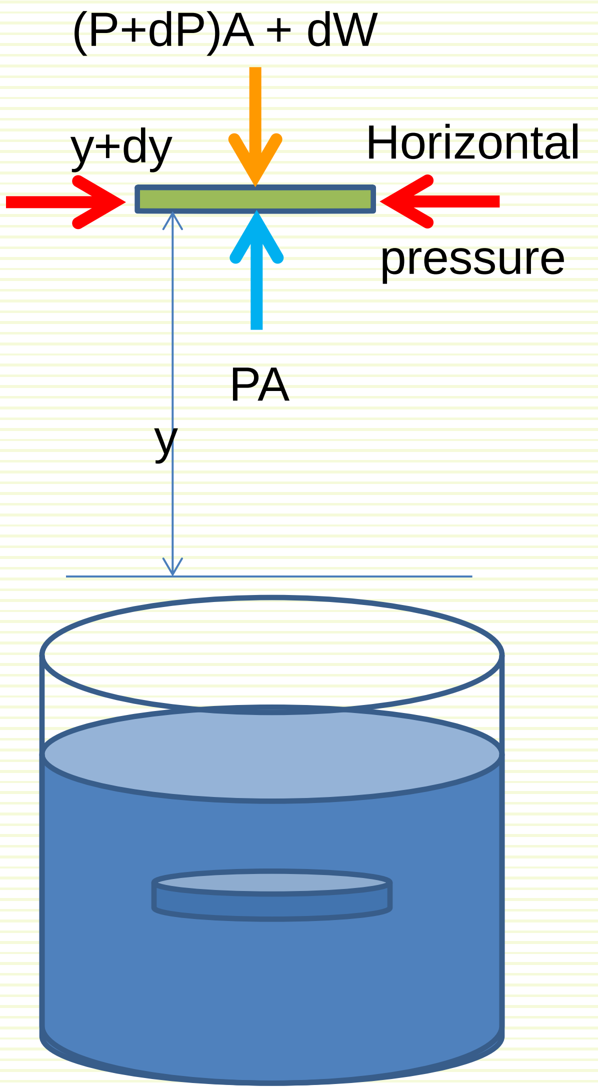
- It is important to know that the pressure in fluid acts equally in all directions, and acts at right angles to any surface, we don't usually notice it.

	Pressure (Pa)
Center of the sun	2×10^{16}
Center of Earth	4×10^{11}
Deepest Ocean trench (bottom)	1.1×10^{8}
Automobile tire	$2 \times 10^{5} + P_{at}$
Best Lab. vacuum	10^{-12}



Variation of pressure in a Fluid at rest

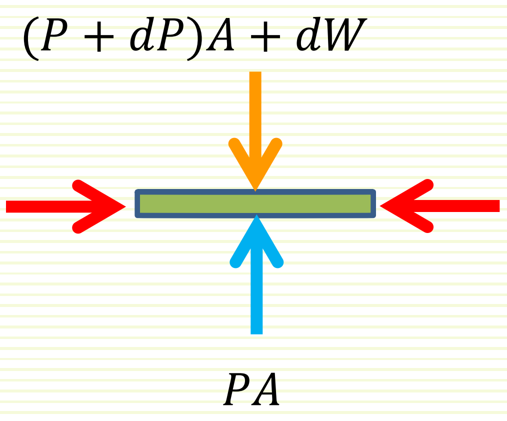
- If the liquid is at rest and equilibrium, then the net force on an element of disk shape (with surface area A and thickness dy) is zero.
- Let the mass of the element is $dm = \rho dV = \rho A dy$,
- and the weight = $dW = \rho\, A\, g\, dy$
- The horizontal forces are due to pressured of the liquid and it is zero due to the symmetry of the pressure.
- The vertical forces are also zero. they are due to the pressure of the liquid on both side of the disk and its weight



Variation of pressure in a Fluid at rest

- Upward force ↑ $= PA$
- Downward force ↓ $= (P + dP)A + \rho A g dy$
- By equating these forces, we get
- $PA = (P + dP)A + \rho A g dy$

$$\Rightarrow \frac{dp}{dy} = -\rho g$$



- This means that the pressure vary linearly with the distance y and decreases with increasing y.
- By Integration from y_1 where $P = P_1$ to y_2 where $P = P_2$
- If P_1 is the pressure at elevation y_1, and P_2 at elevation y_2, then
- $\int_{P_1}^{P_2} dp = -\int_{y_1}^{y_2} \rho g dy$
- $P_2 - P_1 = -\rho g (y_2 - y_1)$

