Denotational Mathematics on Cognitive Informatics with Applications to Computational Intelligence

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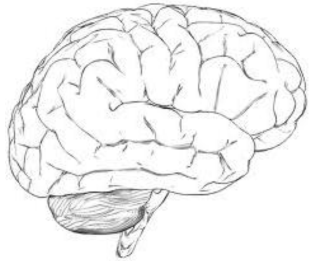
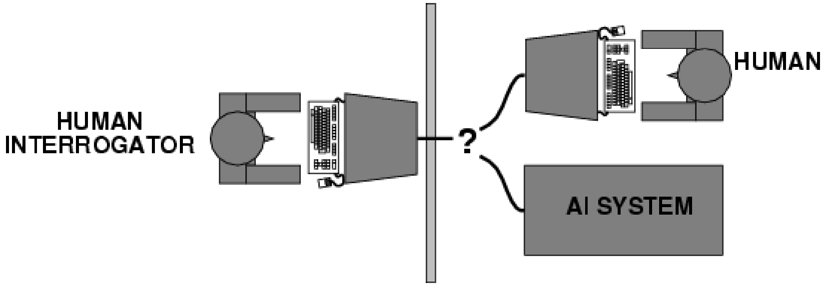
Agenda

- Introduction (Motivations, Aims and Contributions)
- Cognitive Science
- Denotational Mathematics(DM)
- Categorization in Cognitive Psychology
- A Computational Cognitive Model for Binary Categorization
- Experimental Results and Discussions



Intelligence

- Natural intelligence is the Ability to **e** **rule** and **store** it, then **retrieve** it whenev situation is **convenient** to apply that rule.
- The ultimate goal of **Artificial Intellige** give the machine some of those abilities.
- Turing (1950) "Computing machiner intelligence":
- "Can machines think?" → "Can ma behave intelligently?"



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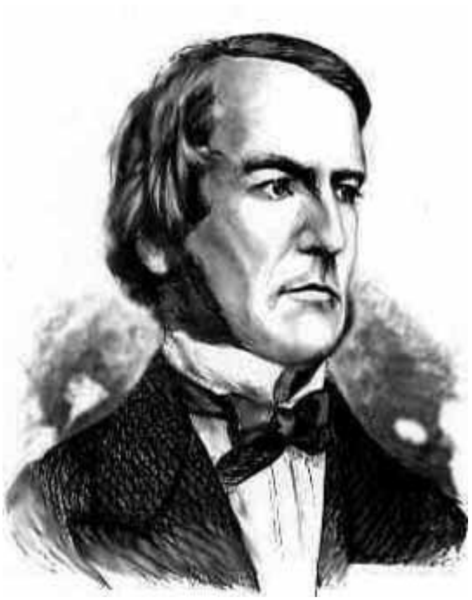
Motivation and Aims

- The rapid progress in cognitive science during the past decade is intimately linked to three exciting and particularly active areas of research:
  - computational and quantitative modeling of cognition,
  - advances in the neurosciences,
  - Bayesian techniques as a tool to describe human behavior and to analyze data
- Cognitive scientists aims to
  - understand how human mind works.
  - describe and predict people's behavior,
  - ultimately wish to explain it

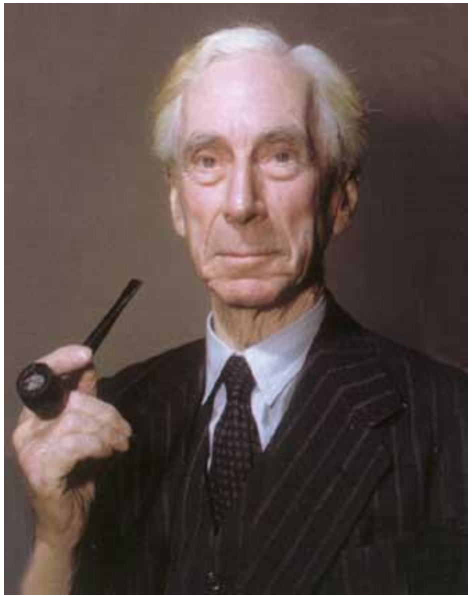
Contributions

- Studied cognitive phenomena such as
  - concept classification
  - knowledge representation
- Aiming to build computational cognitive models for both phenomena.
- This model is formulated by using a mathematical entity known as concept algebra.
- New semantic computational model based on cognitive aspects is proposed to enable a cognitive computer to process the knowledge as human mind and find the representation rules.

