Electrical Engineering Department

Signal Analysis (802321) – G1

Dr. Mouaaz Nahas

First Term (1435-1436), Final Exam, Saturday 12/03/1436 H

الاسم:
↓↓↓↓↓↓↓↓↓↓↓↓↓↓ الرقم الجامعي: ↓↓↓↓↓↓↓↓↓↓↓↓↓↓

Start from here → | | | | | | | | |

Accredited ABET Engineering Accreditation Commission

==

Q 1. Choose the correct answer: 21 *Marks*, 1 *Each*

1. A signal with infinite energy has a finite power if:

a) The signal amplitudes approaches zero as the time goes to infinity
b) The signal is periodic or has a statistical regularity
c) The signal is aperiodic
d) None of the above

2. The unit step function is classified as:

a) Energy signal
b) Neither energy nor power signal
c) Power signal
d) Cannot decide

3. The power value of the sinusoidal signal $g(t) = C \sin(\omega t + \theta)$ depends on:

a) The maximum amplitude only
b) The maximum amplitude and phase angle
c) The maximum amplitude and frequency
d) The maximum phase angle and frequency

4. If $g_2(t) = k\, g_1(t)$, the energy of the signal $g_2(t)$ equals:

a) The energy of g_1(t)
b) The energy of g_1(t) multiplied by $2k$
c) The energy of $g_1(t)$ multiplied by k^2
d) The energy of $g_1(t)$ multiplied by k

5. If $g_2(t) = -\, g_1(-\, t + 3)$, the energy of the signal $g_2(t)$ equals:

a) The energy of $g_1(t)$
b) The energy of $g_1(t)$ plus 3
c) The energy of $g_1(t)$ multiplied by −1
d) The energy of $g_1(t)$ multiplied by 3

6. The signal $g(-2t + 5)$ is:

a) $g(t)$ advanced by 5, then time-inverted, then compressed by 2.
b) $g(t)$ inverted in time, then compressed by 2, then delayed by 5/2.
c) $g(t)$ inverted in time, then compressed by 2, then advanced by 5.
d) a) and b)

7. If the complex exponential signals $x_1(t)$ (with the frequency ω_1) and $x_2(t)$ (with the frequency ω_2) are harmonically related, which relationship is correct for ω_1 and ω_2:

a) $\omega_1 = \omega_2$
b) $\omega_1 = 5/2\ \omega_2$
c) $\omega_1 = 6/2\ \omega_2$
d) $\omega_1 = 2/6\ \omega_2$

8. The complex number $(1 - j) / (1 + j)$ equals:

a) $e^{\,j\pi/2}$
b) $e^{\,j\pi/4}$
c) $e^{\,-j\pi/4}$
d) $e^{\,-j\pi/2}$

9. Which one of the following systems is nonlinear:

a) $y(t) = 2\,x(t) + 3$

b) $y[n] = n\,x[n]$

c) $y(t) = (t-1)\,x(t)$

d) $y(t) = 2\,x(t)$

10. The system $y[n+1] = x^3[1-n]$ is:

a) Linear and time-invariant

b) Linear and time-variant

c) Nonlinear and time-invariant

d) Nonlinear and time-variant

11. The system given in Q.10 is:

a) Memoryless and causal

b) Memoryless and non-causal

c) Non-memoryless and causal

d) Non-memoryless and non-causal

12. In the parallel interconnection of systems:

a) The total output is the output of the last system

b) The input is processed by all systems simultaneously

c) The output of the second system is fed back and added to external input

d) b) and c)