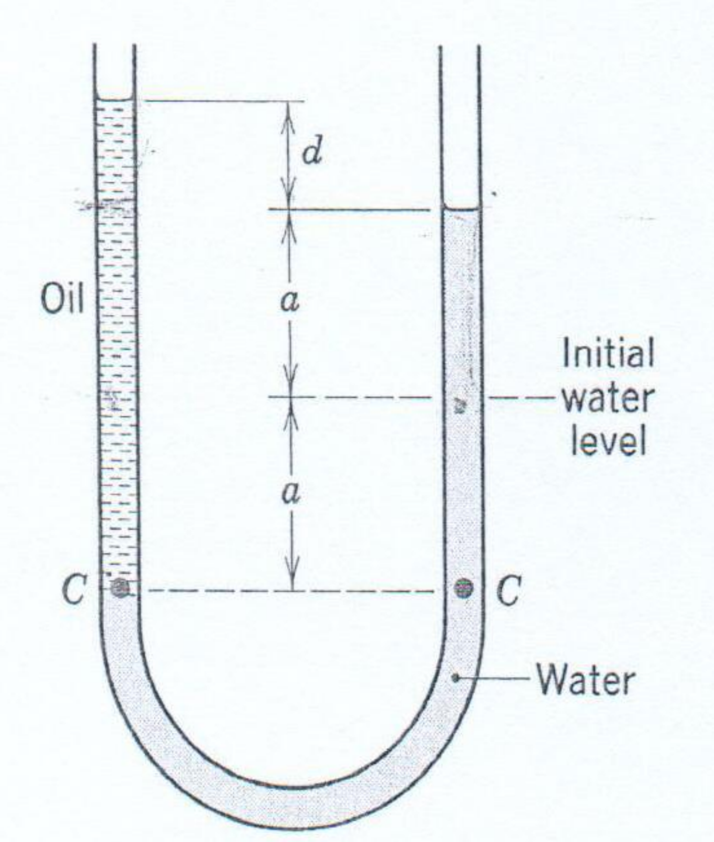
Variation of pressure in a Fluid at rest

- If y_2 was taken at the elevation of the surface, at which $P_2 = Po$ = the atmospheric pressure, then at y_1 to be at any level of the liquid, we find
- $P_o - P_1 = -\rho g(y_2 - y_1)$

- Let $y_2 - y_1 = h$ = the depth below the surface of the liquid, then the pressure
- $P = P_1 = P_o + \rho g(y_2 - y_1)$ (*)

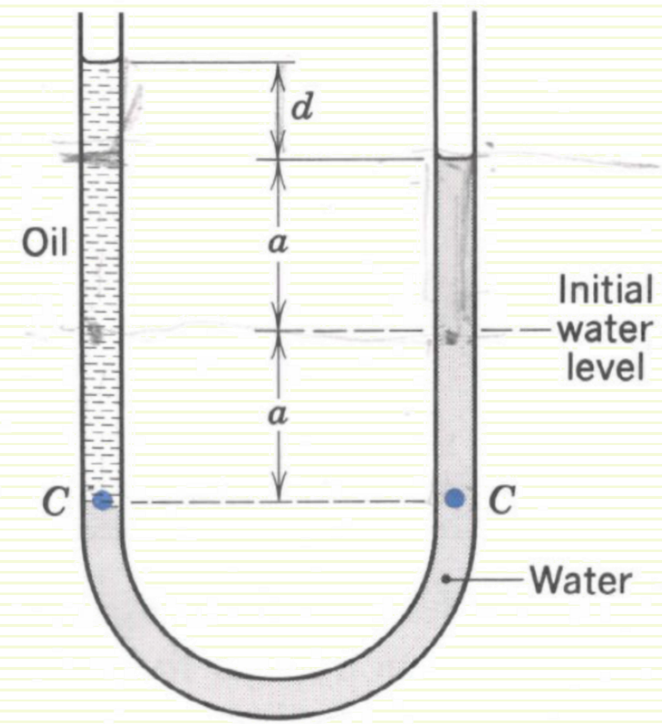
Problem 1

- A U-tube, in which both ends are open to the atmosphere, is partially filled with water. Oil, which does not mix with water is poured into one side of the tube until it stands a distance d=12.3 mm above the water level on the other side which has meanwhile risen a distance a=67.5 mm from its original level. Find the density of the oil.