Pioneers in Computational Mathematics



George Boole



Bertrand Russell

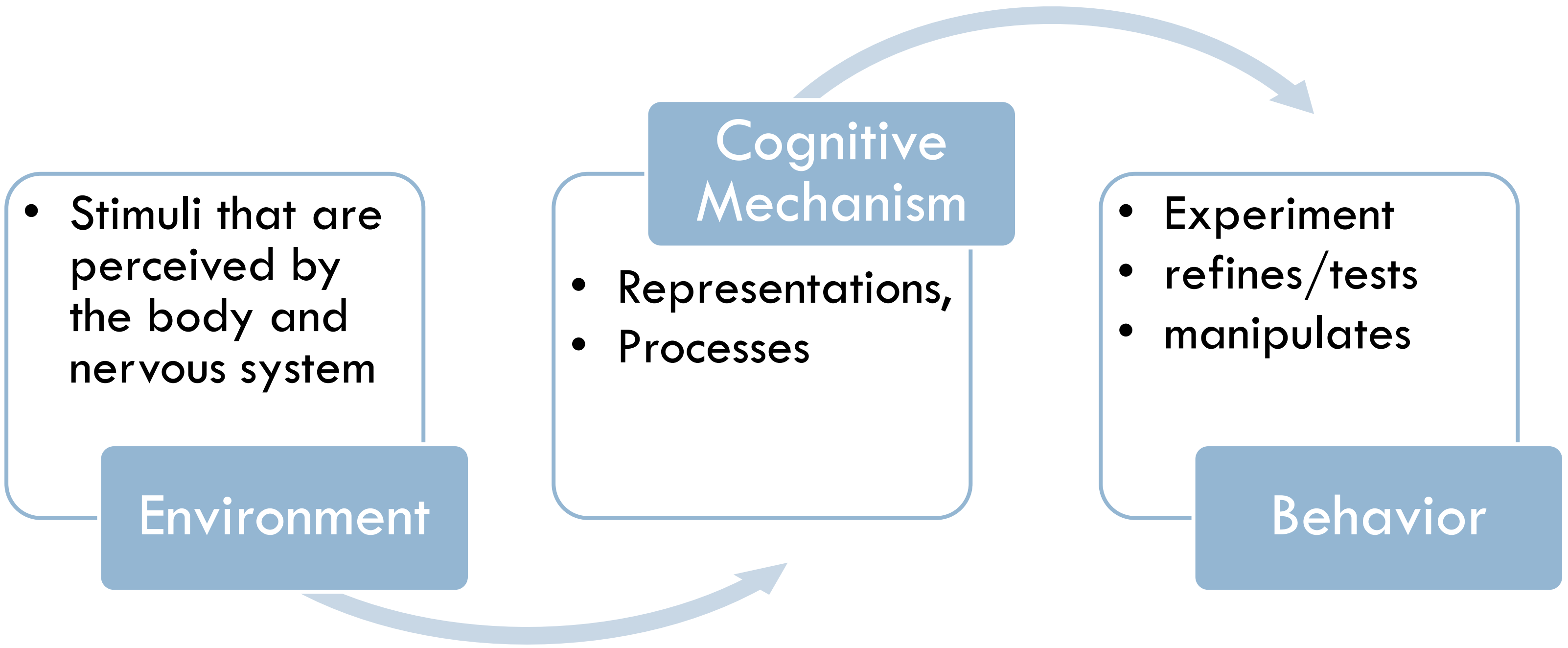
Kurt Gödel

Alan Turing

- **Automata theories**
- **Set theory**
- **Turing machines**
- **Computational theories**
- **Mathematical logic**
- **Formal languages theories**

What is Cognition?

