



Chapter 18

Fluid Dynamics

Units of Chapter 17: Fluid Statics

- GENERAL CONCEPTS OF FLUID FLOW
- STREAMLINES AND THE EQUATION OF CONTINUITY
- BERNOULLI'S EQUATION
- Application of Bernoulli's equation and the equation of continuity.
- VISCOSITY

Learning goals of this chapter

- **On completing this chapter, the student will be able to :**
- Differentiate between steady flow and nonsteady flow.
- Differentiate between compressible and incompressible flow.
- Differentiate between viscous flow and nonviscous flow.
- Define the equation of continuity.
- Define Bernoulli's equation.
- Apply both equation of continuity and Bernoulli's equation to solve problems.
- Define the viscosity.

General Concept of Fluid Flow

- Fluid flow can be steady or nonsteady:

We describe the flow in terms of these parameters; pressure, density, and flow velocity at every point of the fluid. If these variables are constant with time at any given point, the flow is called steady or laminar; if not, the flow is called nonsteady or turbulent flow.

- Fluid Flow can be compressible or incompressible.

If the density of the fluid is constant, independent of x , y , z and time, the flow is incompressible flow ; if not, the flow is called compressible.

- Fluid flow can be viscous or nonviscous.

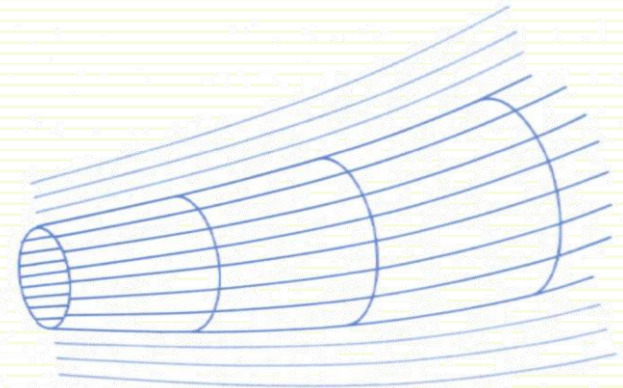
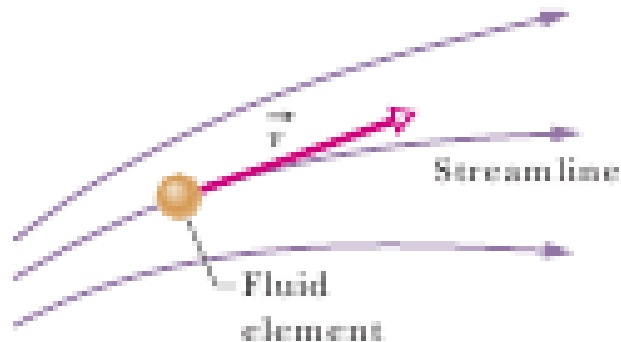
When the fluid flows such that no energy is dissipated through viscous forces, the flow is called nonviscous flow.

- Fluid flow can be rotational or irrotational

If an element of the moving fluid does not rotate about an axis through the center of mass of the element, the flow is called irrotational.

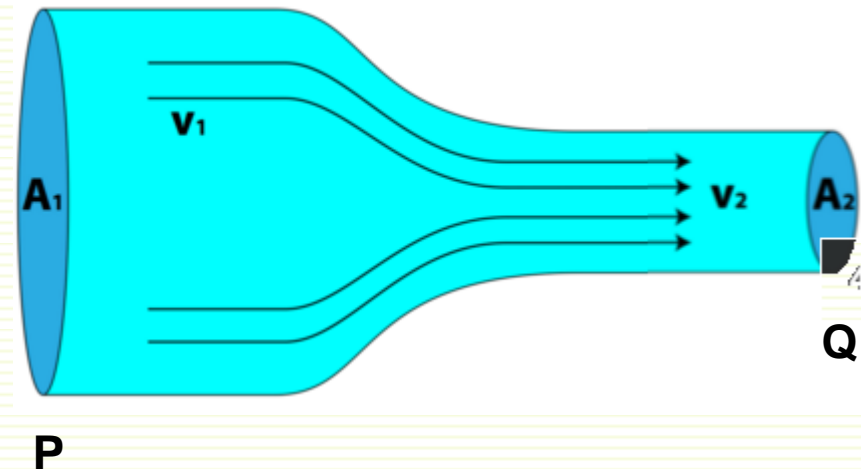
Streamlines and the equation of continuity