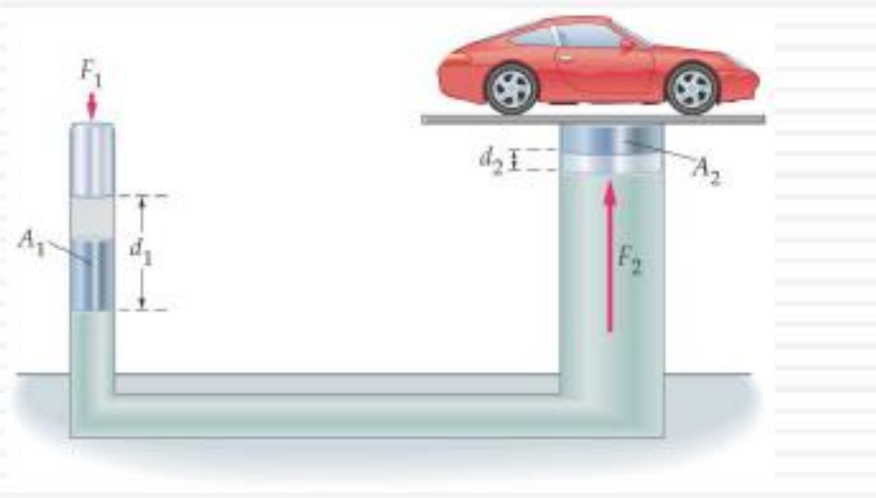
Problem 1

- Solution
- At point C, the pressure is the same at both terminal of the tube.
- So : $\rho_w g 2a + P_o = \rho g(2a+d) + P_o$
- $\rho = \rho_w 2a/(2a+d)$
- $= \rho_w = 1.000\times10^3\ kg/m^3$
- $2a = 2\times67.5\ mm = 135\ mm$
- $2a+d = 135mm + 12.3\ mm = 147.3\ mm$
- $\rho = (1.000\times10^3\ kg/m^3)(135\ mm)/(147.3mm)$
- $\rho = 916\ kg/m^3$



Pascal's Principle and Archimedes' Principle

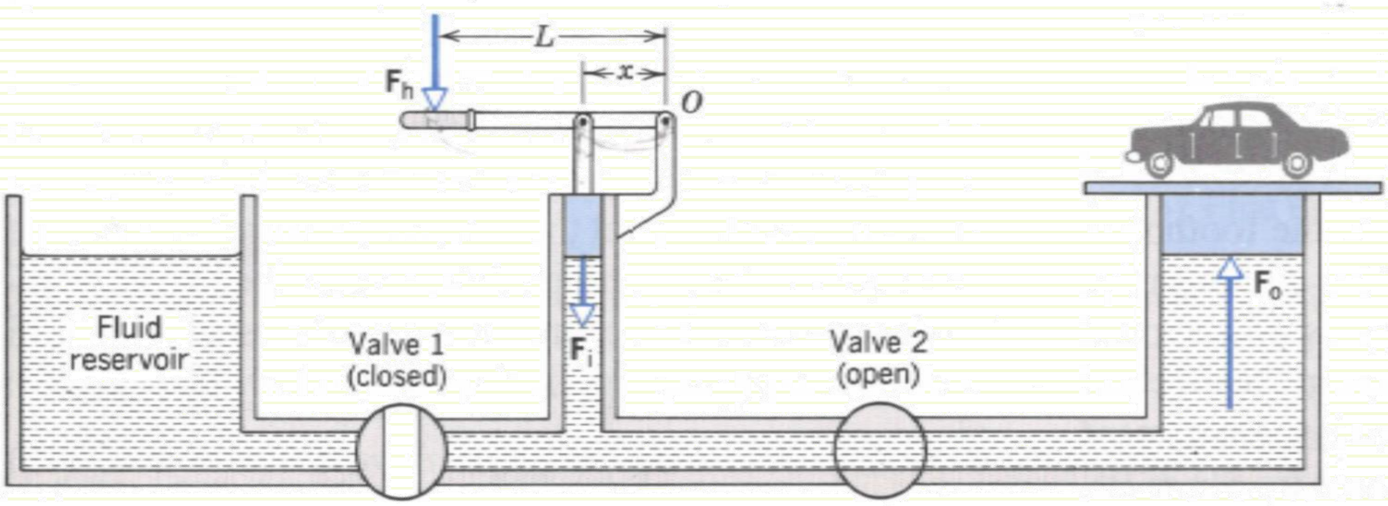
- Pascal's Principle:
- An external pressure applied to an enclosed fluid is transmitted unchanged to every point within the fluid.
- $\Delta P = \frac{F_1}{A_1}$
- $F_2 = \Delta P . A_2$
- $F_2 = \left(\frac{F_1}{A_1}\right) A_2 = \left(\frac{A_2}{A_1}\right) F_1 \;>\; F_1$
- Note that, since $V_1 = V_2$
- $A_1 d_1 = A_2 d_2$
- Pascal principle is the basis for the operation of all hydraulic force-transmitting mechanisms.