- The collection of mental processes and/or activities used in acquiring knowledge, learning, remembering, thinking, and understanding and the act of using those processes



Cognitive Modeling

- A model is a simplified (usually formal) representation of reality
- Cognitive modeling
  - Create formal (e.g. mathematical, algorithmic, symbolic) representations of cognitive processes
  - Then, use these models to simulate, predict or explain behaviour associated with those cognitive processes
- The formal model attempts to mimic human data from the tasks they are modelling
- Computational modeling
  - models usually implemented as computer programs with output corresponding to the predicted behaviour

Denotational Mathematics(DM)

- **Denotational mathematics:** a category of expressive mathematical structures that deals with high level mathematical entities beyond numbers and sets,
  - abstract objects, complex relations, behavioral information, concepts, knowledge, processes, intelligence, and systems.
- Types of DMS
  - Concept algebra, RTPA, System Algebra, Granular algebra, Visual Semantic Algebra (VSA)
- DMs for Software Engineering
  - RTPA unified data models (UDMs): Software architectures (entities)
  - RTPA unified process models (UPMs): Software behaviors (functions)

Denotational vs. Analytic Mathematics

Function	Category	Mathematical Means	
		Conventional	Denotational
Identify *objects* & *attributes*	To *be* (|=)	Logic	Concept algebra
Describe *relations* & *possession*	To *have* (|⊂)	Set theory	System algebra
Describe *status* and *behaviors*	To *do* (|>)	Functions	Real-time process algebra (RTPA)

Abstract Concept in Cognitive Sciences

- **Concepts**
  - The basic unit of cognition that carries certain meanings in almost all cognitive processes such as thinking, learning, reasoning, and system design.
- **Abstract Concepts**
  - A dynamical mathematical structure with internal attributes, objects, and their relations
- **Concept algebra (CA) (Wang, 2006)**
  - A new mathematical structure for the formal treatment of abstract concepts and their algebraic relations, operations, and associative rules for composing complex concepts and knowledge.

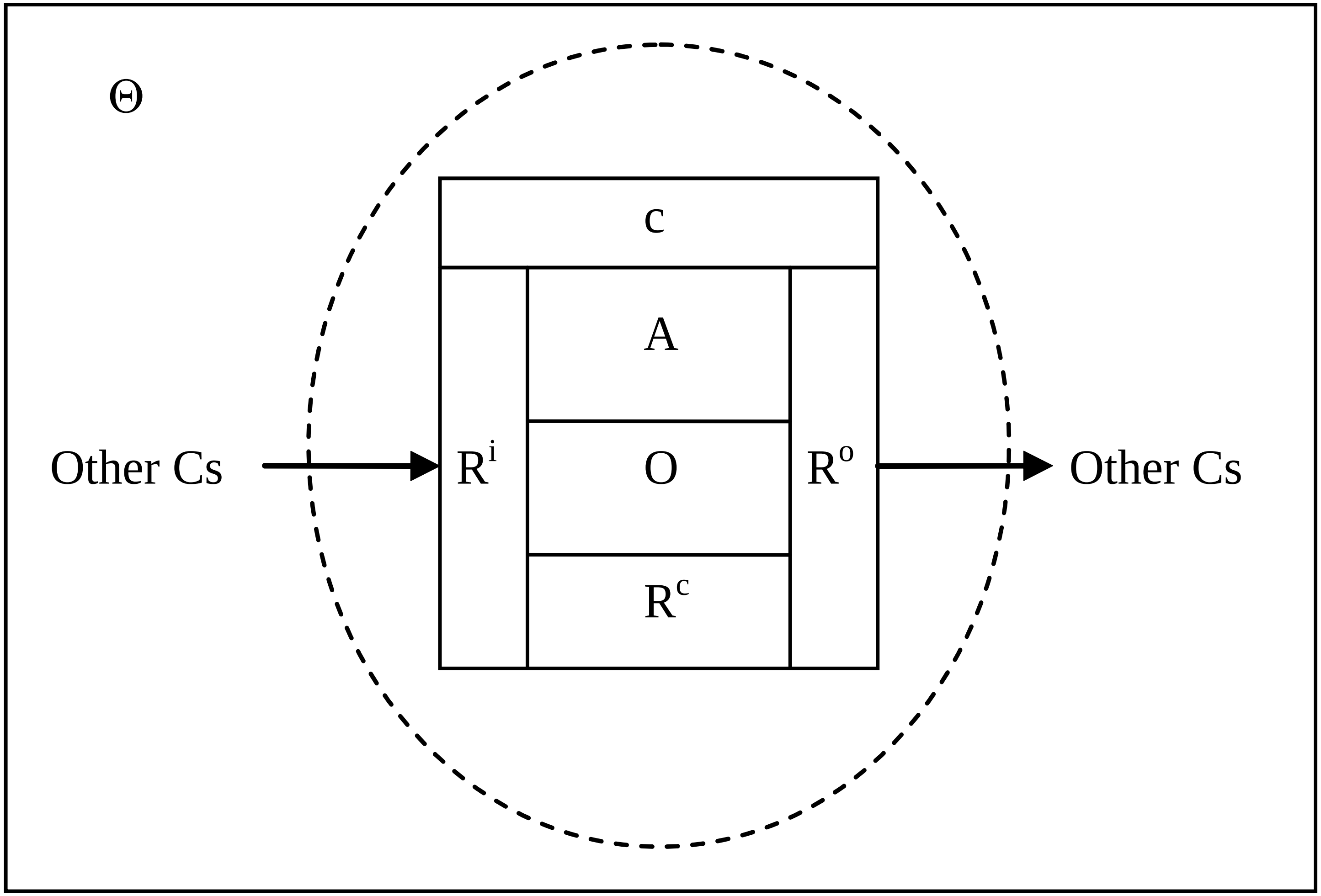
Concept Algebra (CA)

