Elements of Engineering Design

Based on notes from: Thinking like an Engineer and other sources

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# 2.TEAMWORK

No engineering project can be completed without teamwork. Teamwork is performed when a group of persons act together carrying out different tasks to accomplish a common goal. Students working in a class room solving a problem, in a way that each student is doing the same work is not teamwork. However, working on a design problem or an experiment with different tasks distributed to each student so that when all tasks are accomplished the required results are obtained, is an example of teamwork.

## 2.1 Points to agree upon

The first thing to do is to hold meetings of team members and agree on the objective of the team, the ground rules and distribution of work. The following is a list of points to consider:

### 2.1.1 Objective

The objective of the team must be agreed upon and must be documented in as much detail as possible so that the final goal to be achieved is clear to all team members.

### 2.1.2 Team member acceptability criterion

Criterion for acceptable and unacceptable individuals as team member must be discussed and agreed and documented.

### 2.1.3 Decision Making Process

Most of the time things are not agreed unanimously. Difference of opinions exist naturally and team must make decisions to adopt one of the suggested ways. There are different ways a team can make decisions. A democratic way may be to make decisions based on consensus or majority vote done openly with show of hands or secretly. Other ways of making decisions may be considered. Once team members reach a consensus on the decision-making process, it must be documented and followed.

### 2.1.4 Communication

Constructive feedbacks from team members make a team strong and keep it directed towards the objective. Very often projects end up in failure or financial loss and one or more team members may say, “I told you not to do it this way but you didn’t listen to me.” This is obviously due to lack of communication. To avoid such things, process of presenting opinions must be devised, discussed and documented. For example, a team member must inform of his opinion about an issue through email to the team leader. The team leader then circulates it among the team members and calls a team meeting to discuss it. In the meeting a decision is made and documented.

### 2.1.5 Progress Indicator

Among the first things to do for a team includes a way to measure the progress of the team towards the goal. One way may be that each team member reports each week, the progress on the task assigned. For example, percentage of the task competed may be reported. The team can develop a formula to come up with a number that represents the progress of the team and serves as a progress indicator.

### 2.1.6 Roles

A team member may have been assigned a role, in addition to a given task to be finished to complete the project. For example, a team member may be given the role of keeping track of the budget and spending while the other may be given the role of organizing the meetings, seminars, etc. Usually, in a long-term project, roles may be changed so that each team member work load is balanced and all get their share of learning from these roles.

### 2.1.7 Work Distribution

As soon as the objective of team is decided and documented, team must decide the tasks to be accomplished by each team member. Redistribution of the work when a team member is sick or unable to finish the task due to any other reason must also be planned.

## 2.2 Project Timeline

A project time line must include all tasks, all decisions to be made at various points in the project and procurement of all supplies and equipment necessary with dates. For this purpose each task must be assigned a start date and a due date. Once due dates are known, start dates may be estimated by working backwards from the due date.

The most popular chart used by project managers for project timeline is a chart called the Gantt Chart. Henry Gantt came up with the idea in 1910. It so a type of bar chart and can be produced easily using EXCEL. It provides a quick visualization of the order in which tasks need to happen and tells how tasks are dependent on each other and which task must be finished to allow other tasks to start. An example is shown in Figure 2-1.

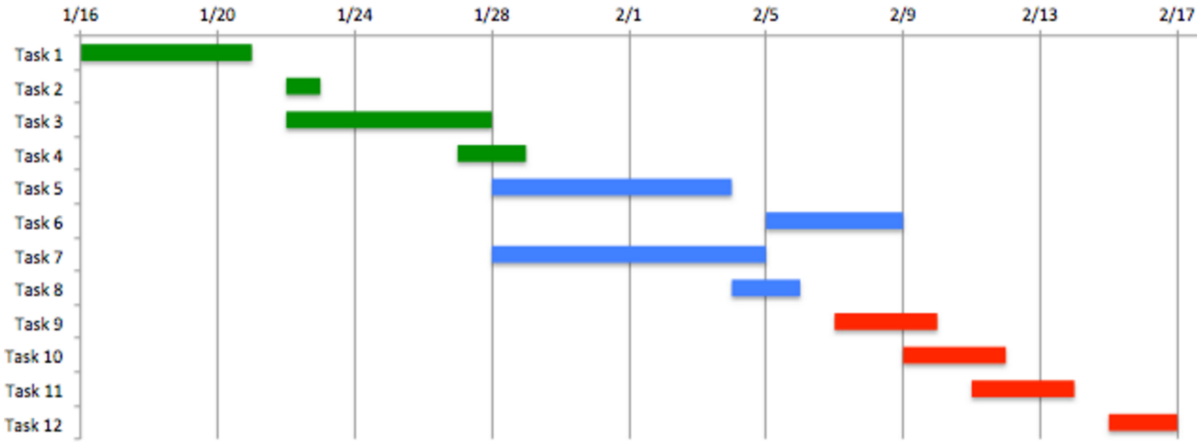



Figure 2-1: An example of timeline using Gantt chart

## 2.3 Brainstorming

Brainstorming means to hold a group discussion to produce ideas. Dictionary.com describes it as: a conference technique of solving specific problems, amassing information, stimulating creative thinking, developing new ideas, etc., by unrestrained and spontaneous participation in discussion. businessdictionary.com describes it as a process for generating creative ideas and solutions through intensive and freewheeling group discussion. Every participant is encouraged to think aloud and suggest as many ideas as possible, no matter seemingly how outlandish or bizarre. Analysis, discussion, or criticism of the aired ideas is allowed only when the brainstorming session is over and evaluation session begins. See also lateral thinking and nominal group technique.