Hydraulic Lift

Problem 2

- The figure shows a schematic view of a hydraulic jack used to lift an automobile. The hydraulic fluid is oil (density = 812 kg/m^3). A hand pump is used in which a force of magnitude F_i is applied to the smaller piston (of diameter 2.2 cm) when the hand applies a force of magnitude F_h to the end of the pump handle. The combined mass of the car to be lifted and the lifting platform is M =1980 kg, and the large piston has a diameter of 16.4 cm. The length L of the pump handle is 36 cm, and the distance x from the pivot to the piston is 9.4 cm. what is the applied force F_h needed to lift the car?



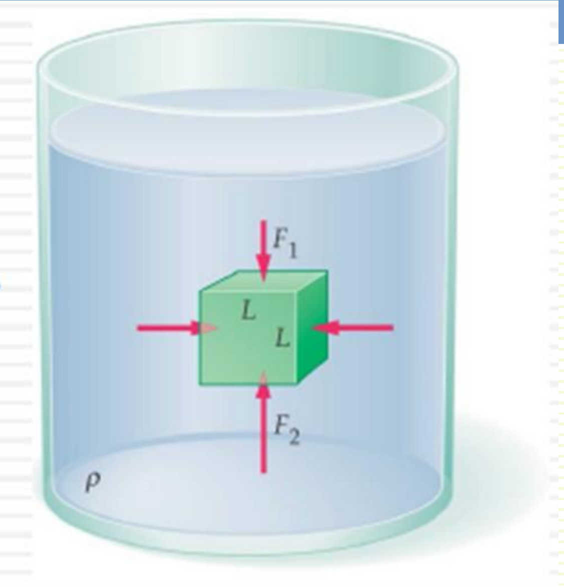
Problem 2

- Solution:
- Since $\frac{F_1}{A_1} = \frac{F_2}{A_2}, then$
- $F_i = Mg\frac{A_i}{A_o} = (1980\ kg)\left(\frac{9.8m}{s^2}\right) \times \frac{\pi(1.1cm)^2}{\pi(8.2cm)^2} = 349\ N$
- From the figure we can write $F_h L = F_i x$ (Newton's third law).
- $F_b = F_i\frac{x}{L} = (349\ N)\frac{9.4\ cm}{36\ cm} = 91\ N$

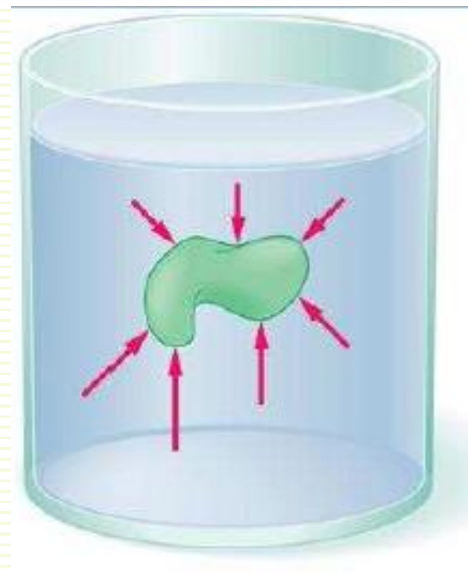
Pascal’s Principle and Archimedes’ Principle