13. For the signal $x(t) = 4\cos\omega_0 t$, the Fourier series coefficients are:

a) $a_0 = 0,\ a_1 = 2,\ a_{-1} = 2$

b) $a_0 = 0,\ a_1 = 2/j,\ a_{-1} = -2/j$

c) $a_0 = 0,\ a_1 = 4,\ a_{-1} = 4$

d) $a_0 = 0,\ a_1 = 2,\ a_{-1} = -2$

14. Which one of the following periodic signals is absolutely integrable:

a) $x(t) = 1/(2t)\ ,\ 0 \le t \le 1$

b) $x(t) = t^{-2}\ ,\ 0 \le t \le 1$

c) $x(t) = \ln t\ ,\ ½ \le t \le 1$

d) $x(t) = \ln t\ ,\ 0 \le t \le 1$

15. Given a periodic square wave which is defined over one period as: $x(t) = rect\left(\frac{t}{2}\right), \quad -2 \le t \le 2$

The Fourier series coefficients a_k are:

a) $a_0 = ½\ ,\ a_k = (1/k\pi)\sin(k\pi/2)$ for $k \ne 0$

b) $a_0 = ½\ ,\ a_k = (1/k\pi)\sin(k\pi)$ for $k \ne 0$

c) $a_0 = 1\ ,\ a_k = (1/k\pi)\sin(k\pi/2)$ for $k \ne 0$

d) $a_0 = 1\ ,\ a_k = (1/k\pi)\sin(k\pi)$ for $k \ne 0$

16. Fourier series is expressed using the formula $x(t) = a_o + 2\sum_{k=1}^{\infty}\frac{a_{-k}}{j}\sin k\omega_0 t$, when:

a) $x(t)$ is real but $\{a_k\}$ are complex

b) $x(t)$ is complex but $\{a_k\}$ are imaginary

c) $x(t)$ is complex but $\{a_k\}$ are real

d) $x(t)$ is real but $\{a_k\}$ are imaginary

17. Given the impulse train signal $\delta_{T_0}(t)$ where $T_0 = ½$ sec, the Fourier series coefficients of $\delta_{T_0}(t)$ are:

a) $a_k = 1/2$ for all values of k

b) $a_0 = 2\ ,\ a_k = 4$ for $k \ne 0$

c) $a_k = 2$ for all values of k

d) $\delta_{T_0}(t)$ does not have Fourier series

18. Inverse Fourier transform is calculated as:

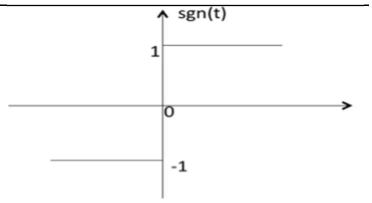
a) $x(t) = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$

b) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$

c) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{-j\omega t}\,d\omega$

d) $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\,dt$

19. The signum function sgn(t) is shown on the right. This function can be defined as:



a) $\text{sgn}(t) = u(t) + u(-t)$

b) $\text{sgn}(t) = 2u(t) - 1$

c) $\text{sgn}(t) = u(t) - u(-t)$

d) b) and c)

20. The Fourier transform of the function sgn(t) is:

a) $1 / j\omega$

b) $2 / j\omega + 2\pi\,\delta(\omega) - 1$

c) $2 / j\omega$

d) $2 / j\omega - \pi\,\delta(\omega)$

21. For the linear time-invariant system, the input and output are related as:

a) $Y(j\omega) = H(j\omega) * X(j\omega)$

b) $y(t) = x(t) * h(t)$

c) $x(t) = h(t) * y(t)$

d) $Y(j\omega) = H(j\omega) / X(j\omega)$

==

Q 2. Find the Fourier series coefficients a_k of the following two periodic signals: 10 *Marks*

a) $y_1(t) = t \qquad -1 \leq t \leq 1$

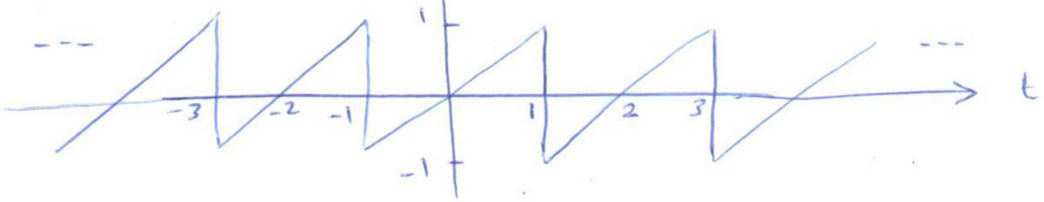
b) $y_2(t) = \frac{1}{2}\, t \qquad 0 \leq t \leq 2$

Hint: you can use Fourier series integral only or Fourier series integral and Fourier series properties.