Brainstorming is required in almost all engineering projects. It helps in generating ideas to solve problems that seem hard to resolve by the team. The book [1] quotes the following story taken from Alex Osborn's *Applied Imagination* (out of print), illustrating the importance of generating ideas before evaluating them.

In November 1952, in Washington State, the local telephone company had to clear the frost from 700 miles of telephone lines to restore long distance service. The company believed strongly enough in the importance of variety in the process that ALL the company's employees were asked to participate in a brainstorming session, executives and secretaries, engineers and linemen.

After some time of idea generation, it was clear that the participants needed a break. One of the sessions overheard one lineman say to another at the water fountain, "Ever had a bear chase you up one of the poles? That would sure shake the ice off the lines!" The facilitator encouraged the lineman to repeat himself when the session reconvened.

The facilitator hoped that the unusual suggestion would encourage new ideas, and the lineman sheepishly offered his suggestion. "How should we encourage the bears to climb all the poles?" asked the facilitator. "We could put honey pots at the tops of all the poles!" shouted someone else in the room. "How should we put honey pots on the tops of all the poles?" asked the facilitator. "Use the company helicopters that the executives use!" piped in another participant. "Hmm," said one of the company secretaries calmly. "When I served in the Korean War, I was impressed by the force of the downdraft off helicopter blades. I wonder if that would be enough to shake the ice off the power lines."

The idea was so intriguing that it was tested immediately. It provided a successful and economically viable solution. This story clearly illustrates the benefit of variety and the value of avoiding criticism in a brainstorming session. The rest of the story illustrates how essential quantity is to the process.

A problem-solving group composed of five veteran air force helicopter pilots was convened to address this same problem. Each was unfamiliar with the solution that had already been implemented. It was hoped that because of their background, they would eventually arrive at the same solution. In fact, they did, but it was the 36th idea on their list. If they had stopped after generating 35 ideas, they may very well have had an acceptable solution, but it might not have been as elegant or as successful as the proven solution.

### 2.3.1 Basic principles of brainstorming

First principle of brainstorming is to invite as many ideas as possible. We don’t go for quality but for quantity. Secondly, we must ensure we get a variety of ideas. This requires that for brainstorming people from varied professions be invited. And thirdly, to let the ideas come forward, criticism on the presented ideas must be avoided. Criticism may discourage the audience from thinking and presenting their ideas.

## 2.4 Evaluation of team members

Team leader has the additional responsibility of evaluating the team members for their performances. The team members evaluation is usually done periodically. In most organizations annual evaluation is common but it may be done more frequently.

A good teamwork evaluation form is available at:

http://www.unm.edu/~bgreen/ME360/Teammate%20Evaluation%20Form.pdf

It is reproduced here.

Team Member Evaluation

Your Name: ___________________________________

| Enter the names of your team members below. Do not enter your name here. | | | |
|---|---|---|---|
| 1 | | 4 | |
| 2 | | 5 | |
| 3 | | 6 | |

1. Put an X in the box under the member's number that best describes the way they contributed to the project.

| Members | | | | | | Rate your teammate on the effort she/he put into researching and gathering background information for the design. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Did not collect any information that relates to the project. |
| | | | | | | Collected very little information that related to the project. |
| | | | | | | Collected a reasonable amount of information and most of it related to the project. |
| | | | | | | Collected a great deal of information and all if it related to the project. |

| Members | | | | | | Rate your teammate on how well she/he shares information with the group. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Did not relay any information to other teammates. |
| | | | | | | Relayed very little information that related to the project to other teammates. |
| | | | | | | Relayed some information and most of it related to the project. |
| | | | | | | Relayed a great deal of information and all of it related to the project. |

| Members | | | | | | Rate your teammate on how punctual she/he was in completing and turning in project assignments. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Did not complete team assignments. |
| | | | | | | Completed few assignments on time other assignments completed late or not completed. |
| | | | | | | Completed most of the team assignments on time. |
| | | | | | | Completed all of the team assignments on time. |

2. Put an X in the box under the member's number that best describes the way they accepted responsibility for the project.

| Members | | | | | | Rate your teammate on how well he/she performed their duties relating to their role in the group. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Did not perform any of the duties of the assigned team role. |
| | | | | | | Performed very few duties. |
| | | | | | | Performed nearly all duties. |
| | | | | | | Performed all duties of assigned team role. |

| Members | | | | | | Rate your teammate on how well he/she shared the work load. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Always relied on others to do the work. |
| | | | | | | Rarely did the assigned work – often needed reminding. |
| | | | | | | Usually did the assigned work – rarely needed reminding. |
| | | | | | | Always did the assigned work without having to be reminded. |

| Members | | | | | | Rate your teammate on how well he/she attended meetings. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Missed most group meetings. Did not inform other group members they would be absent. |
| | | | | | | Frequently missed group meetings and seldom informed others they would be absent. |
| | | | | | | Attended most meetings and informed others when he/she could not attend. |
| | | | | | | Attended all group meetings. |

3. Put an X in the box under the member's number that best describes the way they valued others' ideas.

| Members | | | | | | Rate the team member on how well she/he listened to others in the group. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Was always talking – never allowed anyone else to speak. |
| | | | | | | Usually did most of the talking – rarely allowed others to speak. |
| | | | | | | Listened, but occasionally talked too much. |
| | | | | | | Listened well and spoke without dominating the conservation. |

| Members | | | | | | Rate the team member on how well he/she cooperates with others in the group. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Usually argued with teammates. |
| | | | | | | Sometimes argued. |
| | | | | | | Rarely argued with other team members. |
| | | | | | | Never argued with teammates. |

| Members | | | | | | Rate your team member on how well she/he made fair decisions. |
|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | |
| | | | | | | Usually wanted to have things their way. |
| | | | | | | Often sided with friends instead of considering all views. |
| | | | | | | Usually considered all views. |
| | | | | | | Always helped the team to reach a fair decision. |

