4/1/4. Course Specification:

COURSE SPECIFICATIONS
Form

Course Title: **Variational Calculus**......

Course Code: ...... **4046501-4**......

Course Specifications

Institution: Umm Al-Qura University Date: 20/2/1439
College/Department: Faculty of Applied Science/ Department of Mathematical Sciences

A. Course Identification and General Information

1. Course title and code: Variational Calculus (4046501-4)			
2. Credit hours: 4 Hours			
3. Program(s) in which the course is offered. (If general elective available in many programs indicate this rather than list programs) Master in Mathematics			
4. Name of faculty member responsible for the course: Dr. Wajdi F. Kallel			
5. Level/year at which this course is offered: Level 1/ Master			
6. Pre-requisites for this course (if any): Calculus of several Variables			
7. Co-requisites for this course (if any):			
8. Location if not on main campus: Al-Abidiyah campus and Al-Zahir campus			
9. Mode of Instruction (mark all that apply)			
a. traditional classroom	✓	What percentage?	85
b. blended (traditional and online)		What percentage?	
c. e-learning	✓	What percentage?	15
d. correspondence		What percentage?	
f. other		What percentage?	
Comments:			

B Objectives

1. What is the main purpose for this course? ▪ The role of the course is to introduce concepts and quantitative techniques for the study of Variational Calculus. It is assumed that students entering this course have previously taken courses in differential and integral Calculus for functions of many variables. ▪ The calculus of variations is concerned with finding extrema and, in this sense; it can be considered a branch of optimization. The problems and techniques in this branch, however, differ markedly from those involving the extrema of functions of several variables owing to the nature of the domain on the quantity to be optimized.
2. Briefly describe any plans for developing and improving the course that are being implemented. (e.g. increased use of IT or web based reference material, changes in content as a result of new research in the field) 1. Updating references used in teaching process. 2. Using e-learning facilities more efficiently. 3. Encouraging students to collect problems from web based references and supervise discussions in the class.

C. Course Description (Note: General description in the form used in Bulletin or handbook)

Course Description: The Variational Calculus is nearly as old as the calculus, and the two subjects were developed somewhat in parallel. In 1927 Forsyth noted that the subject "attracted a rather fickle attention at more or less isolated intervals in its growth." In the eighteenth century, the Bernoulli brothers, Newton, Leibniz, Euler, Lagrange, and Legendre contributed to the subject, and their work was extended significantly in the next century by Jacobi and Weierstrafi. Variational principles abound in physics and particularly in mechanics. The application of these principles usually entails finding functions that minimize definite integrals (e.g., energy integrals) and hence the Variational Calculus comes naturally to the fore. Hamilton's Principle in classical mechanics is a prominent example. This course is a 4 credit hours course comprising approximately 52 hours of lectures.

1. Topics to be Covered

List of Topics	No. of Weeks	Contact hours
Chapter 1 - Introduction to Calculus of Variations. • Describe some of the classical problems of Calculus of Variations. - The Brachistochrone problem - The Geodesic problem - The Isoperimetric problem - The Plateau problem • Introduce the concept of a Functional and the associated notation.	1	4
Chapter 2- The Euler-Lagrange equation for functionals of a single variable: • Develop the lemmas to be used in the construction of Euler equations. • Define the variation of a functional. • Derive the Euler-Lagrange equation for the functional I(y)= $\int_a^b f(x,y,y')dx$ • Derive particular results in the cases: -The integrand $f(x,y,y')$ does not depend on y. -The integrand $f(x,y,y')$ does not depend on x. -The integrand $f(x,y,y')$ does not depend on y'. • Solve the Brachistochrone problem and other assorted problems. • Establish the Euler-Lagrange equations for functionals involving n> 1 functions of a single variable with fixed endpoint conditions. • Introduce geodesic lines in two and three dimensions. Develop the distance metric $ds^2 = E(u,v)du^2 + 2F(u,v)du\,dv + G(u,v)dv^2$ • Develop the Euler-Lagrange equations for functionals of type I(y)= $\int_a^b f\left(x,y,y',y'',\ldots,y^{(n)}\right)dx$	3	12