- Semantic environment or context Θ of all concepts

$$\Theta \triangleq (\mathcal{O}, \mathcal{A}, \mathcal{R})$$
$$= \mathcal{R} : \mathcal{O} \to \mathcal{O} | \mathcal{O} \to \mathcal{A} | \mathcal{A} \to \mathcal{O} | \mathcal{A} \to \mathcal{A}$$

- For example: The concept "PEN"
- The intension of connotes the **attributes** of being
  - Writing tool, with a nib, and with ink.
- The extension of the "PEN" denotes
  - All kinds of pens that share the common attributes as specified in the intension of the concept, such as a ballpoint pen, a fountain pen, and a quill pen.

An Abstract Concept



Mathematical Model of an Abstract Concept

- An abstract concept c is a 5-tuple, i.e.:

$$c \triangleq (O, A, R^c, R^i, R^o)$$

where

- O is a nonempty set of **object** of the concept, $O = \{o_1, o_2, \ldots, o_m\}$
- A is a nonempty set of **attributes**, $A = \{a_1, a_2, \ldots, a_n\}$
- $R^c \subseteq O \times A$ is a set of **internal relations**.
- $R^i \subseteq C' \times C$ is a set of **input relations**
- $R^o \subseteq C \times C'$ is a set of **output relations**.

C′ is a set of external concepts.

Concept Algebra (CA)

