Chapter 2

Motion in one Dimension

Units of Chapter 2: Motion in one dimension

- Particle Kinematics
- Descriptions of Motion
- Average Velocity
- Instantaneous Velocity
- Accelerated Motion
- Motion With Constant Acceleration
- Freely Falling Bodies

Learning goals of this chapter

- **On completing this chapter, the student will be able to:**
- Define the concepts of the distance, displacement, velocity , speed, acceleration and acceleration of gravity.
- Differentiate between the fundamental concept of the distance and the velocity.
- Differentiate between the fundamental concept of the velocity and speed.
- Differentiate between the fundamental concepts of displacement, velocity, and acceleration of a moving body.
- Describe the motion of a particle with mathematical equations and with graphs.
- Solve problems concerning with the motion of the body with constant acceleration.
- Describe the motion of Freely Falling Bodies mathematically
- Solve problems concerning with the motion of the free fall body.

2-1 Particle Kinematics

To describe the kinematics of a particle, we need to defined its **position**, **velocity**, and **acceleration**.

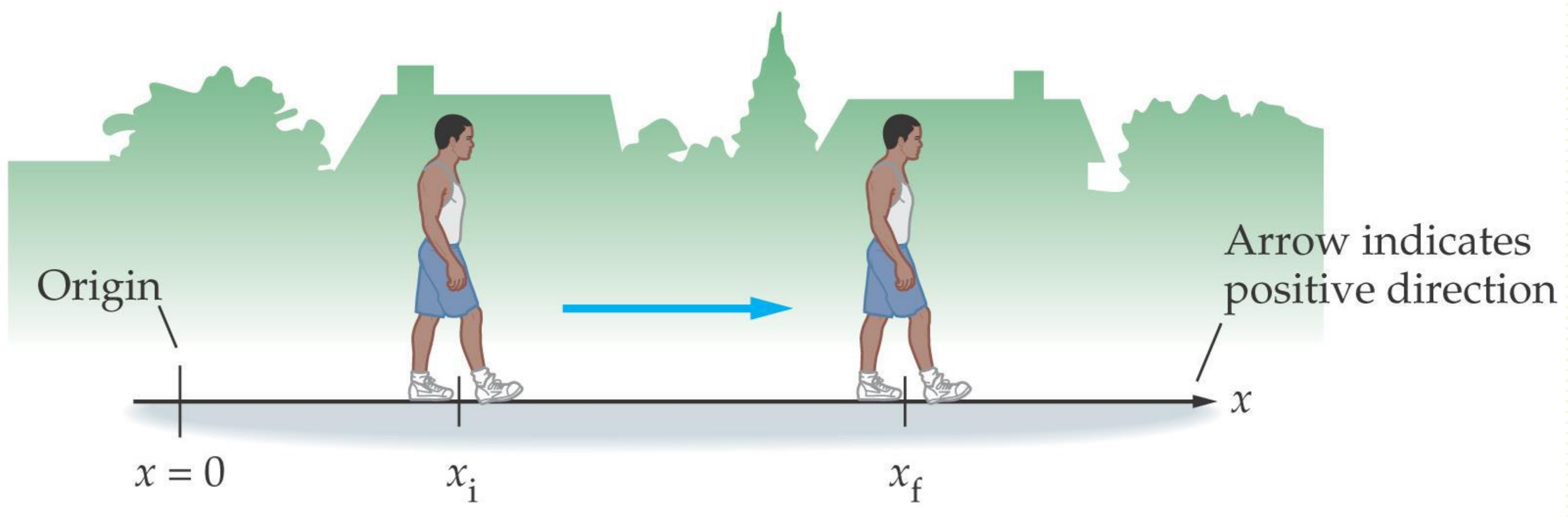
- Two ways to describe the motion of a particle:

  (1) The mathematical approach is usually better for solving problems, because it permits more precision than the graphical sketch.

  (2) The graphical method is helpful because it often provides more physical insight than a set of mathematical equations.