A peer evaluation instrument that is widely used in engineering education is the Comprehensive Assessment of Team-Member Effectiveness (CATME, see www.catme.org). CATME measures five different types of contributions to a team using such a behaviorally anchored rating scale. Each scale includes representative behaviors describing exceptional, acceptable, and deficient performance in each area. Recognizing that an individual team member may exhibit a combination of behaviors, the CATME instrument also includes "in-between" ratings. The five types of contributions are described below the associate behaviors.

*Contributing to the Team's Work* describes a team member's commitment to the effort, quality, and timeliness of completing the team's assigned tasks.

- A student who is exceptional at contributing to the team's work
    - Does more or higher-quality work than expected
    - Makes important contributions that improve the team's work
    - Helps to complete the work of teammates who are having difficulty
- A student who does an acceptable job at contributing to the team's work
    - Completes a fair share of the team's work with acceptable quality
    - Keeps commitments and completes assignments on time
    - Fills in for teammates when it is easy or important
- A student who is deficient at contributing to the team's work
    - Does not do a fair share of the team's work. Delivers sloppy or incomplete work
    - Misses deadlines. Is late, unprepared, or absent for team meetings
    - Does not assist teammates. Quits if the work becomes difficult

*Interacting with Teammates* measures how a team member values and seeks contributions from other team members.

- A student who is exceptional at interacting with teammates
    - Asks for and shows an interest in teammates' ideas and contributions

    - Improves communication among teammates. Provides encouragement or enthusiasm to the team
    - Asks teammates for feedback and uses their suggestions to improve
- A student who does an acceptable job at interacting with teammates
    - Listens to teammates and respects their contributions
    - Communicates clearly. Shares information with teammates. Participates fully in team activities
    - Respects and responds to feedback from teammates
- A student who is deficient at interacting with teammates
    - Interrupts, ignores, bosses, or makes fun of teammates
    - Takes actions that affect teammates without their input. Does not share information
    - Complains, makes excuses, or does not interact with teammates. Accepts no help or advice

*Keeping the Team on Track* describes how a team member monitors conditions that affect the team's progress and acts on that information as needed.

- A student who is exceptional at keeping the team on track
    - Watches conditions affecting the team and monitors the team's progress
    - Makes sure teammates are making appropriate progress
    - Gives teammates specific, timely, and constructive feedback
- A student who does an acceptable job at keeping the team on track
    - Notices changes that influence the team's success
    - Knows what everyone on the team should be doing and notices problems
    - Alerts teammates or suggests solutions when the team's success is threatened
- A student who is deficient at keeping the team on track
    - Is unaware of whether the team is meeting its goals
    - Does not pay attention to teammates' progress
    - Avoids discussing team problems, even when they are obvious

*Expecting Quality* is about voicing expectations that the team can and should do high-quality work.

- A student who is exceptional at expecting quality
    - Motivates the team to do excellent work
    - Cares that the team does outstanding work, even if there is no additional reward
    - Believes that the team can do excellent work
- A student who does an acceptable job at expecting quality
    - Encourages the team to do good work that meets all requirements
    - Wants the team to perform well enough to earn all available rewards
    - Believes that the team can fully meet its responsibilities
- A student who is deficient at expecting quality
    - Is satisfied even if the team does not meet assigned standards
    - Wants the team to avoid work, even if it hurts the team
    - Doubts that the team can meet its requirements

*Having Relevant Knowledge, Skills, and Abilities* accounts for both the talents a member brings to the team and those talents a member develops for the team's benefit.

- A student who has exceptional knowledge, skills, and abilities
    - Demonstrates the knowledge, skills, and abilities to do excellent work

    - Acquires new knowledge or skills to improve the team's performance
    - Is able to perform the role of any team member if necessary.
- A student who has an acceptable level of knowledge, skills, and abilities
    - Has sufficient knowledge, skills, and abilities to contribute to the team's work
    - Acquires knowledge or skills needed to meet requirements
    - Is able to perform some of the tasks normally done by other team members
- A student who has deficient knowledge, skills, and abilities is
    - Missing basic qualifications needed to be a member of the team
    - Unable or unwilling to develop knowledge or skills to contribute to the team
    - Unable to perform any of the duties of other team members

# 3.ENGINEERING DESIGN

## 3.1 The Design Process

The term design refers not only to products but also to processes. Design is an art and not a science because it creates solutions or modifies existing solutions. In this book the focus will be on Engineering Design rather than "Product Design" or "Industrial Design". The **engineering design is a** methodology requiring problem identification, problem definition based on engineering principles and theories, considering various solutions, predicting their behaviors using engineering analysis and then comparing designs based on a set of criteria. Engineering design process requires repeated analysis, modeling and experimentation. This chapter will present various aspects of the engineering design process to enable the students to manage the engineering design activity and compare design options.

### 3.1.1 ABET Definition of Engineering Design

*ABET*[1] has defined the engineering design process as:

"…It is a decision-making process (often iterative) in which the basic sciences, mathematics, and engineering sciences are applied to convert resources optimally to meet a stated objective. Among the fundamental elements of the design process are the establishment of objectives and criteria, synthesis, analysis, construction, testing and evaluation."