- Concept

$$C \triangleq (O, A, R^c, R^i, R^o)$$

- Intension of a concept

$$C^*(O, A, R^c, R^i, R^o) \triangleq A = \bigcap_{i=1}^{\#O} A_{o_i}$$

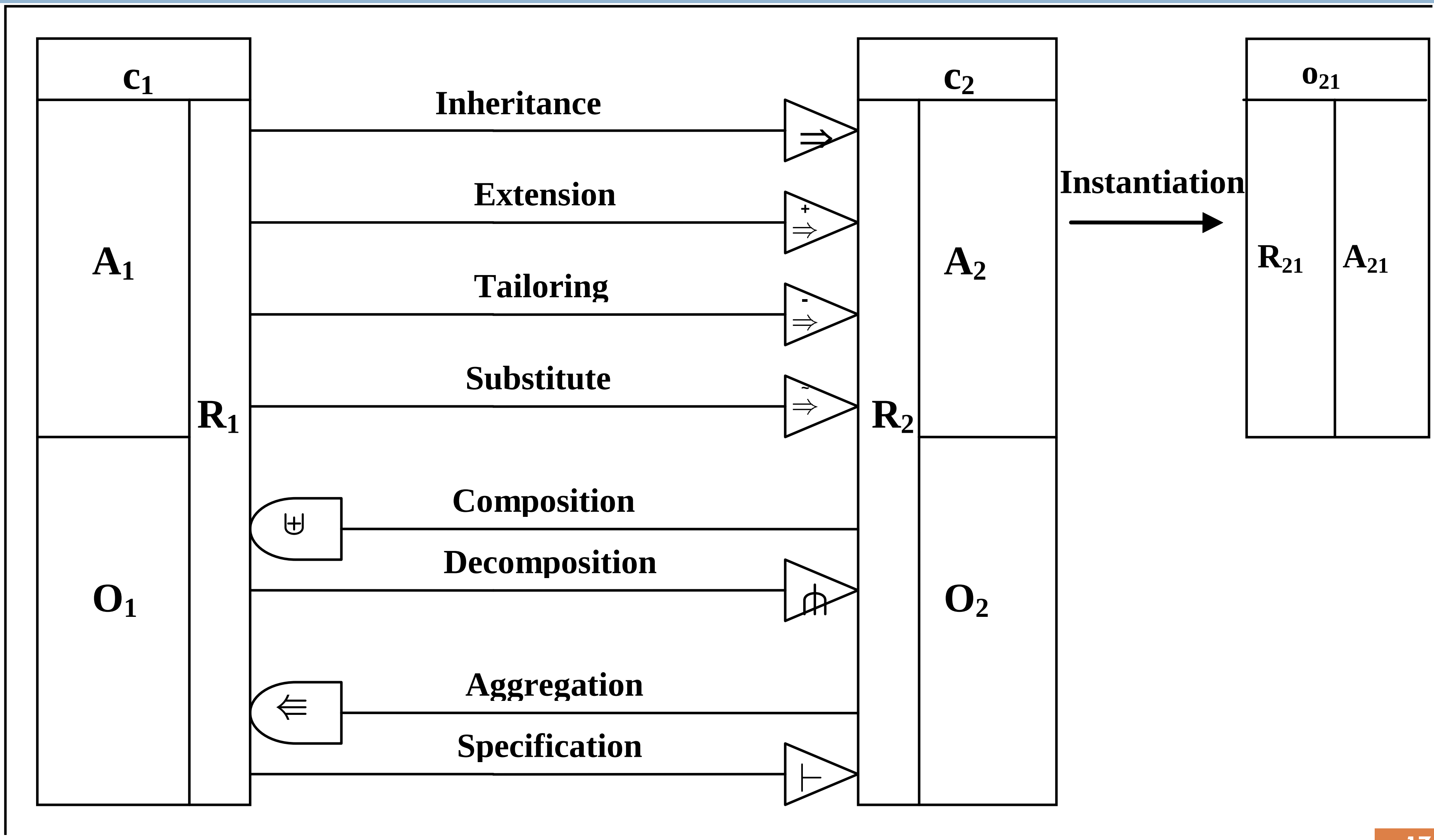
- Extension of a concept

$$C^+(O, A, R^c, R^i, R^o) \triangleq O = \{o_1, o_2, \ldots, o_m\}$$

- Concept algebra (CA)

$$CA \triangleq \left(C = \{O, A, R^c, R^i, R^o\}, op = \{\bullet_r, \bullet_c\}, \Theta\right)$$