Solution:

a) See Note “Fourier Series of Triangular & Sawtooth Waves (Examples)” – Pages 6, 7.

P.3.22 (Pg 255)



$$y_s(t) = t \ , \ -1 \le t \le 1 \qquad T = 2 \text{ sec}$$

$$\omega_0 = \frac{2\pi}{2} = \pi \text{ rad/s}$$

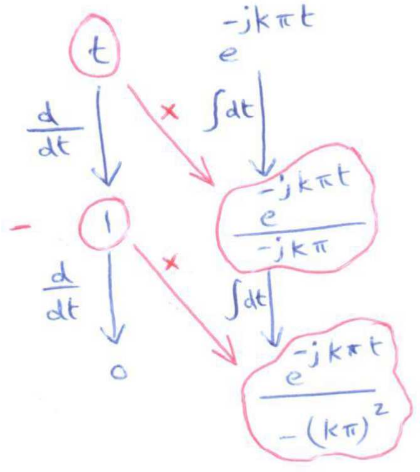
$$a_0 = \frac{1}{T}\int_T y_s(t)\,dt = \frac{1}{2}\int_{-1}^{1} t\,dt = \frac{1}{2}\Big[t\Big]_{-1}^{1} = 0$$

$$a_k = \frac{1}{2}\int_{-1}^{1} t\,e^{-jk\pi t}\,dt \quad , \quad \text{since } \omega_0 = \pi \text{ rad/s}$$

$$= \frac{1}{2}\left[\frac{t\,e^{-jk\pi t}}{-jk\pi} + \frac{e^{-jk\pi t}}{(k\pi)^2}\right]_{-1}^{1}$$

Recall:

$$\int t\,e^{-jk\pi t}\,dt = \frac{t\,e^{-jk\pi t}}{-jk\pi} + \frac{e^{-jk\pi t}}{(k\pi)^2}$$



$$\Rightarrow a_k = \frac{1}{2}\left[\left(\frac{e^{-jk\pi}}{-jk\pi} + \frac{e^{-jk\pi}}{(k\pi)^2}\right) - \left(\frac{-e^{jk\pi}}{-jk\pi} + \frac{e^{jk\pi}}{(k\pi)^2}\right)\right]$$

If k is odd: $e^{-jk\pi} = -1$ & $e^{jk\pi} = -1$

$$\Rightarrow a_k = \frac{1}{2}\left[\left(\frac{-1}{-jk\pi} + \frac{-1}{(k\pi)^2}\right) - \left(\frac{+1}{-jk\pi} + \frac{-1}{(k\pi)^2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{1}{jk\pi} - \cancel{\frac{1}{(k\pi)^2}} + \frac{1}{jk\pi} + \cancel{\frac{1}{(k\pi)^2}}\right]$$

$$= \frac{1}{2}\left[\frac{1}{jk\pi} + \frac{1}{jk\pi}\right] = \frac{1}{2}\left(\frac{2}{jk\pi}\right)$$

$$= \frac{1}{jk\pi} = -j\frac{1}{k\pi}$$

If k is even: $e^{-jk\pi} = 1$ & $e^{jk\pi} = 1$

$$\Rightarrow a_k = \frac{1}{2}\left[\left(\frac{1}{-jk\pi} + \frac{1}{(k\pi)^2}\right) - \left(\frac{-1}{-jk\pi} + \frac{1}{(k\pi)^2}\right)\right]$$