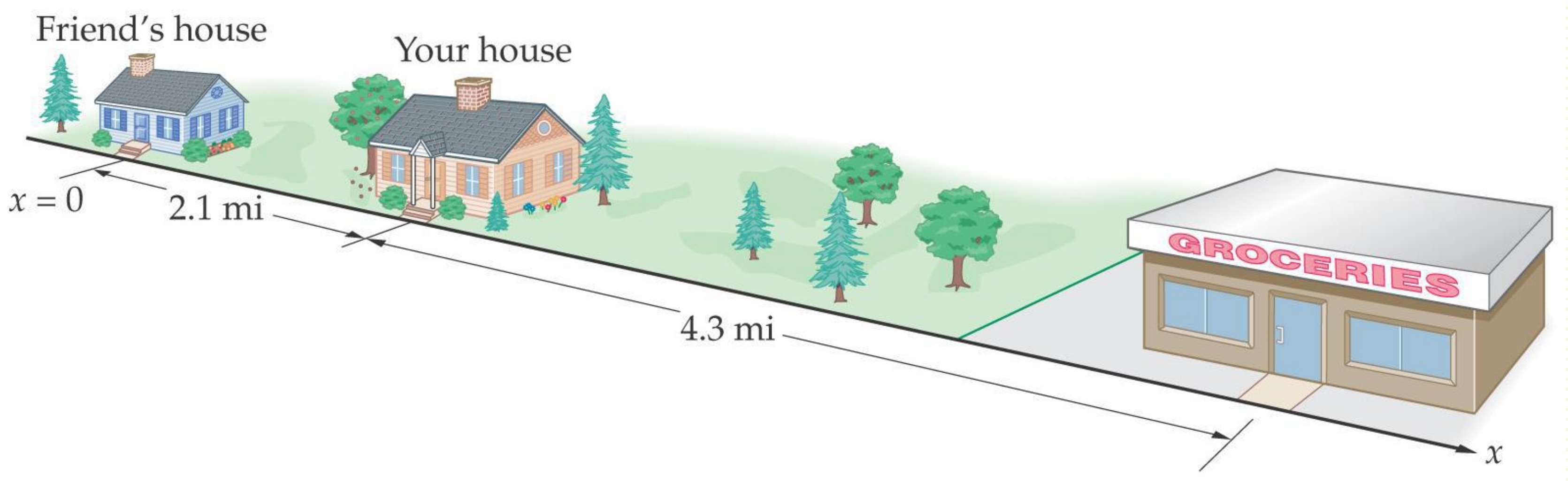
2-1 Particle Kinematics

- **Position**
- Before describing motion, you must set up a coordinate system – define an origin and a positive direction.
- The mathematical dependence of its position *x* on the time *t* is: $\boldsymbol{x(t)}$



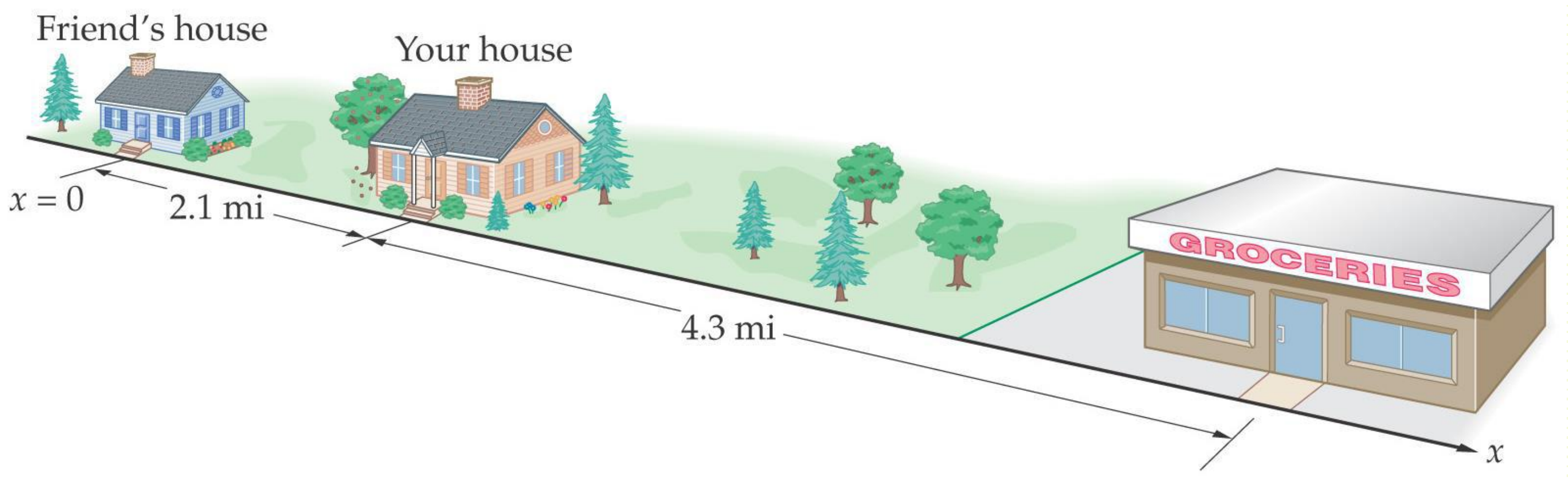
2-1 Particle Kinematics

- **The distance** is the total length of travel;
- It is a scalar quantity and always positive, with SI unit meter (m).
- Ex: if you drive from your house to the grocery store and back, you have covered a distance of 8.6 mi.



2-1 Particle Kinematics

- **Displacement** Δx: is the change in position.
- If you drive from your house to the grocery store and then to your friend's house, your displacement is 2.1 mi and the distance you have traveled is 10.7 mi.
- It is a vector quantity, and it can be positive or negative.