- **Streamlines:**
- In steady flow, the velocity at a given point is constant with time, i.e., the fluid's particle arriving at a given point will pass on with the same speed and same direction.
- The motion of every particle passing through a given point, thus follows the same path, called streamline.
- In principle we can draw a streamline through every point in the fluid. We select a finite number of streamlines to form a bundle.
- This tubular region is called a tube of flow.
- The fluid that enters at one end must leave at the other.



Streamlines and the equation of continuity

- Equation of continuity:
- Let us consider the case of steady flow of incompressible fluid (i.e., $\rho_1 = \rho_2$) through a non-uniform cross-sectional tube, where at the point **P**, the cross-sectional area is A_1 and speed of the flow is v_1 and at point **Q**, the cross-sectional area is A_2 and the speed is v_2 .
- Thus if an amount Δm of the fluid pass through the point **P** through a time interval Δt , then this amount of fluid must pass through point **Q** during the same interval.
- i.e $\Delta m_P = \rho_1 A_1 v_1 \Delta t = \Delta m_Q = \rho_2 A_2 v_2 \Delta t$
- Since the fluid is incompressible, then
- $A_1 v_1 = A_2 v_2$
- or: $Av = R = \text{constnat}$
- This is the equation of continuity.
- R is volume flow rate or volume flux.



Sample Problem 1

- The figure shows how the stream of water emerging from a faucet “necks down” as it falls. The cross-sectional areas are $A_0 = 1.2 \text{ cm}^2$ and $A = 0.35 \text{ cm}^2$. The two levels are separated by a vertical distance $h = 45 \text{ mm}$. What is the volume flow rate from the tap?

- **Solution:**

- From the equation of continuity,

- $A_0 v_0 = A v$

- From the equations of motions:

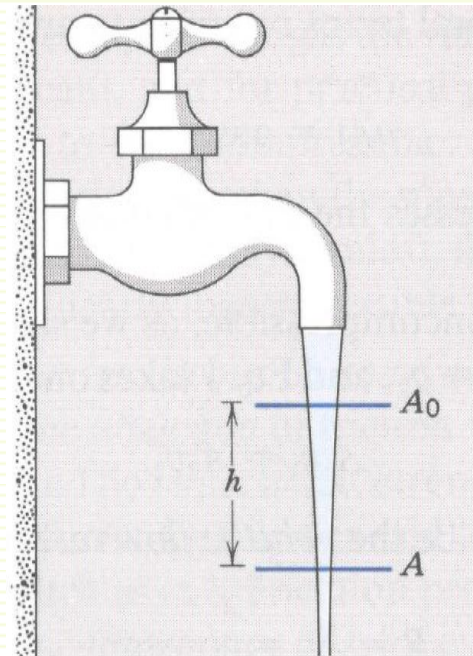
- $v^2 = v_0^2 + 2gh$

- Solving these equations we get

- $$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}} = \sqrt{\frac{2(9.8 \text{ m/s}^2)(0.045 \text{ m})(0.35 \text{ cm}^2)^2}{(1.2 \text{ cm}^2)^2 - (0.35 \text{ cm}^2)^2}} = 0.286 \text{ m/s}$$

- The volume flow rate R is

- $R = A_0 v_0 = (1.2 \text{ cm}^2)(28.6 \text{ cm/s}) = 34 \text{ cm}^3/\text{s}$



Bernoulli's Equation

- Or

$$P + \frac{1}{2}\rho v^2 + \rho g y = \text{constnat}$$

- This equation is called Bernoulli's equation for steady, incompressible nonviscous, and irrotational flow.
- For Horizontal flow, where $y_1 = y_2$ then we get