- Buoyant Force:
- A fluid exerts a net upward force on any object it surrounds, called the buoyant force. The buoyant force is due to the increased pressure at the bottom of the object compared to the top.

$$F_b = F_2 - F_1$$



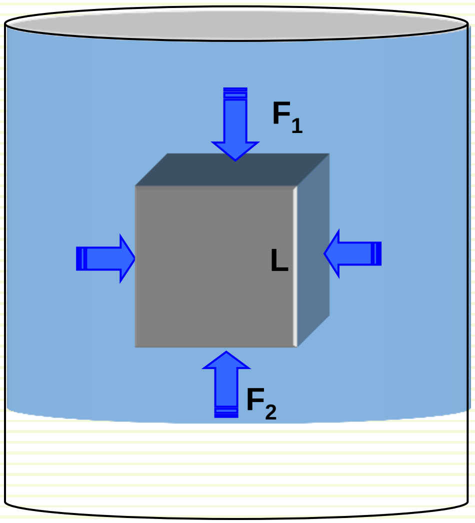
- Archimedes’ Principle:
- An object wholly or partially immersed in a fluid is buoyed up by a force equal in magnitude to the weight of liquid displaced by the object.



Pascal’s Principle and Archimedes’ Principle

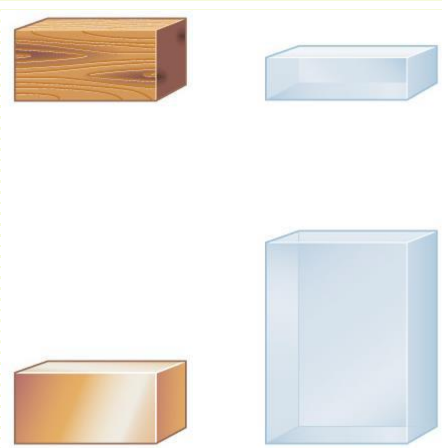
Proof of Archimedes’ Principle

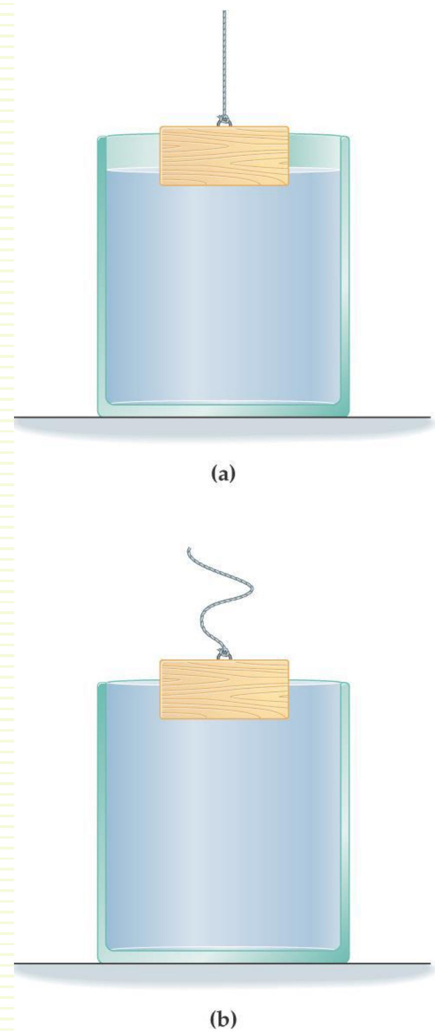
- Let a cube of length L and density ρ_s is immersed in a liquid with density ρ.
- Let the pressure act on the top surface to be P_1, where the force acting on this surface due to the pressure is $F_1 = P_1A = P_1L^2$
- Let the pressure acting on the lower surface to be P_2, then the force acting on this surface is $F_2 = P_2A = P_2L^2$.
- But the pressure $P_2 = P_1 + \rho gL$
- Then $F_2 = P_2L^2 = P_1L^2 + \rho gL^3$.
- Or $F_2 = F_1 + \rho gL^3$
- Then the buoyant force acting on the cube.
- $F_b = F_2 - F_1$.
- Or $F_b = \rho gL^3 = \rho gV$



Pascal's Principle and Archimedes' Principle

- An object floats when it displaces an amount of fluid equal to its weight.
- An object made of material that is denser than water can float only if it has indentations or pockets of air that make its average density less than that of water.