Compositional Operations $\bullet_c$



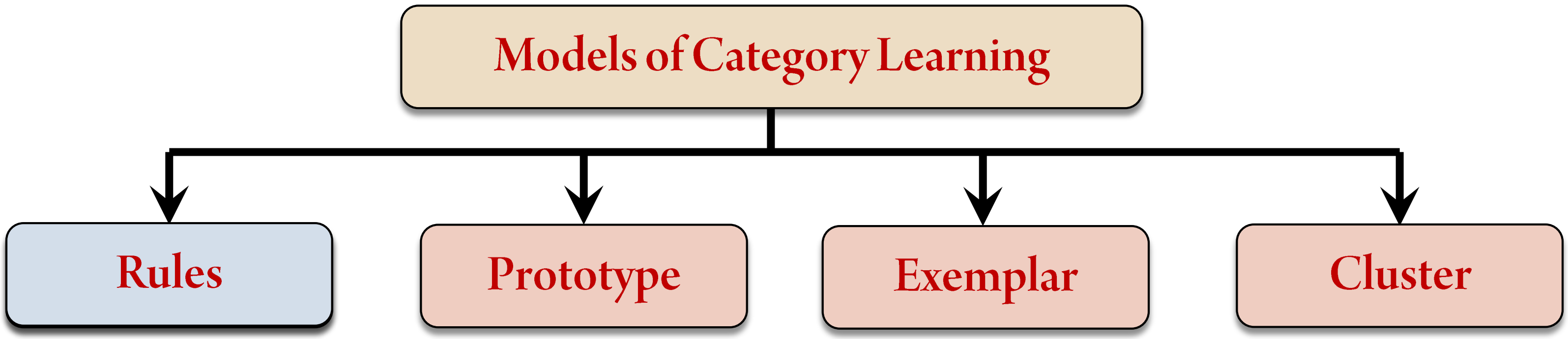
Relational Operations $\bullet_r$

Operation	Symbol	Definition
Related concepts C_1 and C_2	$\leftrightarrow$	$C_1 \leftrightarrow C_2 \triangleq A_1 \cap A_2 \neq \emptyset$
Independent concepts C_1 and C_2	$\nleftrightarrow$	$C_1 \nleftrightarrow C_2 \triangleq A_1 \cap A_2 = \emptyset$
Subconcept C_1 of concept C_2	$\prec$	$C_1 \prec C_2 \triangleq A_1 \supset A_2$
Superconcept C_2 of concept C_1	$\succ$	$C_2 \succ C_1 \triangleq A_2 \subset A_1$
Equivalent concepts C_1 and C_2	$=$	$C_1 = C_2 \triangleq (A_1 = A_2) \wedge (O_1 = O_2)$
Consistent concepts C_1 and C_2	$\cong$	$C_1 \cong C_2 \triangleq (C_1 \succ C_1) \vee (C_1 \prec C_2)$
Comparison between C_1 and C_2	$\sim$	$C_1 \sim C_2 \triangleq \frac{\#(A_1 \cap A_2)}{\#(A_1 \cup A_2)} * 100\%$

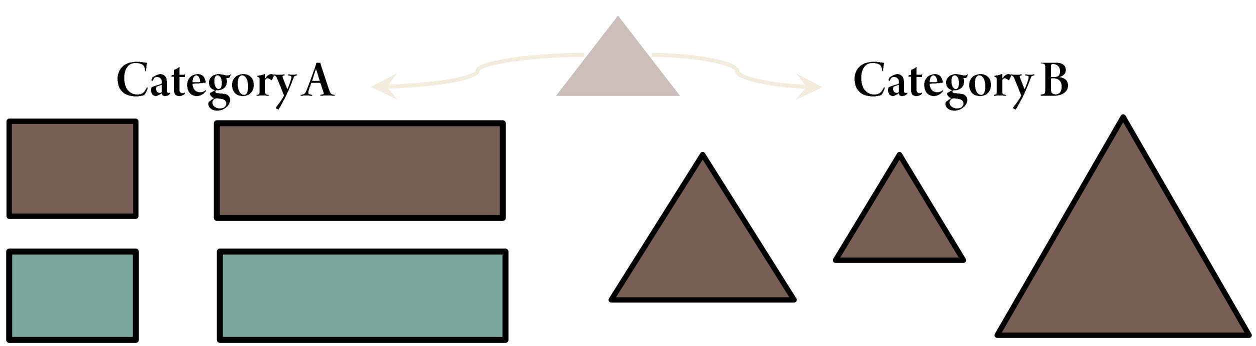
Categorization in Cognitive Psychology

- Bruner, Goodnow and Austin (1967)
  - The searching for and listing of attributes that can be used to distinguish exemplars from non-exemplars of various categories.
- Klausmeier (1980) Suggested 4 levels of concept learning
  - Concrete - recall of critical attributes (prototype),
  - Identity - recall of examples,
  - Classification - generalizing to new intity,
  - Formalization - discriminating new instances (misclassified).
- Tennyson and Cocchiarella (1986) Suggested a 3 stages model
  - Establishing a connection in memory between the concept to be learned and existing knowledge in the form of rules.
  - Improving the formation of concepts in terms of relations.
  - Facilitating the development of classification rules.