Chapter 3 - General Variation of a functional. • Derive the Euler-Lagrange equation for the general variation of the functional • Establish the conditions of transversality for the functional. • Introduce the Weierstrass-Erdmann Corner Conditions. • Do examples using the conditions of transversality including the modified Brachistochrone problem in which the lower endpoint is replace by a horizontal distance to be travelled. • Derive the Euler-Lagrange equations for functionals of many dependent variables but a single independent variable when functional constraints are in force. • Examples on the Isoperimetric problem for a curve of fixed length and the shape of a suspended chain of fixed length among other examples.	2	8
Chapter 4 - Sufficiency conditions for a weak extremum. • Introduce the concept of the second variation of a functional. J$[y]=\int_{x_0}^{x_1} f(x,y,y')dx$ • Calculate the second variation of the functional. • State and prove Legendre's theorem and use it the prove that the extremal of the Brachistochrone problem is in fact a minimum.	2	8
Chapter 5 - Functionals with two or more independent variables. • Derive the Euler-Lagrange equation for the functional. I$[\varphi]=\iint f(x,y,\varphi,p,q)dxdy$ • Derive the Euler-Lagrange equation to be satisfied in the case of the Plateau Problem. • Derive the equation to be satisfied by the optimal shape of a homogeneous membrane stretched to tension T and subjected to uniform loading P by minimizing the energy integral I$[\varphi]=\iint \left(T_p^2+T_q^2-2P\varphi\right)dxdy, \quad \varphi=0 \ on\ \partial A$	3	12

Chapter 6 - Canonical variables • Introduce the concept of momentum for the functional $J[y_1, \dots, y_n] = \int_{x_0}^{x_1} f(x, y_1, \dots, y_n, y_1', \dots, y_n')\,dx$ • Establish the canonical Euler equations, often more commonly called Hamilton's equations of motion and deduce that the Hamiltonian is conserved on extremely curves if the function does not depend explicitly on the independent variable. • Introduce the Principal of Least Action, define the Lagrangian function and construct Lagrange's equations. • Derive the Hamilton-Jacobi equation, and thereafter state and prove Jacobi's Theorem. • Analyse the problem of motion under a central force using (a) Hamilton's equations , (b) The Hamilton-Jacobi equation. • Introduce the concept of canonical transformations, Poisson Bracket and Generating function and show how these are used in the context of Hamilton's equations. • Define a Legendre transformation and illustrate Legendre transformations with reference to connections between the classical functions of Thermodynamics, namely Internal Energy U, Helmholtz Free Energy F, Gibbs Free Energy G and Enthalpy H. • Show that the Legendre transformation of the Lagrangian with respect to velocity is the Hamiltonian.	2	8

Chapter 7 - Assorted matters • Explain the strategy by which the lowest eigenvalue of the system Ly = α y may be approximated by estimating the eigenvalues of a suitable Rayleigh-Ritz quotient. Illustrate the technique, for example, with reference to the eigenvalue problem. • Explain how higher order eigenvalues of the system Ly = α y may also be approximated by the use of the Rayleigh-Ritz quotient. Illustrate the technique by estimating the second eigenvalue of equation (1). • Introduce Pontryagin's maximum (or minimum) principle for moving a dynamical system between states in an optimal way particularly when constraints are in place, for example, there is a maximum force that can be applied to the system. • Illustrate Pontryagin's principle by calculating the minimum time needed to bring a particle of mass m at the origin given that the particle is initially at x = a with velocity V , and that the maximum available force is F but this force can be applied in any direction.	2	8

2. Course components (total contact hours and credits per semester):

	Lecture	Tutorial	Laboratory or Studio	Practical	Other:	Total
Contact Hours	60	--	--	--	--	60
Credit	4	--	--	--	--	4

3. Additional private study/learning hours expected for students per week.

Four hours weekly for homework and revision

4. Course Learning Outcomes in NQF Domains of Learning and Alignment with Assessment Methods and Teaching Strategy

On the table below are the five NQF Learning Domains, numbered in the left column.

First, insert the suitable and measurable course learning outcomes required in the appropriate learning domains (see suggestions below the table). **Second**, insert supporting teaching strategies that fit and align with the assessment methods and intended learning outcomes. **Third**, insert appropriate assessment methods that accurately measure and evaluate the learning outcome. Each course learning outcomes, assessment method, and teaching strategy ought to reasonably fit and flow together as an integrated learning and teaching process. (Courses are not required to include learning outcomes from each domain.)