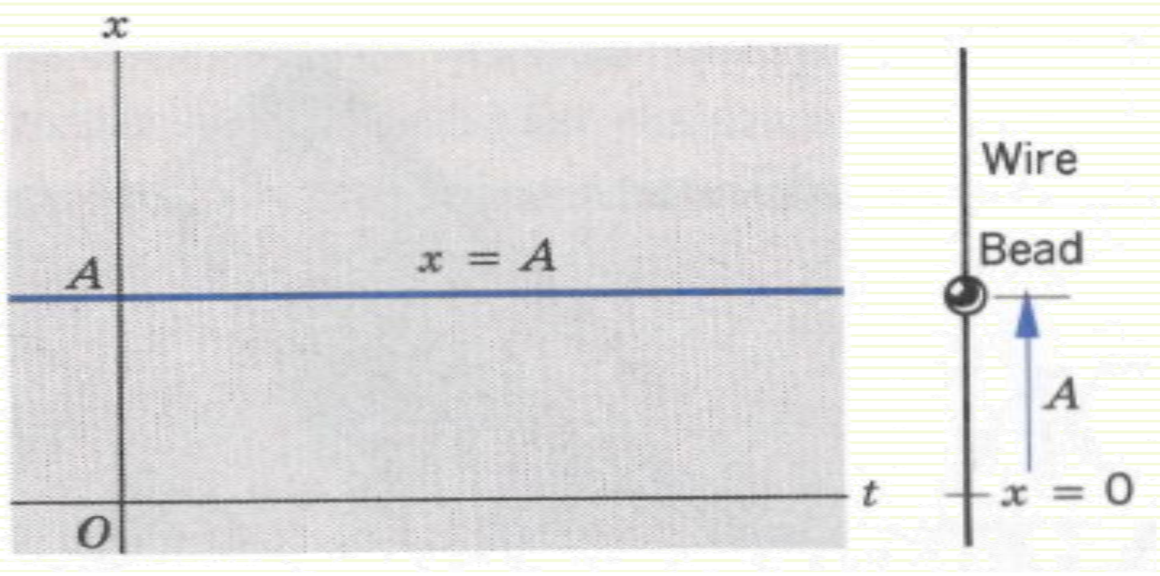
2-2 Descriptions of Motion

For $x(t) = A$, where A is constant

- In this case the particle occupies the position at the coordinate A at all times and **No motion at all.** :
- Notes:

  1- t is the time , it is independent variable

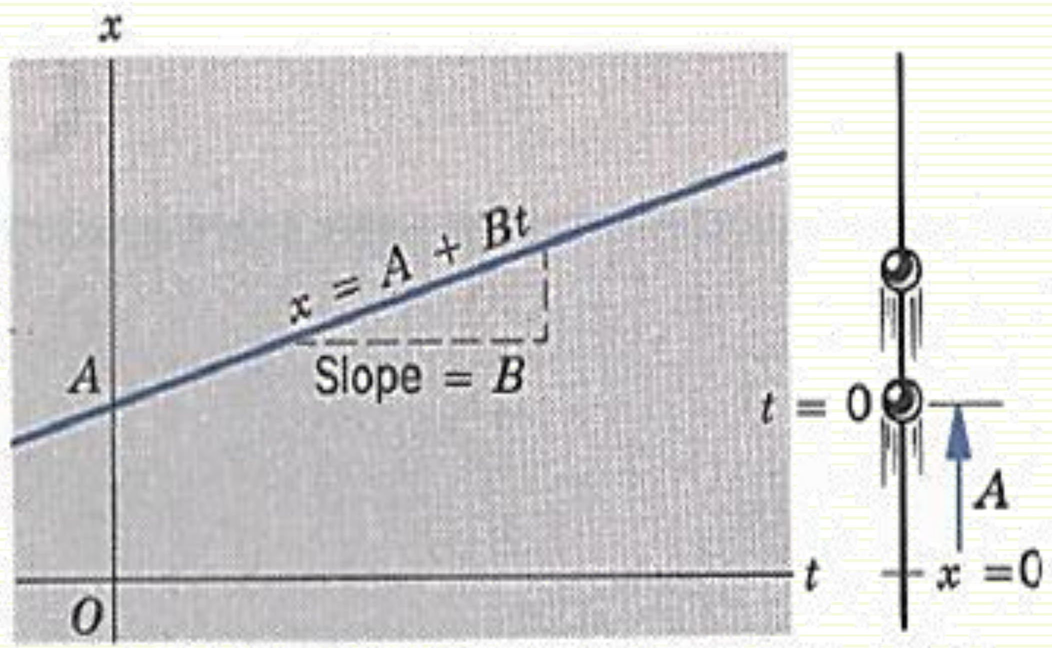
  2- $x = function\ of\ t = x(t)$; it is dependent variable.