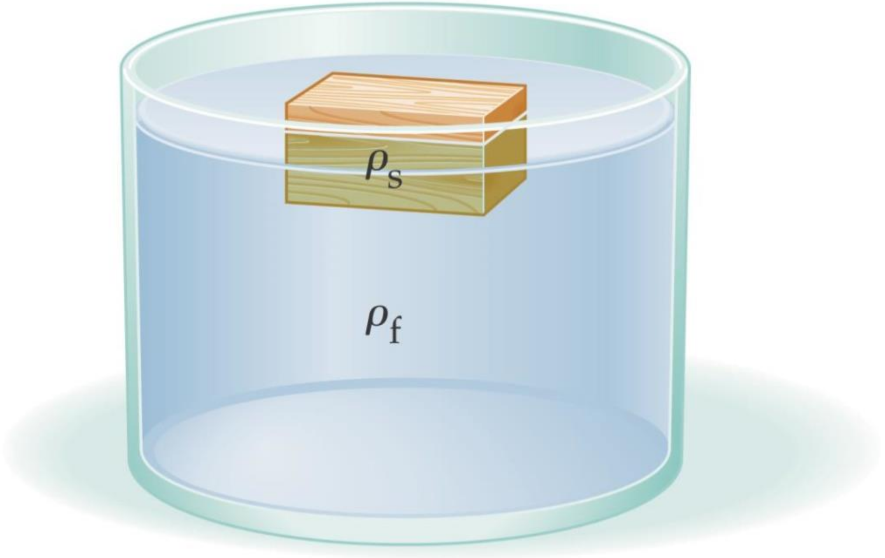
Pascal's Principle and Archimedes' Principle

- The fraction of an object that is submerged when it is floating depends on the densities of the object and of the fluid

Submerged Volume V_{sub} for a Solid of Volume V_s and Density ρ_s Floating in a Fluid of Density ρ_f

$$V_{sub} = V_s(\rho_s/\rho_f)$$

SI unit: m^3

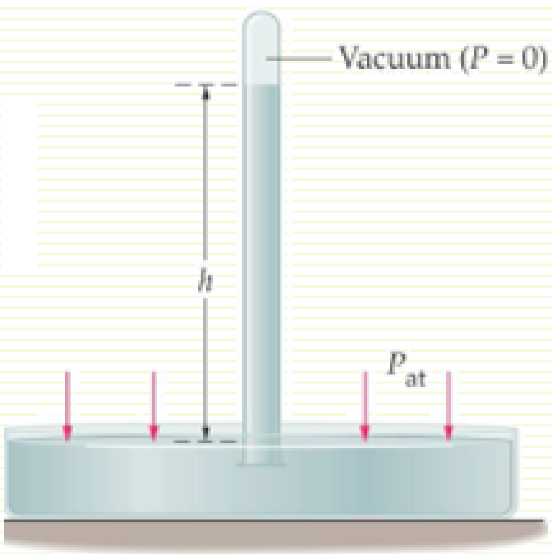


Problem 3

- What fraction of the total volume of an iceberg is exposed?
- Solution:
- The weight of the iceberg is
- $W_{ice} = \rho_{ice} V_{ice}\, g$
- The weight of the volume V_w of seawater displaced is the buoyant force
- $F_b = \rho_w V_w\, g$
- But $F_b = W_{ice}$ because the iceberg in equilibrium, so that
- $\rho_{ice} V_{ice}\, g = \rho_w V_w\, g$
- Then
- $\frac{V_w}{V_{ice}} = \frac{\rho_{ice}}{\rho_w} = \frac{917\frac{kg}{m^3}}{1024\frac{kg}{m^3}} = 0.896 = 89.6\ \%$
- The volume of water displaced V_w is the volume of the submerged portion of the iceberg, so that 10.4% of the iceberg will be in air.