$$= \frac{1}{2}\left[-\frac{1}{jk\pi} + \cancel{\frac{1}{(k\pi)^2}} - \frac{1}{jk\pi} - \cancel{\frac{1}{(k\pi)^2}}\right]$$

$$= -\frac{1}{jk\pi} = j\frac{1}{k\pi}$$

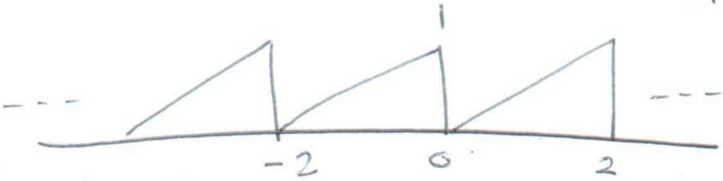
$$\Rightarrow a_k = \begin{cases} 0 & , \ k = 0 \\ -j\frac{1}{k\pi} & , \ k \text{ is odd} \\ j\frac{1}{k\pi} & , \ k \text{ is even} \end{cases}$$

OR:

$$a_k = \begin{cases} 0 & , \ k = 0 \\ j\frac{(-1)^k}{k\pi} & , \ k \neq 0 \end{cases}$$

b) See Note “Fourier Series of Triangular & Sawtooth Waves (Examples)” – Pages 4, 5.

$y_4(t) = \frac{1}{2}t \quad 0 \le t \le 2$



$T = 2$ sec

$\omega_0 = \frac{2\pi}{T} = \pi$ (rad/

$$a_0 = \frac{1}{T}\int_T y_4(t)\,dt = \frac{1}{2}\int_0^2 \frac{1}{2}t\,dt$$

$$= \frac{1}{4}\left[\frac{t^2}{2}\right]_0^2 = \frac{1}{4}\left(\frac{4}{2} - 0\right) = \frac{1}{2}$$

$$a_k = \frac{1}{T}\int_T y_4(t)\,e^{-jk\omega_0 t}\,dt = \frac{1}{2}\int_0^2 \frac{1}{2}t\,e^{-jk\pi t}\,dt$$

$$= \frac{1}{4}\int_0^2 t\,e^{-jk\pi t}\,dt$$

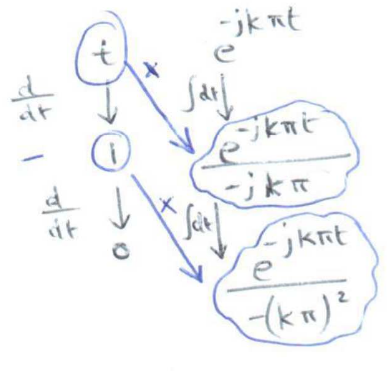
$$\int t\,e^{-jk\pi t}\,dt = t\int e^{-jk\pi t}\,dt - \iint e^{-jk\pi t}\,dt \times \frac{d}{dt}(t)$$

$$= \frac{t\,e^{-jk\pi t}}{-jk\pi} - \int \frac{e^{-jk\pi t}}{-jk\pi}\,dt \times 1$$

$$= \frac{t\,e^{-jk\pi t}}{-jk\pi} - \frac{e^{-jk\pi t}}{-(k\pi)^2} = \frac{t\,e^{-jk\pi t}}{-jk\pi} + \frac{e^{-jk\pi t}}{(k\pi)^2}$$

or:

$$\int t\,e^{-jk\pi t}\,dt = \frac{t\,e^{-jk\pi t}}{-jk\pi} + \frac{e^{-jk\pi t}}{(k\pi)^2}$$



$$a_k = \frac{1}{4}\int t\, e^{-jk\pi t}\, dt = \frac{1}{4}\left[\frac{t\, e^{-jk\pi t}}{-jk\pi} + \frac{e^{-jk\pi t}}{(k\pi)^2}\right]_0^2$$