2-2 Descriptions of Motion

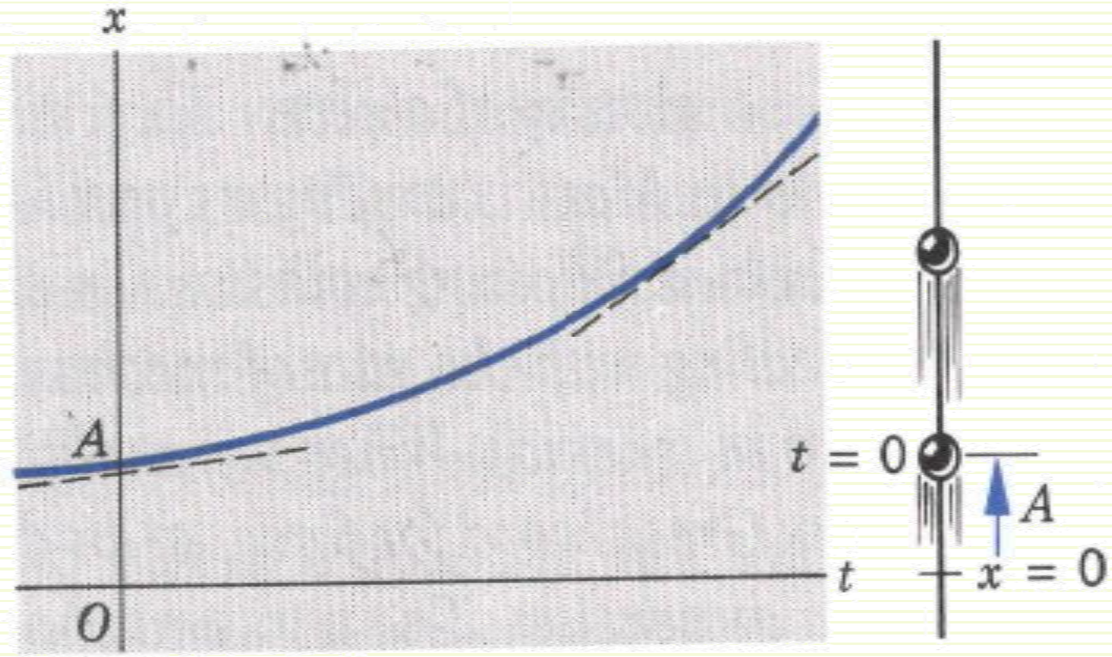
For $x(t) = Bt + A$, where A and B are constant

- In this case the particle moves with **constant speed**, and the rate of motion is described by the **velocity**, where the velocity $v = \frac{dx}{dt}$.
- Also we notice that the velocity $= B$
- The velocity may be positive or negative depending on the direction of motion



2-2 Descriptions of Motion

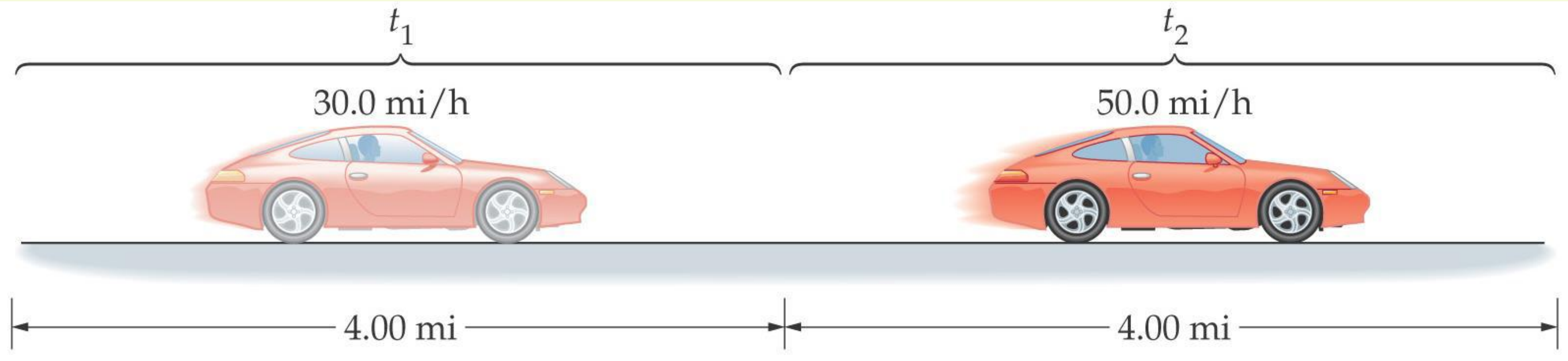
- For $x(t) = A + Bt + Ct^2$ where $A, B, and\ C$ are constant.
- In this case the speed is changing, i.e., it is accelerated motion (acceleration being defined as the rate of change of velocity), and so the slope must change also. These graphs are therefore curves rather than straight lines.



(a)

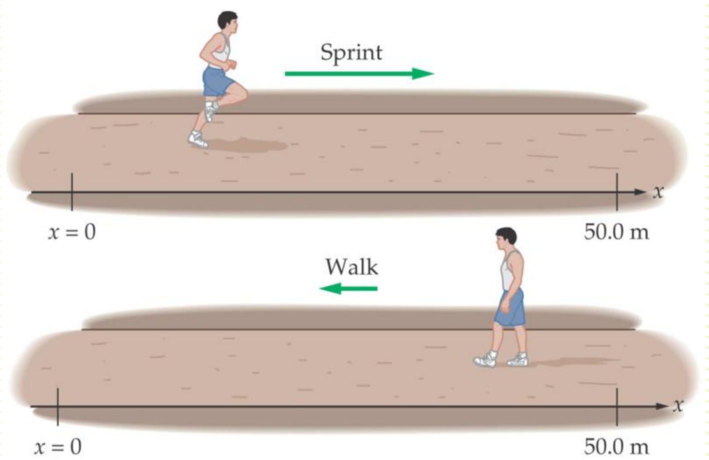
2-3 Average Speed and Velocity

- **The average speed**: is defined as the distance traveled divided by the time the trip took.
- Average speed = distance / elapsed time
- It is scalar and always positive, (In SI unit (m/s)).
- Is the average speed of the red car 40.0 mi/h, more than 40.0 mi/h, or less than 40.0 mi/h?