Measurement of pressure

- Most pressure gauges use atmospheric pressure as a reference level and measure the difference between the actual pressure and atmospheric pressure.
- The actual pressure at a point is called absolute pressure.
- A barometer compares the pressure due to the atmosphere to the pressure due to a column of fluid typically mercury Hg.
- This leads to the definition of atmospheric pressure in terms of millimeters of mercury:
- 1 atmosphere $= Pat = 760\, mmHg$

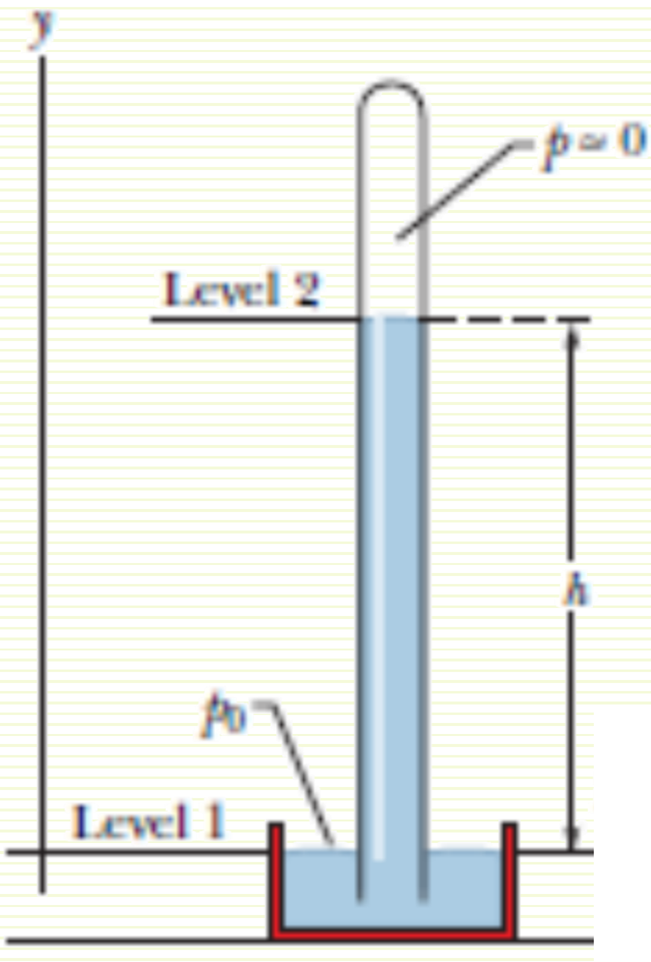


Measurement of pressure

- Barometer is a long glass tube that has been filled with mercury and inverted intro a dish of mercury. It is used to measure the atmospheric pressure.
- To find the atmospheric pressure P_o in terms of the height *h* of the mercury column. We choose level 1 to be that of the air–mercury interface and level 2 to be that of the top of the mercury column, as labeled in the figure.
- then substitute in $P_2 - P_1 = -\rho g(y_2 - y_1)$
- $P_1 = Po\,,\ h = y_2 - y_1\,, P_2 = 0,$
- then $P_2 - P_1 = 0 - Po$
- Then the atmospheric pressure is given by:

