Electrical Engineering Department

Signal Analysis (802321) – G1

Dr. Mouaaz Nahas

First Term (1436-1437), Final Exam

Tuesday 18/03/1437 H

Solution:

Accredited ABET Engineering Accreditation Commission

الاسم:

↓↓↓↓↓↓↓↓↓↓↓↓↓↓ الرقم الجامعي: ↓↓↓↓↓↓↓↓↓↓↓↓↓↓

Start from here →

CLO #:	CLO:	Mark:
CLO 1	*Classify signals in terms of their properties, and calculate the energy and power for a wide range of continuous-time signals.*	5

Q1. Choose the correct answer.	5

1. A signal with infinite energy has a finite power if:

a) The signal amplitudes approaches zero as the time goes to infinity

b) The signal is periodic or has a statistical regularity

c) The signal is aperiodic

d) None of the above

2. The power value of the sinusoidal signal $g(t) = C \sin(\omega t + \theta)$ depends on:

a) C only

b) C and θ

c) C and ω

d) ω and θ

3. Given the signal $x(t) = C_1 \cos \omega_1 t + C_2 \cos \omega_2 t + C_3 \cos \omega_3 t$. The power of $x(t)$ is calculated as:

a) $\frac{1}{2}[C_1^2 + C_2^2 + C_3^2]$

b) $\frac{1}{2}[C_1 + C_2 + C_3]$

c) $\frac{1}{2}[C_1 + C_2 + C_3]^2$

d) $[C_1^2 + C_2^2 + C_3^2]$

4. If $g_2(t) = k\, g_1(t)$, the energy of the signal $g_2(t)$ equals:

a) The energy of $g_1(t)$

b) The energy of $g_1(t)$ multiplied by $2k$

c) The energy of $g_1(t)$ multiplied by k^2

d) The energy of $g_1(t)$ multiplied by k

5. A signal is a power signal if:

a) $0 < \int_{-\infty}^{\infty} |g(t)|^2 dt < \infty$

b) $0 \le \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty$

c) $\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty$

d) $0 < \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |g(t)|^2 dt < \infty$

CLO #:	CLO:	Mark:
CLO 2	***Apply some time operations to signals and analyze the unit impulse, unit step, exponential and sinusoidal functions.***	___ 8

Q2. Choose the correct answer.	___ 8

1. If a signal is time-shifted, scaled and inverted, which order can be followed?

a) Scaling → shifting → inversion.
b) Shifting → inversion → scaling.
c) Inversion → shifting → scaling.
d) All the above.

2. If $g_2(t) = -g_1(-t + 3)$, the energy of the signal $g_2(t)$ equals:

a) The energy of $g_1(t)$
b) The energy of $g_1(t)$ plus 3
c) The energy of $g_1(t)$ multiplied by -1
d) The energy of $g_1(t)$ multiplied by 3

3. Which relation is correct?

a) $\int_{-\infty}^{\infty} \delta(t)dt = u(t)$
b) $\frac{du(t)}{dt} = \delta(t)$
c) $\frac{d\delta(t)}{dt} = u(t)$
d) a) and b)

4. The continuous-time unit step function is classified as:

a) Energy signal
b) Neither energy nor power signal
c) Power signal
d) Cannot decide

5. Which relation is correct?

a) $u[n] = \delta[n] + \delta[n-1]$
b) $\delta[n] = u[n] + u[n-1]$
c) $u[n] = \delta[n] - \delta[n-1]$
d) $\delta[n] = u[n] - u[n-1]$

6. Which one of the following signals is <u>complex</u>?

a) $|7e^{-j5000\pi t}|$
b) $j\left(\frac{1}{j}t + jt^2\right)$
c) $\frac{1}{2}\left[e^{j50\pi t} + e^{j50\pi t}\right]$
d) $\frac{1}{2}\left[e^{j50\pi t} + e^{-j50\pi t}\right]$

7. If the complex exponential signals $x_1(t)$ (with the frequency ω_1) and $x_2(t)$ (with the frequency ω_2) are harmonically related, which relationship is correct for ω_1 and ω_2:

a) $\omega_2 = \omega_1$
b) $\omega_2 = 5/2\ \omega_1$
c) $\omega_2 = 6/2\ \omega_1$
d) $\omega_2 = 2/3\ \omega_1$

8. The signal $\sin \omega_0 t$ can be expressed in terms of complex exponential signal as:

a) $\frac{1}{2}[e^{j\omega_0 t} - e^{-j\omega_0 t}]$
b) $\frac{1}{j2}[e^{j\omega_0 t} - e^{-j\omega_0 t}]$
c) $\frac{1}{2}[e^{j\omega_0 t} + e^{-j\omega_0 t}]$
d) $\frac{1}{j2}[e^{j\omega_0 t} + e^{-j\omega_0 t}]$