$$= \frac{1}{4}\left[\left(\frac{2\, e^{-jk(2\pi)}}{-jk\pi} + \frac{e^{-jk(2\pi)}}{(k\pi)^2}\right) - \left(0 + \frac{1}{(k\pi)^2}\right)\right]$$

$$= \frac{1}{4}\left[\frac{2}{-jk\pi} + \frac{1}{(k\pi)^2} - \frac{1}{(k\pi)^2}\right]$$

$$= \frac{1}{-j2k\pi} = j\frac{1}{2k\pi}$$

$$\Rightarrow \quad a_k = \begin{cases} \frac{1}{2} & , \quad k = 0 \\ j\frac{1}{2k\pi} & , \quad k \neq 0 \end{cases}$$

OR Page 8.

$$y_4(t) = \frac{1}{2}\, y_5(t-1) + \frac{1}{2}$$

$$b_k = \frac{1}{2}\, a_k\, e^{-jk\pi} \quad , \qquad k \neq 0$$

$$b_0 = 0 + \frac{1}{2} = \frac{1}{2}$$

$$b_k = j\, \frac{1}{2}\, \frac{(-1)^k}{k\pi}\, e^{-jk\pi} \quad , \quad k \neq 0$$

$$= \begin{cases} j\, \frac{1}{2k\pi} \times -1 \times -1 & , \quad k \text{ is odd} \\ j\, \frac{1}{2k\pi} \times 1 \times 1 & , \quad k \text{ is even} \end{cases}$$

$$= \begin{cases} j\, \frac{1}{2k\pi} & , \quad k \text{ is odd} \\ j\, \frac{1}{2k\pi} & , \quad k \text{ is even} \end{cases}$$

$$\Rightarrow \quad b_k = \begin{cases} \frac{1}{2} & , \quad k = 0 \\ j\, \frac{1}{2k\pi} & , \quad k \neq 0 \end{cases}$$

==

Q 3. Find and sketch the Fourier transform $X(j\omega)$ of the following signals: $\boxed{8 \textit{ Marks}}$

a) $x_1(t) = rect\left(\frac{t}{2T_1}\right)$ b) $x_2(t) = e^{-a|t|} \quad a > 0$

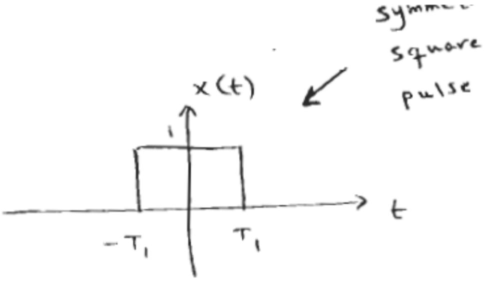
Please simplify your answer as much as you can.

Solution:

a) See Note 12 – Pages 8, 9, 10.

Solution:

$$x(t) = \begin{cases} 1 & -T_1 < t < T_1 \\ 0 & \text{elsewhere} \end{cases}$$



$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$= \int_{-T_1}^{T_1} e^{-j\omega t}\, dt = -\frac{1}{j\omega}\left[e^{-j\omega t}\right]_{-T_1}^{T_1}$$

$$= -\frac{1}{j\omega}\left(e^{-j\omega T_1} - e^{j\omega T_1}\right) = \frac{1}{j\omega}\left(e^{j\omega T_1} - e^{-j\omega T_1}\right)$$

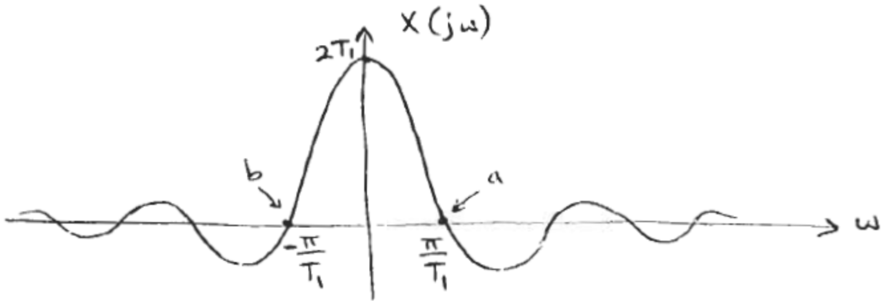
$$\Rightarrow X(j\omega) = \frac{2}{\omega}\sin(\omega T_1) = \frac{2\sin(\omega T_1)}{\omega} \quad \text{- - - - - - - - (III)}$$