2-3 Average Speed and Velocity

- **Average velocity** = displacement / elapsed time
- The displacement $\Delta x = x_2 - x_1 = x_f - x_i$,
- and elapsed time $\Delta t = t_2 - t_1$



- $average\ velocity = \dfrac{Displacement}{elapsed\ time}$
- $\bar{v} = v_{av} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{x_f - x_i}{t_2 - t_1}\ \ m/s$
- If you return to your starting point, your average velocity is zero since $x_f \rightarrow x_i$, $then$, $\Delta x = x_2 - x_1 = x_f - x_i = 0$.
- The average velocity is a vector quantity, and can be positive or negative (SI unit m/s).

Problem 1

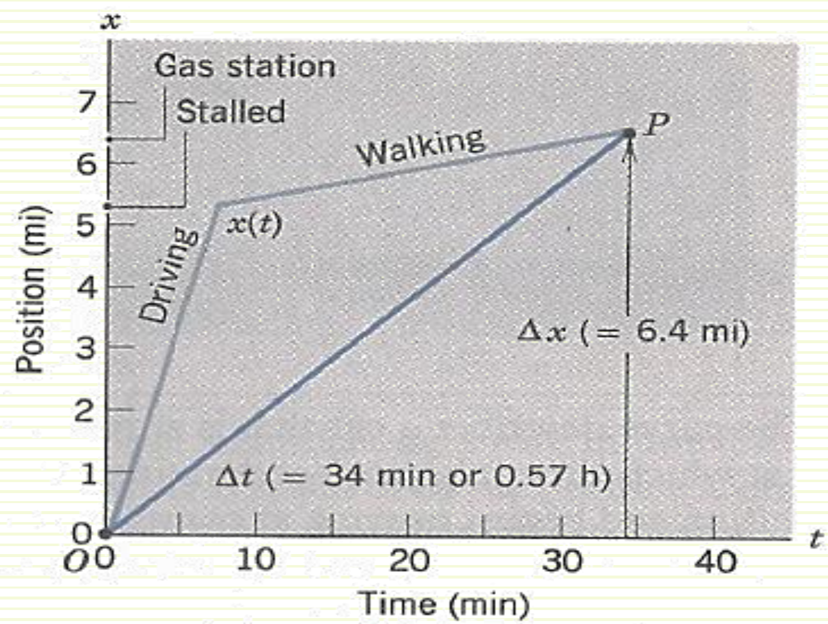
- You drive your BMW down a straight road for 5.2 mi at 43 mi/h, at which point you run out of gas. You walk 1.2 mi farther, to the nearest gas station, in 27 min. What is your average velocity from the time that you started your car to the time that you arrived at the gas station?
- Solution:

$$\Delta x = 5.2\ \mathrm{mi} + 1.2\ \mathrm{mi} = 6.4\ \mathrm{mi}$$

and

$$\Delta t = \frac{5.2\ \mathrm{mi}}{43\ \mathrm{mi/h}} + 27\ \mathrm{min}$$
$$= 7.3\ \mathrm{min} + 27\ \mathrm{min} = 34.3\ \mathrm{min} = 0.57\ \mathrm{h}.$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{6.4\ \mathrm{mi}}{0.57\ \mathrm{h}} = 11.2\ \mathrm{mi/h}$$



2-4 Instantaneous Velocity

- It would be more appropriate to obtain a mathematical function v(t), which gives the velocity at every point in the motion. This is the *instantaneous velocity*; from now on, when we use the term "velocity" we understand it to mean instantaneous velocity.