Code #	NQF Learning Domains And Course Learning Outcomes	Course Teaching Strategies	Course Assessment Methods
1.0	**Knowledge**		
1.1	Have an enhanced knowledge and understanding of Variational calculus.	Lectures- Discussion- solve problems	Short quizzes, periodical and final exams.
1.2	Have the ability to recall the learned material of the course	Lectures- Discussion- solve problems	Short quizzes, periodical and final exams.
2.0	**Cognitive Skills**		
2.1	Be able to apply the learned material of the course in real life problems.	Lectures- Discussion- solve problems	Short quizzes, periodical and final exams.
2.2	Be able to integrate related topics from separate parts of the course	Lectures – Discussion- solve problems	Short quizzes, periodical and final exams
3.0	**Interpersonal Skills & Responsibility**		
3.1	Have the ability to prove theorems and develop lemmas using different techniques	Lectures – Discussion- solve problems	Short quizzes, periodical and final exams
3.2	Be able to describe and analyze models using related equations	Lectures – Discussion- solve problems	Short quizzes, periodical and final exams
4.0	**Communication, Information Technology, Numerical**		

4.1	Have the ability to use computers programs in obtaining numerical solutions and carrying out statistical tests.	Discussion - Use Matlab or Mathematica to solve some problems numerically.	Homework projects
4.2			
5.0	**Psychomotor**		
5.1	Not applicable	Not applicable	Not applicable

5. Schedule of Assessment Tasks for Students During the Semester			
	Assessment task (e.g. essay, test, group project, examination, speech, oral presentation, etc.)	Week Due	Proportion of Total Assessment
1	Periodic exam (1)	6	20
2	Periodic exam (2)	10	20
3	Home work	Over all weeks	20
4	Final exam	End of semester	40

D. Student Academic Counseling and Support

1. Arrangements for availability of faculty and teaching staff for individual student consultations and academic advice. (include amount of time teaching staff are expected to be available each week) - Office hours are specified throughout the week (6 hours/week) - Contacts with students by e-mail, SMS, and e-learning facilities.

E Learning Resources

1. List Required Textbooks: • University Mathematical Texts: Calculus of Variations, J.C. Clegg: Oliver and Boyd Ltd Ed-inburgh, New York, (1968). • Calculus of Variations, I.M. Gelfand and S.V. Fomin: Prentice-Hall, Inc. Englewood Cliffs, N. J, (1963). • A history of Calculus of Variations from the 17th through the 19th century, H.H. Goldstine: Springer-Verlag, New York, (1980).
2. List Essential References Materials (Journals, Reports, etc.)
3. List Recommended Textbooks and Reference Material (Journals, Reports, etc)

4. List Electronic Materials, Web Sites, Facebook, Twitter, etc.
5. Other learning material such as computer-based programs/CD, professional standards or regulations and software. Matlab and Maple software

F. Facilities Required

Indicate requirements for the course including size of classrooms and laboratories (i.e. number of seats in classrooms and laboratories, extent of computer access etc.)
1. Accommodation (Classrooms, laboratories, demonstration rooms/labs, etc.) Properly equipped classroom
2. Computing resources (AV, data show, Smart Board, software, etc.) Matlab and Mathematica software
3. Other resources (specify, e.g. if specific laboratory equipment is required, list requirements or attach list) Non

G Course Evaluation and Improvement Processes

1 Strategies for Obtaining Student Feedback on Effectiveness of Teaching Student feedback on effectiveness of teaching is arranged electronically at the end of the term by the University.
2 Other Strategies for Evaluation of Teaching by the Instructor or by the Department The evaluation of teaching is composed by the Department.
3 Processes for Improvement of Teaching Several workshops on the improvement of teaching are conducting yearly by the University.
4. Processes for Verifying Standards of Student Achievement (e.g. check marking by an independent member teaching staff of a sample of student work, periodic exchange and remarking of tests or a sample of assignments with staff at another institution) Non
5 Describe the planning arrangements for periodically reviewing course effectiveness and planning for improvement. Reviewing process of courses for improvement and development is done normally every five years.

Name of Instructor: Dr. Wajdi F. Kallel
Signature:___________________Date Report Completed: 20/2/1439
Name of Field Experience Teaching Staff ____________________________
Program Coordinator:____
Signature: ____________________________ Date Received:____________________