$\boxed{1}$

$$\Rightarrow X(j\omega) = 2T_1 \sin c(\omega T_1)$$ $\boxed{1}$

OR

$$= 2T_1 \operatorname{sinc}\left(\frac{\omega T_1}{\pi}\right)$$

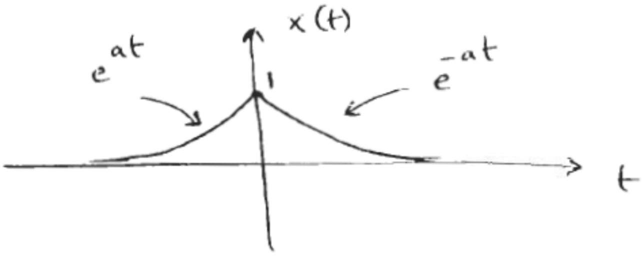


$\boxed{2}$

b) See Note 12 – Pages 6, 7.

$$x(t) = e^{-a|t|} \qquad a > 0$$

$$\Rightarrow \quad x(t) = \begin{cases} e^{at} & t < 0 \\ e^{-at} & t \geq 0 \end{cases}$$



remember that at $t = 0 \Rightarrow x(t) = e^{-0} = 1$

at $t = \infty \Rightarrow x(t) = e^{-\infty} = 0$

at $t = -\infty \Rightarrow x(t) = e^{-\infty} = 0$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt = \int_{-\infty}^{0} e^{at}\, e^{-j\omega t}\, dt + \int_{0}^{\infty} e^{-at}\, e^{-j\omega t}\, dt$$

$$= \int_{-\infty}^{0} e^{(a-j\omega)t}\, dt + \int_{0}^{\infty} e^{-(a+j\omega)t}\, dt$$

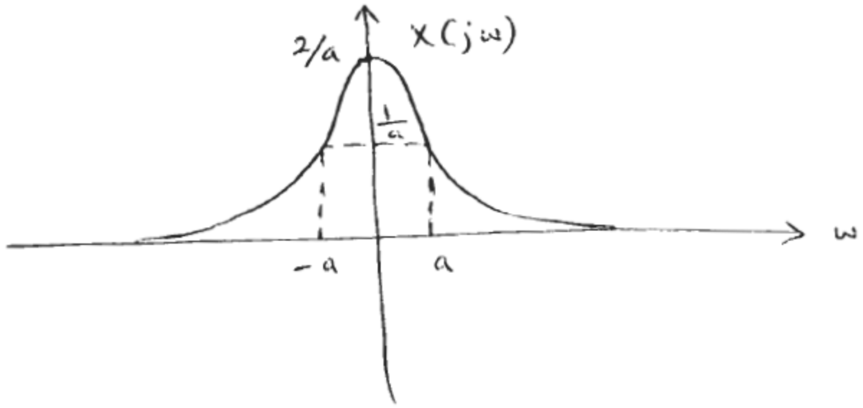
$$= \frac{1}{a-j\omega}\left[e^{(a-j\omega)t}\right]_{-\infty}^{0} - \frac{1}{a+j\omega}\left[e^{-(a+j\omega)t}\right]_{0}^{\infty}$$

$$= \frac{1}{a-j\omega}(1-0) - \frac{1}{a+j\omega}(0-1)$$

$$= \frac{1}{a-j\omega} + \frac{1}{a+j\omega} = \frac{a+\cancel{j\omega}+a-\cancel{j\omega}}{a^2+\omega^2} = \frac{2a}{a^2+\omega^2}$$