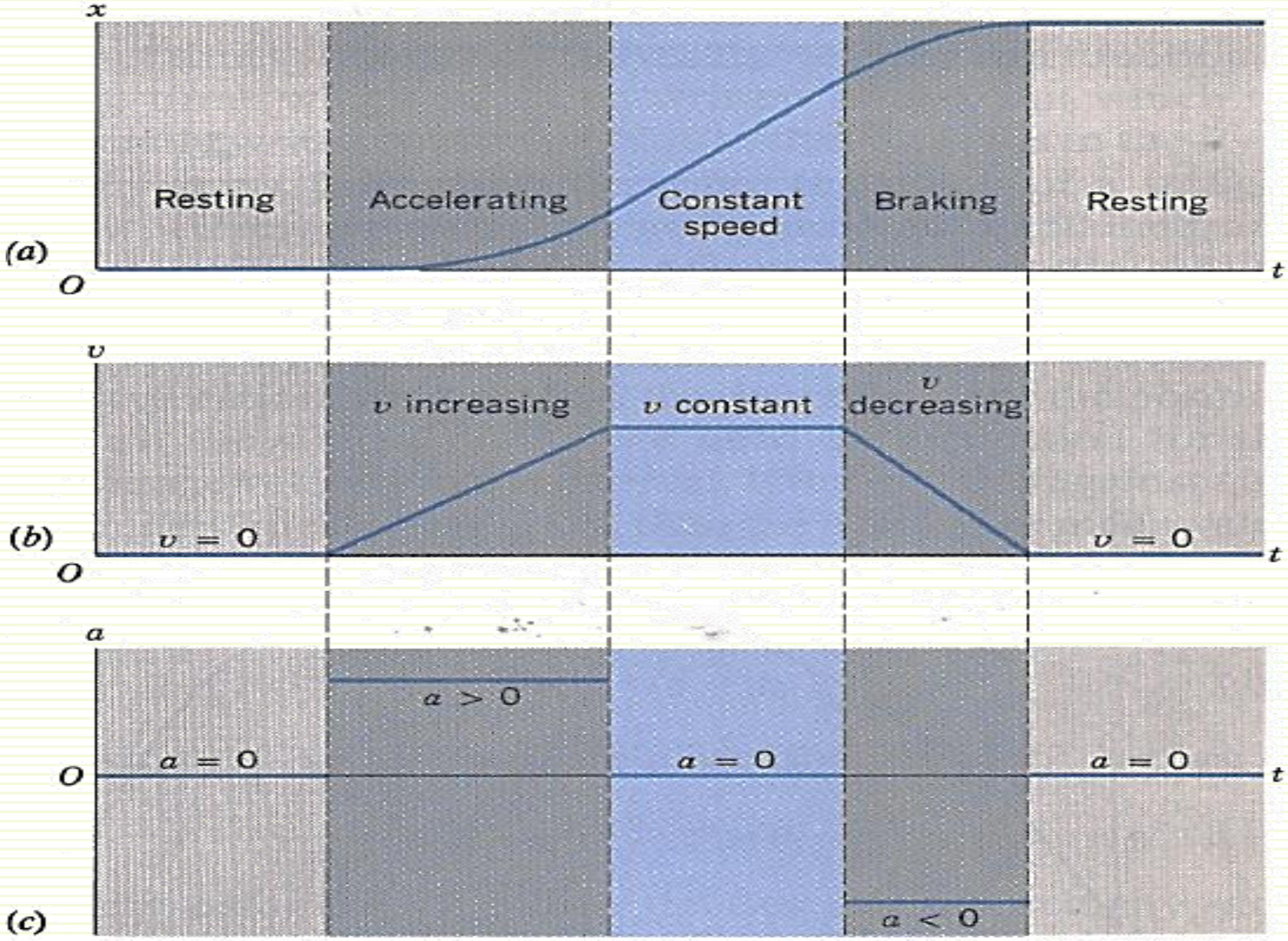
- Definition:
- $v = \lim_{\Delta t \to 0} \left(\frac{\Delta x}{\Delta t} \right) = \frac{dx}{dt}$
- SI (m/s)
- This means that we evaluate the average velocity over a shorter and shorter period of time; as that time becomes infinitesimally small, we have the instantaneous velocity.

2-5 Accelerated Motion

- The change in velocity with time is called *acceleration*.
- Definition: The acceleration is the rate of change the velocity with time.
- It is a vector, can be positive or negative, or zero.
- SI unit: meter per square second, m/s^2.
- Average acceleration
- $\bar{a} = a_{av} = \frac{\Delta v}{\Delta t} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{v_f - v_i}{t_2 - t_1} \quad m/s^2$
- Instantaneous acceleration
- $a = \lim_{\Delta t \to 0} \left(\frac{\Delta v}{\Delta t}\right) = \frac{dv}{dt}$

2-5 Accelerated Motion

Graphical Interpretation of position, velocity, and Acceleration



2-6 Motion with Constant Acceleration

- The equations describing the motion of particles moving **with constant acceleration:**
- 1- Average velocity
- $v_{av} = \frac{1}{2}(v_o + v)$
- 2- Velocity, time, and acceleration
- $v = v_o + at$
- 3- Position, time, and velocity
- $x = x_o + \frac{1}{2}(v_o + v)t$
- 4- Position, time, velocity, and acceleration
- $x = x_o + v_o t + \frac{1}{2}at^2 \quad , \quad x = x_o + vt - \frac{1}{2}at^2$
- 5- Position, velocity, and acceleration
- $v^2 = v_o^2 + 2a\Delta x \quad , \quad v^2 = v_o^2 + 2a(x - x_o)$

2-6 Equations of Motion with Constant Acceleration

Equation No.	Equation	Missing Quantity
1	$v = v_o + at$	$x - x_o$
2	$x - x_o = v_o t + \frac{1}{2} a t^2$	v
3	$x - x_o = v t - \frac{1}{2} a t^2$	v_o
4	$x - x_o = \frac{1}{2} (v_o + v) t$	a
5	$v^2 = v_o^2 + 2a(x - x_o)$	t

Problem 4

- An alpha particle (the nucleus of a helium atom) travels along the inside of a straight hollow tube 2.0m long which forms part of a particle accelerator.
- (a) If one assumes uniform acceleration, what is the acceleration of the particle, if it enters at a speed of 1.0×10^4 m/s and leaves at 5.0×10^6 m/s?
- (b) How long is it in the tube?