CLO #:	CLO:	Mark:
CLO 3	*Demonstrate the understanding of orthogonal signals.*	___ 3

Q3. Choose the correct answer.	___ 3

1. The inner product of the two real signals $g(t)$ and $x(t)$ is defined as:

a) $\int x(t)dt \int x(t)dt$
b) $g(t)\, x(t)$
c) The area under the curve $g(t) + x(t)$
d) The area under the curve $g(t)\, x(t)$

2. The signals $g(t)$ and $x(t)$ are said to be orthogonal over the period $[t_1: t_2]$ if:

a) $\int_{t_1}^{t_2} x(t)g(t)dt = 0$
b) $\int_{t_1}^{t_2} g(t)x(t)dt = 0$
c) Their inner product equals to zero.
d) All the above

3. Which two signals are not orthogonal?

a) sin t and cos t over the interval $[0, \frac{1}{2}\pi]$
b) sin t and cos t over the interval $[0, \pi]$
c) sin t and cos t over the interval $[0, 2\pi]$
d) sin t and cos t over the interval $[-\pi/2, 3\pi/2]$

CLO #:	CLO:	Mark:
CLO 5	*Characterize systems in terms of their interconnections and properties, and analyze LTI systems.*	___ 12

Q4. Choose the correct answer.	___ 6

1. In the parallel interconnection of systems:

a) The total output is the output of the last system only
b) The input is processed by all systems simultaneously
c) The output of the second system is fed back and added to external input
d) a) and b)

2. Which one of the following systems is nonlinear?

a) $y(t) = 2\,x(t) + 3$
b) $y[n] = n\,x[n]$
c) $y(t) = (t - 1)\,x(t)$
d) $y(t) = 2\,x(t)$

3. The system $y[n + 1] = x^3[1 - n]$ is:

a) Linear and time-invariant
b) Linear and time-variant
c) Nonlinear and time-invariant
d) Nonlinear and time-variant

4. The system given in the previous question is:

a) Memoryless and causal
b) Memoryless and non-causal
c) Non-memoryless and causal
d) Non-memoryless and non-causal

5. Which one of the following systems is causal?

a) $y(t) = x(-t+2)$

b) $y(t) = x(-2t)$

c) $x(t) = \sum_{n=t-1}^{\infty} x(n)$

d) $x(t) = \sum_{n=-\infty}^{t-1} x(n)$

6. In memoryless systems, the output at any instant t depends on input samples at:

a) The same and / or past time

b) The same and / or future time

c) The same time only

d) The past time only

Q5. Show mathematically that the system $y(t) = t\,x(t)$ is time-variant (i.e. not time-invariant).	$\overline{\quad}$ 3

① $x_1(t) \longrightarrow y_1(t) = t\,x_1(t)$

$x_2(t) = x_1(t-t_0)$

$\Rightarrow \quad y_2(t) = t\,x_2(t) = t\,x_1(t-t_0) \; \ldots\ldots (1)$

② $y_1(t-t_0) = (t-t_0)\,x_1(t-t_0) \; \text{------} (2)$

$\Rightarrow \quad y_2(t) \neq y_1(t-t_0) \Rightarrow$ The system is time-variant

Q6. Show mathematically that the system $y(t) = x^2(t)$ is nonlinear.	$\overline{\quad}$ 3

Let $x_1(t) \longrightarrow y_1(t) = x_1^2(t)$ & $x_2(t) \longrightarrow y_2(t) = x_2^2(t)$

$x_3(t) = a\,x_1(t) + b\,x_2(t)$

$\Rightarrow y_3(t) = x_3^2 = \left[a\,x_1(t) + b\,x_2(t)\right]^2 = a^2 x_1^2(t) + b^2 x_2^2(t) + 2ab\,x_1(t)\,x_2(t)$

$\text{------} (1)$

$a y_1(t) + b y_2(t) = a\,x_1^2(t) + b\,x_2^2(t) \; \text{------} (2)$

$(1) \neq (2) \Rightarrow$ The system is nonlinear.

CLO #:	CLO:	Mark:
CLO 6	*Express a periodic signal in terms of Exponential and Trigonometric Fourier Series.*	$\frac{}{14}$

Q7. In Fourier series theory:

a) The exponential Fourier series is expressed as:

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \quad \underline{\underline{\text{or}}} \quad \sum_{k=-\infty}^{\infty} a_k e^{jk\left(\frac{2\pi}{T}\right)t}, \quad k = 0, \pm 1, \ldots$$

b) The constant coefficient of the Fourier series is calculated as:

$$a_0 = \frac{1}{T}\int_T x(t)\,dt$$

c) The other Fourier series coefficients are calculated as:

$$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt$$

d) For real signal $x(t)$ and real coefficients a_k, the Fourier series is simplified to:

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} a_k \cos k\omega_0 t$$

$\frac{}{4}$

Q8. Given the impulse train signal $\delta_{T_0}(t)$ where $T_0 = 2$ sec:

a) Find the Fourier series constant coefficient a_0. 1 Mark

b) Find the Fourier series coefficients a_k. 2 Mark

c) Write the Fourier series using complex exponential form. 1 Mark

$\frac{}{4}$

a) $$a_0 = \frac{1}{T}\int_T x(t)\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{1/2} \delta_{T_0}(t)\,dt = \frac{1}{2}(1) = \frac{1}{2}$$

b) $$a_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_{-\frac{1}{2}}^{1/2} \delta_{T_0}(t)\, e^{-jk\omega_0 t}\,dt$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} \delta_{T_0}(t)\, e^{-jk\omega_0 (0)}\,dt$$

$\delta(t)\,g(t) = g(0)\,\delta(t)$

$$= \frac{1}{2}\int_{-1/2}^{1/2} \delta_{T_0}(t)\,dt = \frac{1}{2}(1) = \frac{1}{2}$$

c) $$x(t) = \frac{1}{2}\sum_{k=-\infty}^{\infty} e^{jk\omega_0 t} = \frac{1}{2}\sum_{k=-\infty}^{\infty} e^{jk\left(\frac{2\pi}{T_0}\right)t} = \frac{1}{2}\sum_{k=-\infty}^{\infty} e^{jk(\pi)t}$$

Q9. Choose the correct answer.

$\overline{}$
6

1. Which diagram is correct to convert a periodic signal $x(t)$ from time-domain to frequency-domain?

a) $x(t) \overset{F}{\Rightarrow} X(j\omega)$

b) $x(t) \overset{FS}{\Rightarrow} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t} \overset{F}{\Rightarrow} X(j\omega)$

c) $x(t) \overset{F}{\Rightarrow} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

d) $x(t) \overset{F}{\Rightarrow} X(j\omega) \overset{FS}{\Rightarrow} \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$

2. For the signal $x(t) = 4 \cos \omega_0 t$, the Fourier series coefficients are:

a) $a_0 = 0, a_1 = 2, a_{-1} = 2$

b) $a_0 = 0, a_1 = 2/j, a_{-1} = -2/j$

c) $a_0 = 0, a_1 = 4, a_{-1} = 4$

d) $a_0 = 0, a_1 = 2, a_{-1} = -2$

3. Which one of the following periodic signals is absolutely integrable?

a) $x(t) = 1/(2t)$, $0 \le t \le 1$

b) $x(t) = t^{-2}$, $0 \le t \le 1$

c) $x(t) = \ln t$, $½ \le t \le 1$

d) $x(t) = \ln t$, $0 \le t \le 1$

4. Fourier series is expressed using the formula $x(t) = a_o + 2\sum_{k=1}^{\infty} \frac{a_{-k}}{j} \sin k\omega_0 t$, when:

a) $x(t)$ is real but $\{a_k\}$ are complex

b) $x(t)$ is complex but $\{a_k\}$ are imaginary

c) $x(t)$ is complex but $\{a_k\}$ are real

d) $x(t)$ is real but $\{a_k\}$ are imaginary

5. Given the signal $x(t) = \cos^2 3\pi t$. If the fundamental frequency in radian per second is 3π:

a) $a_0 = ½$, $a_1 = 0$, $a_2 = ½$

b) $a_0 = ½$, $a_1 = ¼$, $a_2 = 0$

c) $a_0 = 0$, $a_1 = ¼$, $a_2 = 0$

d) $a_0 = ½$, $a_1 = 0$, $a_2 = ¼$

6. If a_k are the Fourier series coefficients of a signal $g(t)$, the Fourier series coefficients of the signal $dx(t)/dt$ are:

a) $(1/jk\omega_0)\, a_k$

b) $jk\omega_0\, a_k$

c) $(-1/jk\omega_0)\, a_k$

d) $-jk\omega_0\, a_k$

CLO #:	CLO:	Mark:
CLO 7	*Determine the Fourier Transform and the Inverse Fourier Transform of periodic and aperiodic signals, and apply some properties of the Fourier Transform to determine FT of complex signals.*	14

Q10. The continuous-time Fourier transform and inverse Fourier transform equations are:

a) $X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

b) $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$

2

Q11. Choose the correct answer.