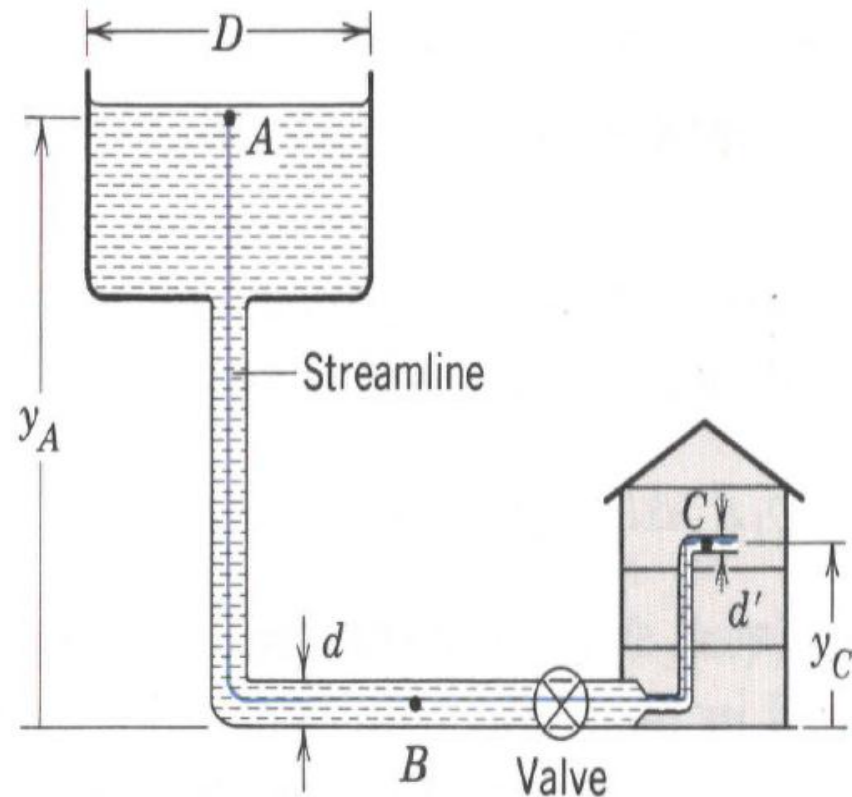
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

- If the fluid at reset, then $v_1 = v_2$, , and

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

Sample Problem 2

- A storage tower of height $h = 32\text{ m}$ and diameter $D = 3.0\text{ m}$ supplies water to a house. A horizontal pipe at the base of the tower has a diameter $d = 2.54\text{ cm}$. To satisfy the needs of the home, the supply pipe must be to deliver water at rate $R = 0.0025\text{ m}^3/\text{s}$, if water were flowing at the maximum rate, what would be the pressure in the horizontal pipe (at point B)?



Sample Problem 2

□ Solution

- In this case, $y_1 = h = 32\text{m}$, $y_2 = 0$ and $P_1 = P_o = \text{atmospheric pressure}$ and $P_2 = P_B$, and $v_1 = v_A$, $v_2 = v_B$.
- From the continuity equation, we get:

$$A_A v_A = A_B v_B = R$$

- Then

$$v_A = \frac{R}{A_A} = \frac{0.0025 \text{ m}^3}{\pi (1.5 \text{ m})^2} = 3.5 \times 10^{-4} \frac{\text{m}}{\text{s}}, \quad v_B = \frac{R}{A_B} = \frac{0.0025 \text{ m}^3}{\pi (0.0127 \text{ m})^2} = 4.9 \text{ m/s}$$

- Applying Bernoulli's equation: $P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$

$$P_B = P_o + \frac{1}{2} \rho (v_A^2 - v_B^2) + \rho g h \approx P_o - \frac{1}{2} \rho v_B^2 + \rho g h$$