### 3.1.2 Objective in Engineering Design

Design process yields several solutions and not just one solution because a design has to satisfy several criteria. While one of the solutions addresses some of the criteria very well, other criteria might not be well satisfied. The final objective in engineering design is to identify the design that is best in terms of the criteria satisfaction and objectives.

### 3.1.3 Engineering Design Applies to Process Design

Engineering design applies not only to a material thing like a building structure, bridge, tunnel, water distribution systems, highways, traffic systems, water treatment plants, etc. but it also applies to processes. Though the material things and processes differ a lot but have similar design processes which will be described in this chapter.

### 3.1.4 Role of Analysis in Engineering Design

Analysis is used in the design process. As compared to the design process, it is a rigorous process to study various design options requiring attention and details with precisions and therefore it is laborious. While design is an art, analysis is a science because it gives the behavior of various solutions and tell whether it will work or not. Analysis tells whether the given set of criteria are satisfied or not for each design option and may give data to rate design options for each criterion.

### 3.1.5 Distinct Stages of Engineering Design

Analysis does not have well defined stages because it depends on the nature of the problem. Electrical Engineering analysis of a design may have entirely different steps as compared to Civil or Mechanical Engineering analysis. In contrast engineering design process has five distinct stages regardless of the field of engineering it is related to. These stages are as follows:

1. Problem identification: Observe the problem and note items of interest to understand the problem exactly. Discuss, take opinions.

2. Problem definition: Hypothesize an explanation based on whatever knowledge, theories, you know or can learn from others. Use brain storming.
3. Developing possible solutions: Bases on each acceptable problem definition suggest possible solutions.
4. Modeling: Model each solution to predict its behavior
5. Experimentation & tests: Carry out experimentation to see the behavior of each solution or to resolve issue. Th experimentation and tests may be in the laboratories or the field or by using computer simulations and analyses.
6. Evaluation: Check criteria satisfaction. Compare design options.

### 3.1.6 Example of Problem Definition

Suppose a problem is identified by the students of a class as follows:

"In the morning, light streams in the windows of a classroom and the image on the projector screen is not clear." There are various ways of **defining** this problem, and the definition will affect the solutions. Some examples of problem definition are given here:

| | |
|---|---|
| Problem Definition 1. | Glare is the issue. The screen reflects the light. The solution may be to make the screen anti-glare |
| Problem Definition 2. | Light coming in is the issue. The solution may be to cover the window with brick, blinds, shades |
| Problem Definition 3. | Bad screen location is the issue: The solution may be to move the screen and relocate |
| Problem Definition 4. | Classes are held when sun shines in windows is the issue. The solution may be to reschedule the classes. |

This shows that this stage of design requires brainstorming as explained in Chapter 2.

### 3.1.7 ABET Accreditation Process as a Design Process

The book "Thinking Like an Engineer" [1] explains the distinct stages of engineering design process using an example of the design of the ABET accreditation process which is used to design the engineering education process. The explanation from [1] is reproduced here for students of this course only because the students are using this book as their main reference book.

The design process for engineering programs is made up of two iterative processes, shown in Figure 3-1. The iterative loop on the left comes first because a new program would begin there; it includes getting input from constituencies, determining educational objectives, and evaluating or assessing how well those objectives are being achieved. In other engineering processes, these steps might be called something like "problem definition." "Constituencies" may also be called users, clients, stakeholders, or other terms. The process is iterative because it is important to confirm that the constituencies are pleased with the results, to adapt to changing needs, and to achieve the continuous improvement expected by the engineering profession.

With the problem identified (knowing our educational objectives), the iterative process on the right side of Figure 3-1 begins. This process occurs primarily in the designer's workspace, whatever that is. In the case of designing engineering curriculum, the process takes place within the walls of the university or college. Knowing the objectives, the design team determines the outcomes that will accomplish those objectives, how those outcomes will be achieved, how they will be assessed, and what indicators will demonstrate success before any students are actually taught. Once students have had learning experiences (including extracurricular experiences), evaluation and assessment guide both processes into another cycle.