6

1. The Fourier transform of the signal $x(t) = e^{-at}u(t)$ equals to:

a) $F\{e^{-a|t|} - e^{at}u(-t)\}$
b) $F\{e^{-a|t|}\} - F\{e^{at}u(-t)\}$
c) $F\{e^{at}u(-t)\} - F\{e^{-a|t|}\}$
d) $F\{e^{-a|t|}\} + F\{e^{at}u(-t)\}$

2. Which Fourier transform pair is correct?

a) $1 \overset{F}{\Leftrightarrow} 2\pi\delta(\omega)$
b) $\sin 20\pi t \overset{F}{\Leftrightarrow} \frac{\pi}{j}[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)]$
c) $\delta(t) \overset{F}{\Leftrightarrow} 2\pi$
d) $\cos 20\pi t \overset{F}{\Leftrightarrow} \frac{\pi}{j}[\delta(\omega - 20\pi) + \delta(\omega + 20\pi)]$

3. The signum function sgn(t) can be defined as:

a) $\text{sgn}(t) = u(t) + u(-t)$
b) $\text{sgn}(t) = 2u(t) - 1$
c) $\text{sgn}(t) = u(t) - u(-t)$
d) b) and c)

4. The Fourier transform of the function sgn(t) is:

a) $1 / j\omega$
b) $2 / j\omega + 2\pi\, \delta(\omega) - 1$
c) $2 / j\omega$
d) $2 / j\omega - \pi\, \delta(\omega)$

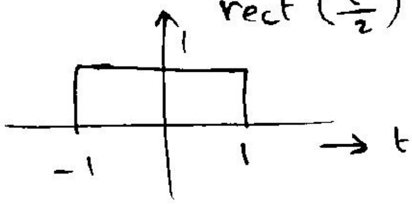
6. The Fourier transform of the signal $x(t) = e^{3t}\, u(-t)$ is:

a) $X(j\omega) = \frac{1}{3+j\omega}$
b) $X(j\omega) = \frac{1}{3-j\omega}$
c) $X(j\omega) = \frac{6}{9+\omega^2}$
d) $X(j\omega) = \frac{3}{9+\omega^2}$

Q12. Given the signal: $x(t) = rect\left(\frac{t}{2}\right)$

a) Sketch $x(t)$.	1 Mark	6
b) Find the Fourier transform $X(j\omega)$ in terms of sinc function.	3 Marks	
c) Sketch the Fourier transform $X(j\omega)$.	2 Marks	

(a)



(b) $$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-1}^{1} e^{-j\omega t}\, dt = -\frac{1}{j\omega}\left[e^{-j\omega t}\right]_{-1}^{1}$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega} - e^{j\omega}\right) = \frac{1}{j\omega}\left(e^{j\omega} - e^{-j\omega}\right) = \frac{2\sin(\omega)}{\omega}$$

since $\mathrm{sinc}\,\theta = \frac{\sin\theta}{\theta} \Rightarrow \boxed{X(j\omega) = 2\,\mathrm{sinc}(\omega)}$

(c) To sketch $X(j\omega)$: if $\omega = 0 \Rightarrow X(j\omega) = \frac{2\sin 0}{0} = \frac{0}{0}$

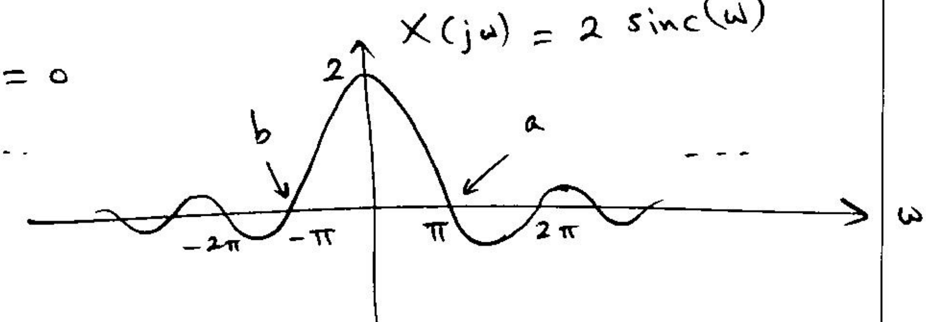
$$\Rightarrow X(j\omega) = \lim_{\omega \to 0}\left[\frac{\frac{d}{d\omega}(2\sin\omega)}{\frac{d}{d\omega}(\omega)}\right] = \lim_{\omega \to 0} \frac{2\cos\omega}{1} = \frac{2}{1} = 2$$

L'Hopital's rule

For a & b:

$2\,\mathrm{Sinc}(\omega) = 0 \Rightarrow \sin\omega = 0$

$\Rightarrow \omega = \pm\pi, \pm 2\pi, \ldots$



Recall that at $\omega = 0 \Rightarrow X(j\omega) = 2T_1 \quad (T_1 = 1) \Rightarrow X(j\omega) = 2$

& $X(j\omega) = 0$ at $\omega = \pm\pi/T_1, \pm 2\pi/T_1, \ldots \Rightarrow \omega = \pm\pi, \pm 2\pi, \ldots$

GOOD LUCK