Sample Problem 2

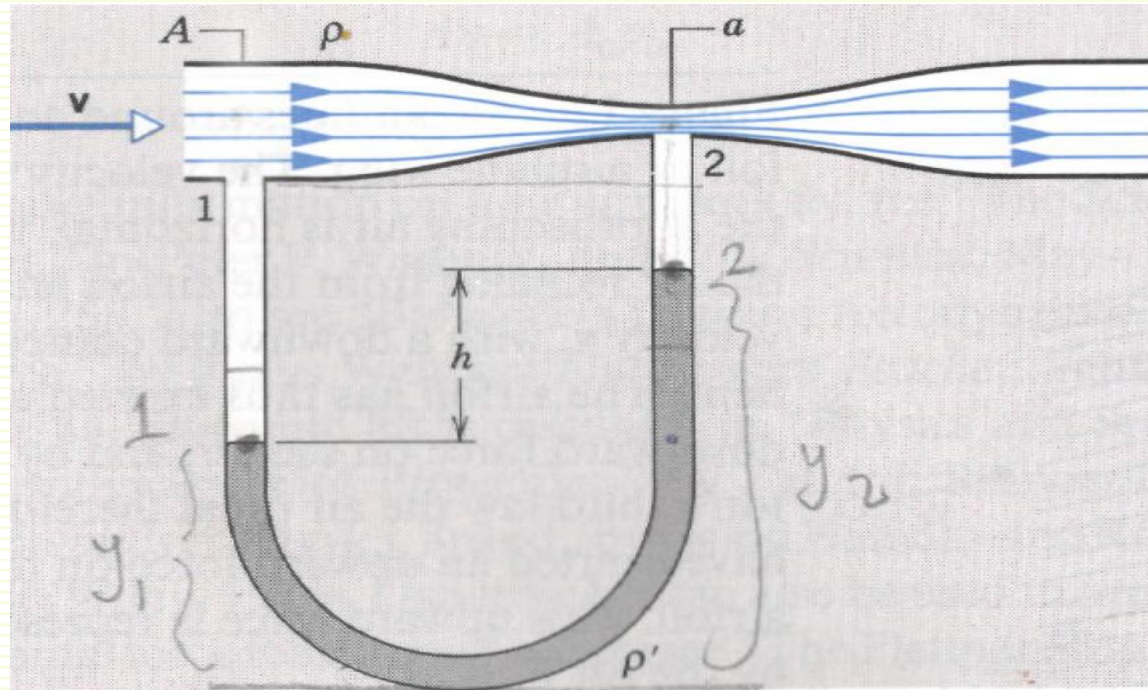
$$\begin{aligned} p_B &= p_0 + \rho gh - \frac{1}{2}\rho v_B^2 \\ &= 1.01 \times 10^5 \text{ Pa} + (1.0 \times 10^3 \text{ kg/m}^3)(9.8 \text{ m/s}^2)(32 \text{ m}) \\ &\quad - \frac{1}{2}(1.0 \times 10^3 \text{ kg/m}^3)(4.9 \text{ m/s})^2 \\ &= 1.01 \times 10^5 \text{ Pa} + 3.14 \times 10^5 \text{ Pa} - 0.12 \times 10^5 \text{ Pa} \\ &= 4.03 \times 10^5 \text{ Pa}. \end{aligned}$$

If the water in the horizontal pipe were not flowing (that is, if the valve were closed), the *static* pressure at *B* would include only the first two terms above, which give $4.15 \times 10^5 \text{ Pa}$. The pressure when the water is flowing is reduced from this static value by the amount of the dynamic pressure.