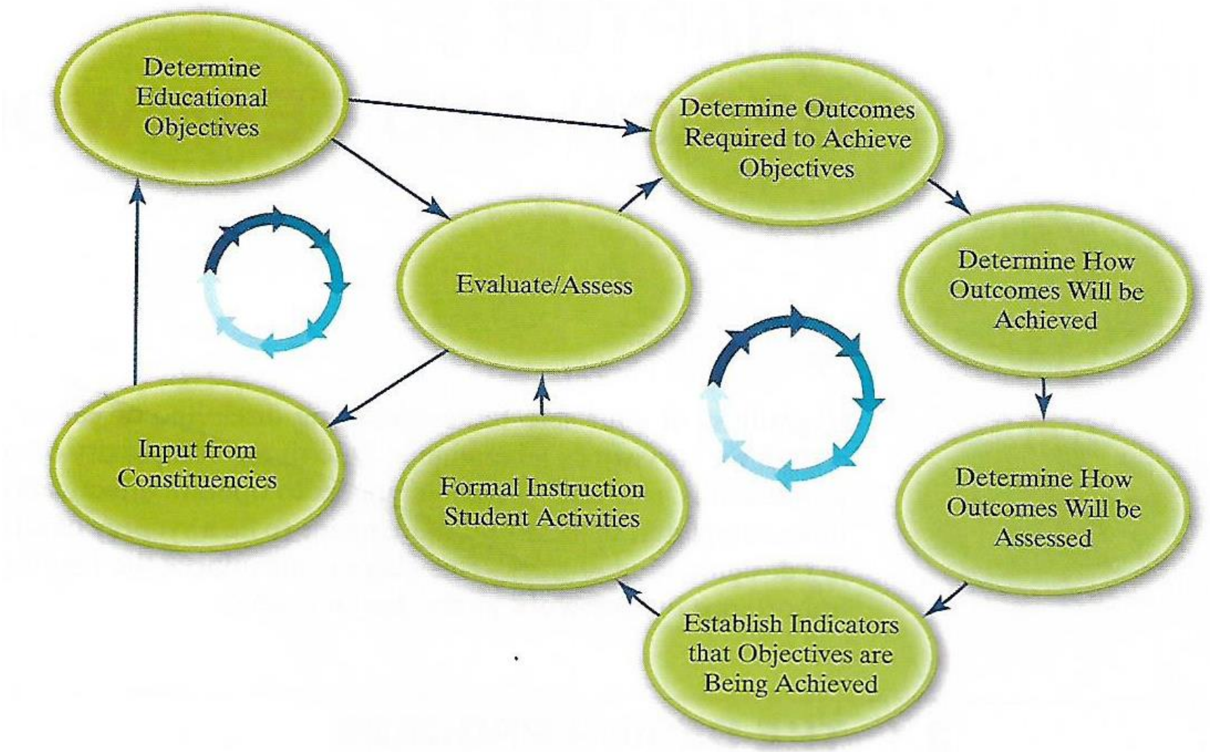



Figure 3-1: The ABET design approach (reproduced from [1]

The step "determine outcomes required to achieve objectives" is called "problem definition" or "specification" in many other design processes. This step is critical because it shapes all the others, "Determine how the outcomes will be achieved" is a particularly creative step in the process and is commonly referred to as "generating ideas," "innovating," "developing possible solutions," or something similar, and might include "research." "Determine how the outcomes will be assessed" is a step that might not be mentioned if most agree on how to measure the success of a design, but in designs with more complex objectives, designers must think carefully about what will be measured and how, finishing up with determining indicators of success. These steps are commonly called "analysis" in a more general design process, breaking down the design to examine its assumptions, benefits, and risks. The remaining part of the process is one of "proto typing," "implementation," and "testing."

## 3.2 Criteria and Evaluation

Criteria are important because the designs can only be compared based on some criteria. For example, safety is a criterion. Comparing solution options or designs will require a rating on safety for each design option. Similarly, other criteria like cost, weight, appearance, etc. may be considered. The question: “Which the best design from a set of proposed design?” can only be answered by asking, “what is your criterion for the best?”

### 3.2.1 Identifying Good Criteria

1) Criteria should be clear: “A good room temperature” is not clear.
2) Criteria should distinguish options: “I’ll buy a car with side mirrors” does not distinguish options.
3) Criteria should be measurable: “A car that is “fun” to drive.” is not measurable.

Write a clear criterion to replace each of these vague criteria for the products in Table 3-1.

Table 3-1: Examples of Good Criteria

| Product | Computer | Automobile | Bookshelf |
|---|---|---|---|
| Inexpensive | Less than $300 | | |
| Small | | | |
| Easy to assemble | | | Requires only a screwdriver |
| Aesthetically pleasing | | | |
| Lightweight | | | |
| Safe | | | |
| Durable | | | |
| Environmentally friendly | | Has an estimated MPG of at least 50 | |

Table 3-2: Examples of Good Criteria (Solution to exercise in Table 3.1)

| Criterion | Computer | Automobile | Bookshelf |
|---|---|---|---|
| **Inexpensive** | Less than $300 | Less than $^1/_5$ the median annual U.S. family income | Less than $30 |
| **Small** | Folds to the size of a DVD case | Two can fit in a standard parking space | Collapses to size of briefcase |
| **Easy to assemble** | No assembly: just turn it on | All parts easily replaceable | Requires only a screwdriver |
| **Aesthetically pleasing** | Body color options available | Looks like the Batmobile© | Blends well with any decor |
| **Lightweight** | Less than one pound | Less than one ton | Less than 5% of the weight of the books it can hold |
| **Safe** | Immune to malware | Receives 5 star rating in NCAP crash tests | Stable even if top-loaded |
| **Durable** | Survives the "Frisbee® test" | 200,000 mile warranty | Immune to cat claws |
| **Environmentally friendly** | Contains no heavy metals | Has an estimated MPG of at least 50 | Made from recycled materials<br>Low VOC finish |

### 3.2.2 Keep Fewer Criteria

In general, using fewer criteria keeps things simple. A rule of thumb is that if you can think of 10 criteria that are meaningful, you should consider only the two most important criteria to compare solutions. In this way, you can be sure that the important criteria maintain their importance in your decision-making. Seek consensus on what the most important criteria are. Experts, specialists, managers, customers, research, etc., can help you focus on the most important criteria.

### 3.2.3 Comparison & Evaluation

Simply identifying criteria is not enough information for a decision-each proposed solution must be evaluated against the criteria. The first step will be to eliminate any solutions that do not meet the minimum requirements-those solutions are out-of-bounds and need not be considered further. Once the minimum requirements have been satisfied, there are many ways of applying the remaining criteria to select a solution.