$\boxed{2}$

we sketch $X(j\omega)$ as:



for $\omega = 0 \Rightarrow X(j\omega) = \dfrac{2a}{a^2} = \dfrac{2}{a}$

for $\omega = \infty \Rightarrow X(j\omega) = \dfrac{2a}{\infty} = 0$

for $\omega = -\infty \Rightarrow X(j\omega) = \dfrac{2a}{\infty} = 0$

for $\omega = a \Rightarrow X(j\omega) = \dfrac{2a}{a^2+a^2} = \dfrac{2a}{2a^2} = \dfrac{1}{a}$

for $\omega = -a \Rightarrow X(j\omega) = \dfrac{2a}{a^2+(-a)^2} = \dfrac{1}{a}$

$\boxed{2}$

==

Q 4. Find the Fourier transform $G(j\omega)$ of the signal $g(t) = \dfrac{2}{1+t^2}$ using "duality property". $\boxed{8 \textit{ Marks}}$

Solution:

See Note “Properties of Fourier Transform (More Examples)” – Pages 2, 3.

Recall that $x(t) = e^{-a|t|} \xleftrightarrow{F} X(j\omega) = \dfrac{2a}{a^2+\omega^2}$

For $a = 1 \Rightarrow x(t) = e^{-|t|} \xleftrightarrow{F} X(j\omega) = \dfrac{2}{1+\omega^2}$

$$\Rightarrow x(t) = e^{-|t|} = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \left(\frac{2}{1+\omega^2}\right) e^{j\omega t}\, d\omega$$

Multiplying both sides by 2π:

$$\Rightarrow 2\pi\, e^{-|t|} = \int_{-\infty}^{\infty} \left(\frac{2}{1+\omega^2}\right) e^{j\omega t}\, d\omega$$

Replacing t by $-t$ (because the signal $x(t)$ is even):

$$\Rightarrow 2\pi\, e^{-|t|} = \int \left(\frac{2}{1+\omega^2}\right) e^{-j\omega t}\, d\omega$$

Interchanging the names of variables t and ω:

$$\Rightarrow 2\pi\, e^{-|\omega|} = \int_{-\infty}^{\infty} \left(\frac{2}{1+t^2}\right) e^{-j\omega t}\, dt$$

$$\Rightarrow \quad x(t) = \frac{2}{1+t^2} \xleftrightarrow{F} X(j\omega) = 2\pi\, e^{-|\omega|}$$

==

Q 5. For an LTI system, if the input signal is $e^{-at}u(t)$ (where $a > 0$) and the impulse response of the system is $u(t)$, find: $\boxed{8\ \textit{Marks}}$

a) $X(j\omega)$. b) $H(j\omega)$. c) $Y(j\omega)$. d) $y(t)$.

Solution:

See Note 13 – Pages 4, 5

$$x(t) = e^{-at} u(t) \xleftrightarrow{F} X(j\omega) = \frac{1}{a + j\omega} \quad \text{(Ex. 4.1)}$$

$$h(t) = u(t) \xleftrightarrow{F} H(j\omega) = \frac{1}{j\omega} + \pi\,\delta(\omega) \quad \text{(Ex.4.11)}$$

$$Y(j\omega) = H(j\omega)\,X(j\omega) = \left[\frac{1}{j\omega} + \pi\,\delta(\omega)\right] \cdot \frac{1}{a+j\omega}$$

$$= \frac{1}{j\omega}\left(\frac{1}{a+j\omega}\right) + \pi\,\delta(\omega)\left(\frac{1}{a+j\omega}\right)$$

$$= \frac{1}{j\omega}\underbrace{\left(\frac{1}{a+j\omega}\right)}_{G(j\omega)} + \pi\,\delta(\omega)\underbrace{\left(\frac{1}{a+j\omega}\right)\Bigg|_{\omega=0}}_{G(0)}$$