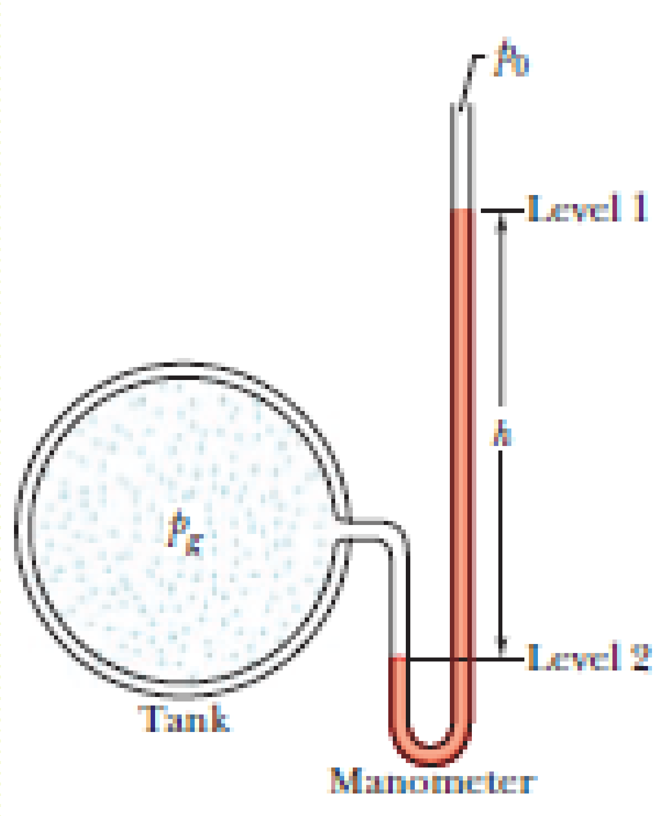
$$P_o = \rho g h$$

- At sea level, the height of the mercury is 760 mm.
- Therefore
  $P_o = 13.6 \times 103\ (kg/m^3) \times 9.8\ (mls^2) \times 0.76\ (m)$
  $= 1.013 \times 105\ N/m^2 = 1.013 \times 105\ Pa = 1\ atm.$
- $1\ torr = 1\ mm\ Hg$
  $= 13.6 \times 103\ (kg/m^3) \times 9.8\ (mls^2) \times 0.001(m)$
  $= 133.3\ Pa$



Measurement of pressure

- Open tube Manometer
- It measures the pressure P_g of a gas. It consists of a U-tube containing a liquid, with one end of the tube connected to the vessel whose gauge pressure we wish to measure and the other end open to the atmosphere.
- Let us choose levels 1 and 2 as shown which are a measure of the pressure, then the pressure of the gas is
- $P - Po = \rho g h.$



Problem 4

- The mercury column in a barometer has a measured height h of 740.35 mm. the temperature is – 5.0 ^{o}C, at which temperature the density of mercury is 1.3608×10^4 kg/m^3. the free-fall acceleration g at the site of the barometer is 9.7835 m/s^2. What is the atmospheric pressure?
- Solution
- $P_o = 1.3608 \times 104\ (kg/m^3) \times 9.7835\ (mls^2) \times 0.74035\ (m)$
$= 9.8566 \times 104\ Pa = 739.29\ torr.$

Summary of Chapter 17

- Density: $\rho = \frac{M}{V}$
- Pressure: $P = \frac{F}{A}$
- Atmospheric pressure: $P_{at} = 1.01 \times 10^5 \text{ N/m}^2 \approx 14.7 \text{ lb/in}^2$
- Gauge pressure: $P_g = P - P_{at}$
- Pressure with depth : $P_2 = P_1 + \rho g h$

Summary of Chapter 17

- Pascal's principle: An external pressure applied to an enclosed fluid is transmitted unchanged to every point within the fluid
- Archimedes' principle: An object completely immersed in a fluid experiences an upward buoyant force equal in magnitude to the weight of fluid displaced by the object.
- Volume of submerged part of object:

$$V_{sub} = V_s \left(\frac{\rho_s}{\rho_f} \right)$$

Homework

1. Find the pressure exerted on the skin of a balloon if you press with a force of 2.1 N using (a) your finger (area 10^{-4} m^2, (b) a needle (area = 2.5×10^{-7} m^2. (c) find the minimum force necessary to pop the balloon with the needle, given that the balloon pops with a pressure of 3.0×10^{-5} N/m^2.
2. Find the force exerted on the pam of your hand by atmospheric pressure. Assume your palm measures 0.080 m by 0.10 m.
3. What is the diameter of a basketball with a $7.0\text{x}10^4$ Pa gauge pressure is pushed down with a force of 22.0 N.
4. The Titanic was found in 1985 laying on the bottom of the North Atlantic at a depth of 2.5 miles What is the pressure at this depth?
5. A cubical box 2.00 cm on a side is completely immersed in a fluid. At the top of the box the pressure is 105.0 kPa; at the bottom the pressure is 106.8 kPa. What is the density of the fluid?
6. To inspect a 14.500 N car, it is raised with a hydraulic lift. If the radius of the small piston is 4.0 cm, and the radius of the large piston is 17 cm, find the force that must be exerted on the small piston to lift the car.