#### *3.2.3.1 Pairwise Comparison*

One approach is to make **pairwise comparisons**. In this approach, you use a table for each criterion to summarize how each of the solutions compares with others. An example is shown in Table 3-3 for the criterion "safety." Among the table entries, 0 indicates that that option in that column is worse than the option in that row; 1 indicates that both options rank equally; and 2 indicates that that option in that column is better than the option in that row. To summarize:

- Put a criterion at the top left of table: Example "Safety" is a criterion
- Column => an "Option" or "Design"
- Row => an "Option" or "Design"
- Column option is better than row => Column score = 2
- Column option is same as row => Column score = 1
- Column option is worse than row => Column score = 0

Table 3-3: Comparing options

| Safety | Option1 | Option2 | Option3 |
|---|---|---|---|
| Option1 | | 0 | 1 |
| Option2 | 2 | | 0 |
| Option3 | 1 | 2 | |
| | | | |
| Total | 3 | 2 | 1 |

Since a high level of safety is preferred, "better" means "safer." In the example, the first column indicates that option 1 is safer than option 2, but the same as option 3. The resulting totals indicate that option 1 ranks best in terms of safety.

A disadvantage of the pairwise comparisons approach is that all criteria have equal weight, whereas some criteria are likely to be more important than others.

### *3.2.3.2 Weighted Benefit Analysis*

An alternative approach is to use **weighted benefit analysis,** shown in Table 3-3. In this approach, each option is scored against each of the criteria.

- Column 1: Criteria
- Column 2: Weights: Criterion importance
- Column 3 onward: Design rating on criterion
- Add up the weighted values for each design
- Use judgment to compare the designs

Table 3-4: Options with a weighted benefit analysis

| | Weight | Option 1 | Option 2 | Option 3 |
|---|---|---|---|---|
| Cost | 2 | 2 | 6 | 10 |
| Safety | 8 | 10 | 4 | 6 |
| Weight | 10 | 7 | 7 | 2 |
| Wow | 5 | 2 | 4 | 6 |
| Totals | 4 + 80 + 70 + 10 = 164; 12 + 32 + 70 + 20 = 134; 20 + 48 + 20 + 30 = 118 | | | |

In Table 3-4, the weights are in the first column and each option has been assigned a score from 0 to 10, indicating how well that option meets each criterion. This approach may be inconsistent in that a "7" for one rater may be different from a "7" for another rater, so it can help to better define the scale. For example, the options may be scored on the scale shown in Table 3-3 as to how well the option fits each criterion.

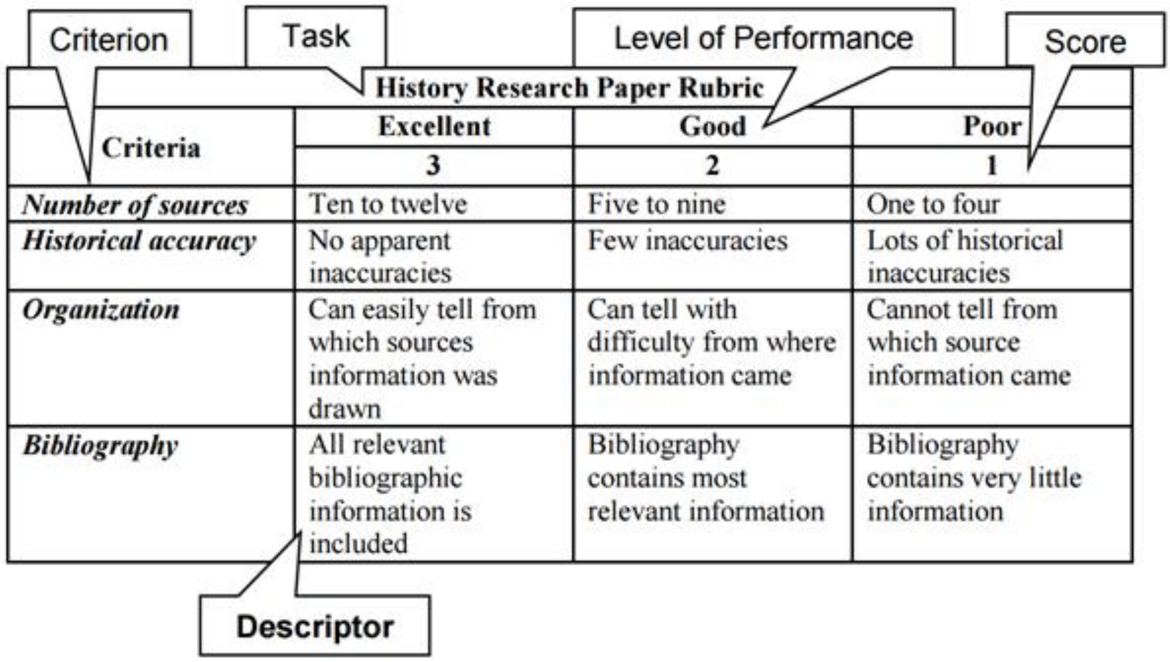
### 3.2.4 Think it over

When a solution has been identified as the best fit to the criteria, it is best to stop for a reality check before moving forward. After the evaluation process, some ideas will be left on the cutting room floor. Do any of these merits further consideration? Are there important elements of those ideas that can be incorporated into the chosen design? If the reality check reveals that an idea really should not have been eliminated, then a change in the selection criteria may be appropriate.

After deciding, you will implement your chosen solution. Undoubtedly, both carrying out the design and using the design once it is complete will provide new information about how the design might be improved. In this way, design tends to be iterative-design, build, test, redesign, and so on. Even when a design performs as expected, it may be important to test a model extensively or even build multiple models to be sure that the design is reliable.