Models of Concept Learning

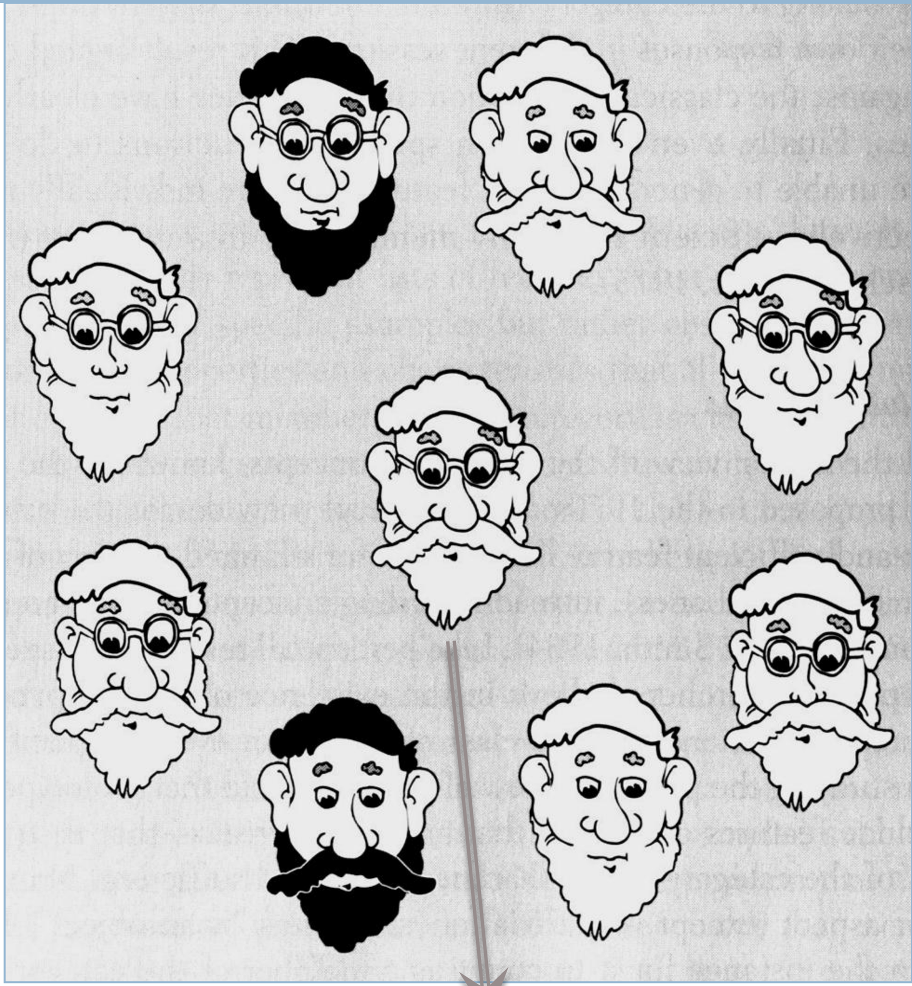


- Grouping objects based on their similar properties.
- These groups called categories that defined by logical rules.



Prototype Based Models

- Developed by Rosch et al, in the 1970s.
- Represent information about all the possible properties, instead of focusing on only a few properties like rule models do.
- A summary of all of its members.



Prototype

Prototype Based Models

- Mathematically, the average or the central tendency of all category members.
- A novel item is classified as a member of the category whose prototype it is most similar to prototype of the category.
- After a novel item classification, the position of the prototype shifts towards the newest category member.

A Computational Cognitive Model for Binary Categorization