The right-hand side represents the ~~integration of~~ Fourier series of the integration of the function $F^{-1}\left\{\frac{1}{a+j\omega}\right\}$.

$$e^{-at} u(t) \xleftrightarrow{F} \frac{1}{a+j\omega}$$

$$\underbrace{\int_{-\infty}^{t} e^{-a\tau} u(\tau)\,d\tau}_{y(t)} \xleftrightarrow{F} \underbrace{\frac{1}{j\omega}\left(\frac{1}{a+j\omega}\right) + \pi\,\delta(\omega)\left(\frac{1}{a+j\omega}\right)\Bigg|_{\omega=0}}_{Y(j\omega)}$$

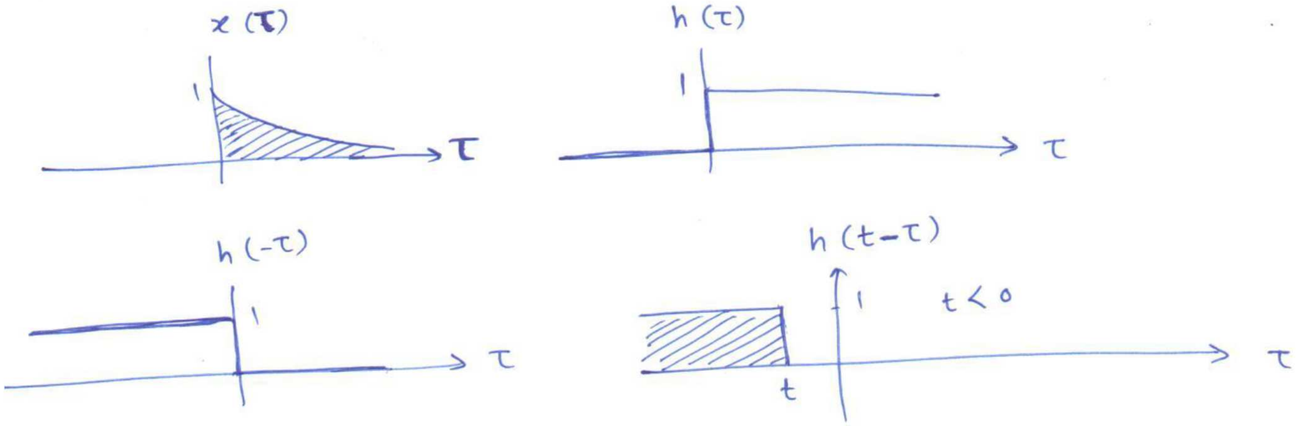
$$\Rightarrow \quad y(t) = \int_{-\infty}^{t} e^{-a\tau} u(\tau)\,d\tau$$

$$= \int_{0}^{t} e^{-a\tau}\,d\tau = -\frac{1}{a}\left[e^{-a\tau}\right]_0^t$$

$$= -\frac{1}{a}\left(e^{-at} - 1\right) = \frac{1}{a}\left(1 - e^{-at}\right)$$

since $t > 0 \Rightarrow y(t) = \frac{1}{a}\left(1 - e^{-at}\right) u(t)$

OR Pages 1, 2 for $y(t)$.



$$\Rightarrow \quad y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

For $t>0$, $y(t) = \displaystyle\int_0^t e^{-a\tau}\, d\tau = -\frac{1}{a}\left[e^{-a\tau}\right]_0^t = \frac{1}{a}\left(1-e^{-at}\right)$

For $t<0$, no overlap between $h(t-\tau)$ and $x(\tau)$

For $t>0$, $x(\tau)\,h(t-\tau) = x(\tau) = e^{-a\tau}$

$$\Rightarrow \quad \text{For all } t, \quad y(t) = \frac{1}{a}\left(1-e^{-at}\right)u(t)$$

==

GOOD LUCK

==