### 3.2.5 Rubrics

Rubrics are used to rate or grade the design based on a set of criteria. Figure 3-2 shows an example of rubrics for determining students' scores in a particular assessment. Table 3-5 shows another set of rubrics for scoring.




| History Research Paper Rubric | | | |
|---|---|---|---|
| Criteria | Excellent 3 | Good 2 | Poor 1 |
| ***Number of sources*** | Ten to twelve | Five to nine | One to four |
| ***Historical accuracy*** | No apparent inaccuracies | Few inaccuracies | Lots of historical inaccuracies |
| ***Organization*** | Can easily tell from which sources information was drawn | Can tell with difficulty from where information came | Cannot tell from which source information came |
| ***Bibliography*** | All relevant bibliographic information is included | Bibliography contains most relevant information | Bibliography contains very little information |

Figure 3-2: Example of Rubrics

Table 3-5: Sample scoring rubric

| Score | Meaning |
|---|---|
| 0 | Not satisfactory |
| 1 | Barely applicable |
| 2 | Fairly good |
| 3 | Good |
| 4 | Very good; ideal |

# 4.SUSTAINABE DESIGN

For about the last two hundred years, engineering development has caused the earth's climate, water and other natural resources to deteriorate. It is becoming apparent to engineers, planners and developers that with the traditional approach of designing without the consideration of sustainability, there won't be enough natural resources for future generations. Therefore, sustainability in design, development and engineering are now being considered as important. Lack of sustainability consideration in design of buildings has resulted in the following:

- People spend 90% of their time indoors
- Indoor pollutant levels may be 2 – 5 times higher than outdoor levels
- Costs billions in health care and lost productivity
- High building maintenance cost
- High energy use, electricity consumption
- High greenhouse gas emissions (water vapor, carbon dioxide, methane, nitrous oxide, and ozone)

Responsible homeowners and industrialists are now implementing Sustainable Design in their homes and businesses and Sustainable Engineering has become an important part of industry.

### 4.1.1 Definition

The capacity of the earth's natural systems and human cultural systems to survive, flourish, and adapt to changing environmental conditions into the very long-term future is called sustainability. Wikipedia defines: "Sustainable engineering is the process of designing or operating systems such that they use energy and resources sustainably, in other words, at a rate that does not compromise the natural environment, or the ability of future generations to meet their own needs." Another common definition of sustainability is "meeting the needs of the present without compromising the ability of future generations to do the same. Sustainability has three key aspects: environmental, social and economic. All three of these aspects must be considered for truly sustainable engineering designs.

Sustainable development is development that meets the needs of the present without compromising the ability of future generations to meet their own needs. It contains within it two key concepts: (1) the concept of "needs", in particular the essential needs of the world's poor, to which overriding priority should be given; (2) the idea of limitations imposed by the state of technology and social organization on the environment's ability to meet present and future needs. Sustainable Engineering is the **science of designing and engineering** systems that safeguard the availability of natural resources for future needs. To understand it clearly, a simple example of Sustainable Engineering is when a Sustainable engineer is designing a house site. The engineer does not just design site improvements and location, but he or she takes into consideration the materials that will be used, the impact it will produce on the land, and other things that can limit the availability of our **natural resources**

### 4.1.2 Holistic Approach

Sustainable design uses a holistic approach that optimizes the overall system performance, not just the product or service itself. Sustainability is not a stand-alone topic. It cannot be bolted onto an engineering design at the end of the project. Assuming the building project is necessary, some of the best sustainability opportunities occur early on in the project, during project planning and design. This is where engineers play a key role. Sustainable solutions require consideration of multiple issues. These basic ideas apply across disciplines, whether you are designing a building, an engine, or a new material. Table 4-1 compares traditional versus sustainable engineering.

Table 4-1: Traditional vs sustainable engineering

| | Traditional Design | Sustainable Design |
|---|---|---|
| 1 | Not holistic | Holistic |
| 2 | Considers technical issues only | Considers non-technical issues |
| 3 | Solves the immediate problem | Solves for generations to come |
| 4 | Based on local context | Based on global context |
| 5 | Ignores political issues | Considers pollical issues |
| 6 | Ignores societal issues | Considers societal issues |
| 7 | Ignores ethical issues | Considers ethical issues |

### 4.1.3 Design for Sustainability (D4S)

Design for sustainability should have the following three characteristics:

1) It should not cause irreversible change to the environment – locally and globally,

2) It should be economically viable while being functional and practical.

3) It should benefit the society.

Basic D4S objectives for products and processes are:

1) increasing energy efficiency,
2) using recycled materials,
3) designing for recyclability,
4) reducing toxic materials,
5) extending product life, and
6) providing services for sustainability.

Life cycle analysis and supply chain management are tools for evaluating material flows and environmental impacts in a product's life cycle and can help designers achieve D4S.

### 4.1.4 Sustainability in Construction

[Note this section has been taken from: https://theconstructor.org/construction/sustainability-in-construction-civil-engineering/9492/]

Construction involves activities like use of building materials from various sources, use of machineries, demolition of existing structures, use of green fields, cutting down of tress etc. which can impact environment in one or more ways. Sustainability in construction is the optimization of construction activities in a way that does not have harmful effects on resources, surroundings and living ecosystem. It is a way of minimizing harmful environmental impacts of construction projects.

Construction has a direct impact on the environment due to following reasons:

1. Generation of waste materials
2. Emissions from vehicles, machineries

3. Noise pollution due to use of heavy vehicles and construction machineries.
4. Releases of wastes and pollutants into water, ground and atmosphere.