Problem 4

(a) We choose an x axis parallel to the tube, its positive direction being that in which the particle is moving and its origin at the tube entrance. We are given v_0, v, and x, and we seek a. Rewriting Eq. (2.6.6), with $x_0 = 0$,

$$a = \frac{v^2 - v_0^2}{2x}$$
$$= \frac{(5.0 \times 10^6 \text{ m/s})^2 - (1.0 \times 10^4 \text{ m/s})^2}{2(2.0 \text{ m})}$$
$$= +6.3 \times 10^{12} \text{ m/s}^2.$$

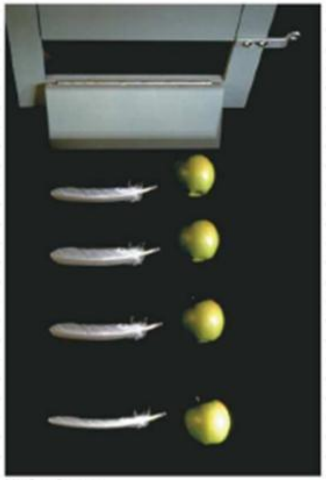
(b)

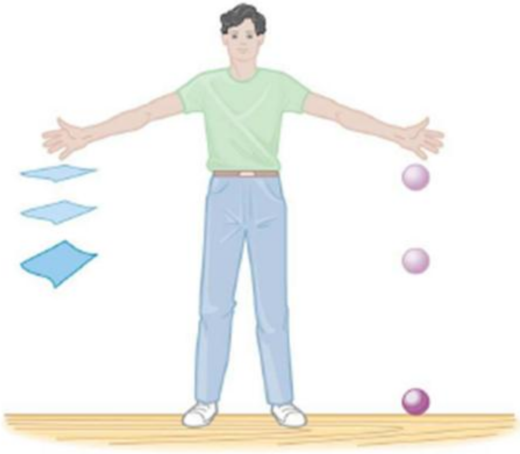
$$t = \frac{2x}{v_0 + v}$$
$$= \frac{2(2.0 \text{ m})}{1.0 \times 10^4 \text{ m/s} + 5.0 \times 10^6 \text{ m/s}}$$
$$= +8.0 \times 10^{-7} \text{ s} = 0.80 \ \mu\text{/s}.$$

2-7 Freely Falling Objects

- Free fall is the motion of an object subject only to the influence of gravity.
- The acceleration due to gravity is a constant, where
- $a = -g$
- And $g = 9.81\ m/s^2$

- An object falling in air is subject to air resistance (and therefore is not freely falling)

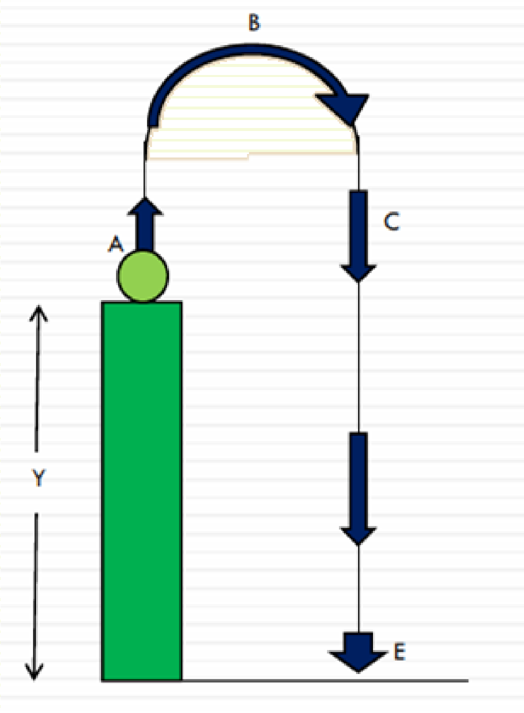




(a)

2-7 Freely falling objects

- The figure show the motion of a ball thrown vertically upward from point A at ground surface (y=0), to reach the maximum height at B then fall down to a well of depth -y at E.



- We can describe all cases as follows:
- At Point A; the time $t = 0 \; and \; y = 0$
- At point B; the velocity $v_B = 0$
- At point C; $t_C = 2tB \; , y = 0 \; , and \; v_C = -v_A$
- At point E; $y = -y$

2-7 Freely falling objects

- Two changes in the equations of motion should be applied for freely falling:
- 1- The direction of motions are now along a vertical $y\ axis$ instead of $x\ axis$, with positive direction of y upwards.
- 2- The free-fall acceleration is now negative $(-)$, that is, downward on the $y - axis$, towards Earth's center, and it has the value $(-g)$ in the equation $(g = 9.81\ m/s^2)$.