Applications of Bernoulli's equation and the equation of continuity

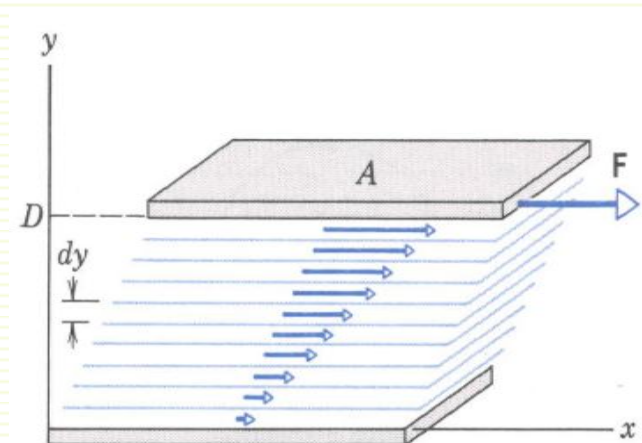
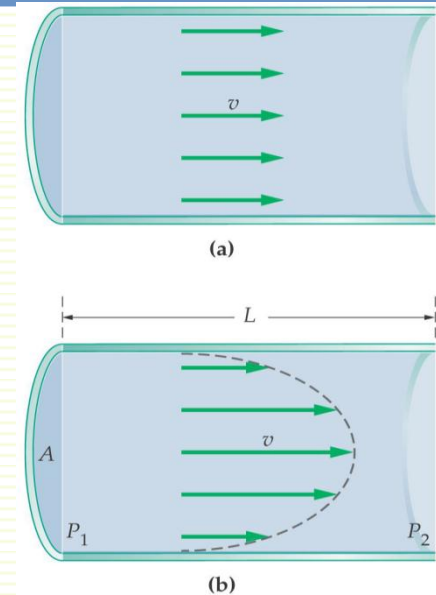
- Venturi Meter.
- It is a gauge to measure the flow speed of a fluid of density ρ in a pipe with cross-sectional area A , as shown in the figure. At the throat the area is reduced to a , and the manometer tube (filled with liquid of density ρ') is attached as shown. Applying Bernoulli's equation at points 1 and 2, we get

$$\square v = a \sqrt{\frac{2\rho'gh}{\rho(A^2 - a^2)}}$$



Viscosity

- Viscosity is a form of friction felt by fluids as they flow along surfaces.
- A viscous fluid will have zero velocity next to the walls and maximum velocity in the center.
- Consider the case of steady flow of a liquid, and let the liquid to flow in a shape of parallel plates sliding on each other.
- When a force F is applied to the upper plate, so that it is in motion at constant velocity v relative to lower plate which is in contact to the wall of the container, will be at rest.
- Let the area of the layer of the fluid is A , then we can define the coefficient of viscosity η as:
$$\eta = \frac{F/A}{dv/dy}$$
- Where dv is the velocity difference between two adjacent layers and dy is their separation.



Viscosity

- SI unit of viscosity is $\text{N}\cdot\text{s}/\text{m}^2$.
- cgs unit of viscosity is $\text{dyne}\cdot\text{s}/\text{cm}^2$, which is called the poise.
- 1 poise = $0.1 \text{ N}\cdot\text{s}/\text{m}^2$

TABLE 1 VISCOSITIES OF SELECTED FLUIDS

<i>Fluid</i>	η ($\text{N}\cdot\text{s}/\text{m}^2$)
Glycerine (20°C)	1.5
Motor oil ^a (0°C)	0.11
Motor oil ^a (20°C)	0.03
Blood (37°C)	4.0×10^{-3}
Water (20°C)	1.0×10^{-3}
Water (90°C)	0.32×10^{-3}
Gasoline (20°C)	2.9×10^{-4}
Air (20°C)	1.8×10^{-5}
CO_2 (20°C)	1.5×10^{-5}

Homework

1. Briefly describe what is meant by each of the following (a) Steady fluid flow; (b) non steady fluid flow; (c) Rotational fluid flow; (d) irrotational fluid flow; (e) compressible fluid flow; (f) incompressible fluid flow; (g) viscous fluid flow; (h) nonviscous fluid flow.
2. Water travels through a 9.6 cm diameter fire hose with speed of 1.3 m/s. At the end of the hose, the water flows out through a nozzle whose diameter is 2.5 cm. What is the speed of the water coming out of the nozzle?
3. In the last question, suppose the pressure in the fire hose is 350 kPa. (a) is the pressure in the nozzle greater than, less than, or equal 350 kPa. (b) Find the pressure in the nozzle.
4. A pipe of diameter 34.5 cm carries water moving at 2.62 m/s. How long will it take to discharge 1600 m³ of water?
5. How much work is done by pressure in forcing 1.4 m³ of water through a 13-mm internal diameter pipe if the difference in pressure at the two ends of the pipe is 1.2 atm?