Equation No.	Equation	Missing Quantity
1	$v = v_o - gt$	$y - y_o$
2	$y - y_o = v_o t - \frac{1}{2} g t^2$	v
3	$y - y_o = vt + \frac{1}{2} g t^2$	v_o
4	$y - y_o = \frac{1}{2}(v_o + v)t$	$-g$
5	$v^2 = v_o^2 - 2g(y - y_o)$	t

Problem 5

- A body is dropped from rest and falls freely. Determine the position and velocity of the body after 1.0, 2.0, 3.0, and 4.0 s have elapsed.
- Solution:
- We choose the starting point as the origin. We know the initial speed (zero) and the acceleration, and we are given the time. To find the position, we use

$$y_o = 0 \; and \; v_o = 0$$

t	y	v	a
s	m	m/s	m/s^2
0	0	0	−9.8
1.0	−4.9	−9.8	−9.8
2.0	−19.6	−19.6	−9.8
3.0	−44.1	−29.4	−9.8
4.0	−78.4	−39.2	−9.8

Problem 5

- *Then,* $y = -\frac{1}{2}gt^2$
- Putting $t = 1.0$ s, we obtain

$$y = -\frac{1}{2}(9.8\,\mathrm{m/s^2})(1.0\,\mathrm{s})^2 = -4.9\,\mathrm{m}.$$

To find the velocity, we use Eq. ??, again with $v_0 = 0$:

$$v = -gt = -(9.8\,\mathrm{m/s^2})(1.0\,\mathrm{s}) = -9.8\,\mathrm{m/s}.$$

- After falling for 1:0 s, the body is 4.9 m *below* (y is negative) its starting point and is moving *downward* (v is negative) with a speed of 9.8 m/s^2. Continuing in this way, we can find the positions and velocities at t = 2.0 , 3.0 , and 4.0 s, which are shown in the figure.

Problem 6

- A ball is thrown vertically upward from the ground with a speed of 25.2m/s.
- (a) How long does it take to reach its highest point?
- (b) How high does it rise?
- (c) At what times will it be 27.0m above the ground?

- Solution:

(a) At its highest point its velocity passes through the value zero. Given v_o and $v(= \ 0)$, we wish to find t

$$t = \frac{v_0 - v}{g} = \frac{25.2\text{ m/s} - 0}{9.8\text{ m/s}^2} = 2.57\text{ s.}$$

Problem 6

(b) Let us use only the original data for this part, to keep from compounding any error that might have been made in part (a). y_o assigned as 0, allows us to solve for *y* when we know the other quantities

$$y = \frac{v_0^2 - v^2}{2g} = \frac{25.2\text{ m/s} - 0}{2(9.8\text{ m/s}^2)} = 32.4\text{ m}.$$

(c) *t* is the only unknown, $y_o = 0$,

$$\frac{1}{2}gt^2 - v_0 t + y = 0$$

$$\frac{1}{2}(9.8\text{ m/s}^2)t^2 - (25.2\text{ m/s})t + 27.0\text{ m} = 0.$$

Using the quadratic formula, we find the solutions to be $t = 1.52\text{ s}$ and $t = 3.62\text{ s}$. At $t = 1.52\text{ s}$, the velocity of the ball is

$$v = v_0 - gt = 25.2\text{ m/s} - (9.8\text{ m/s}^2)(1.52\text{ s}) = 10.3\text{ m/s}.$$

At $t = 3.62\text{ s}$, the velocity is

$$v = v_0 - gt = 25.2\text{ m/s} - (9.8\text{ m/s}^2)(3.62\text{ s}) = -10.3\text{ m/s}.$$

Ch.2 Summary

- **Position:** The position x of a particle on an x axis locates the particle with respect to the origin, or zero point, of the axis. The position is either positive or negative, according to which side of the origin the particle is on.
- **Distance**: total length of travel
- **Displacement**: change in position
- **Average speed**: distance / time
- **Average velocity**: displacement / time
- **Instantaneous velocity: average velocity** measured over an infinitesimally small time

Ch.2 Summary

- **Instantaneous acceleration**: average acceleration measured over an infinitesimally small time
- **Average acceleration**: change in velocity divided by change in time
- **Deceleration**: velocity and acceleration have opposite signs
- **Constant acceleration**: equations of motion relate position, velocity, acceleration, and time
- **Freely falling objects**: constant acceleration g = 9.81 m/s^2

Homework

1. Landing with a speed of 81.9 m/s, ad traveling due south, a jet comes to east in 949 m. Assuming the jet shows with constant acceleration, find the magnitude and direction of its acceleration.
2. Starting from rest, a boat increases its speed to 4.12 m/s with constant acceleration, it takes the boat 4.77 s to reach this speed, how far has it traveled?
3. An animal can accelerate from rest to 25.0 m/s in 6.22 s. Assuming constant acceleration, (a) how far has the animal run in this time (b) calculate the distance covered by the animal in the first 3.11 s and the second 3.11 s.
4. A ball is launched from the ground vertically upward with an initial velocity of 19.6 m/s. Find the time that this ball spends before it reaches the ground again.
5. Saab advertises a car that goes from 0 to 60 mi/h in 6.2 a. what is the average acceleration of this car?
6. An airplane has an average acceleration of 5.6 m/s2 during takeoff. Hoe long does it take for the plane to reach a speed of 150 mi/h?
7. A ball is thrown straight upward with an initial velocity of +8.2 m/s. If the acceleration of the ball is -9.81 m/s^2, what is its velocity after (a) 0.50 s, and (b) 1.0 s?
8. A person steps off the end of a 3.00 m high diving board and drops to the water below. (a) How long does It take for the person to reach the water? (b) What is the person's speed on entering the water?