Sustainability assessment of construction projects is essential to the fact that it does not create any harmful effects on the living ecosystem while optimizing the cost of construction. This is to ensure the availability of resources for the future generations. Following are the important construction activities which have large impacts on sustainability in construction and civil engineering:

**1. Wastes from demolition of building and structures:**

Over billions of tons of construction and demolition waste are generated worldwide annually. These wastes can be hazardous to environment is not disposed off at suitable place without environmental impact assessment of such wastes. The other alternate is to recycle and reuse of the demolished building materials to minimize the risk of harmful impacts.

**How to make construction waste sustainable?**

Following are the steps which need to be followed to make construction waste more sustainable:

- Eliminate – avoid producing construction waste in the first place.
- Reduce – minimize the amount of waste you produce.
- Reuse – reuse the construction wastes in other works.
- Recover (recycling, composting, energy) – recycle what you can only after you have reused it.
- Dispose – dispose of what is left in a responsible way.

Use of durable construction materials and quality control at site for durability of structure is one step towards minimization of construction waste generation.

**2. Use of Sustainable Building Materials:**

Building Materials such as sand and gravel have been used for thousands of years in construction. The demand for these is increasing day by day as demand for infrastructure development is increasing.

Uses of construction materials such river sand and gravels also have negative impact on environment. Excessive sand-and-gravel mining causes the degradation of rivers. Instream mining lowers the stream bottom, which may lead to bank erosion. This results in the destruction of aquatic and riparian habitat through large changes in the channel morphology. Impacts include bed degradation, bed coarsening, lowered water tables near the streambed, and channel instability.

There are many harmful impacts of using river sand and mining of gravels and a detailed study is required to list all the negative impacts. The use of alternate building materials can reduce the impact of this on environment.

The alternate to river sand is Manufactured Sand (M-Sand) which can be used in construction works reduce impacts of mining river sand.

**3. Energy Consumption and Green House:**

Around 40% of total energy consumption and greenhouse gas emissions are directly due to construction and operation of buildings. The best way to reduce this impact is the use of green buildings construction techniques. The use of transparent concrete in buildings also helps to reduce the use of energy for lighting during daytime.

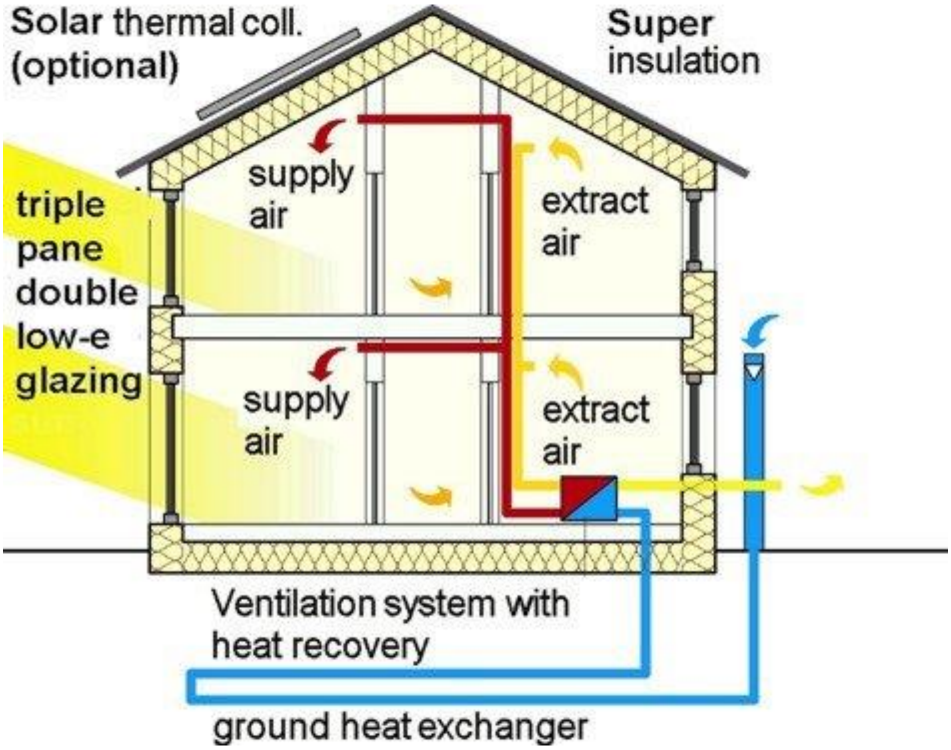



Figure 4-1: Example of a Sustainable Building Construction

**How to Ensure Sustainable Construction?**

**Following steps should be taken to for better sustainability of construction activities:**

- Reduce the supply chains to reduce transport costs
- Exercise waste minimization and recycling construction
- Building orientation – Choose the building orientation in a way to reduce energy utilization.
- Durability and quality of building components, generally chosen to last for the appropriate refurbishment or demolition cycle.
- Use construction materials which are locally available.
- Design buildings and structures as per local topological, climatic and community demands.
- Select appropriate construction methods – prefabrication, wood or concrete structures.
- Reuse of existing buildings or structures can reduce the construction waste. Reutilizing by strengthening and rehabilitation of buildings can also save construction cost.
- Make site waste management plans not only during construction but also during use or operation.
- Minimize energy in construction.

### 4.1.5 Examples of Sustainable Design of Buildings

Sustainable design of buildings is becoming more popular. Some examples are given below:

1) An example of home design is shown in Figure 4-2. The basic ideas used are as follows:
   - Use orientation of building to control heat gain/loss
   - Use solar energy for heating
   - Use building shape (floor plan) to control air flow
   - Proper ventilation for cooling
   - Use materials to control heat
   - Mass thickness to absorb and store heat
   - Select colors of floors/walls/ceilings for heat losses
   - Use proper surface coverings materials
   - Use Insulation/Air Tightness

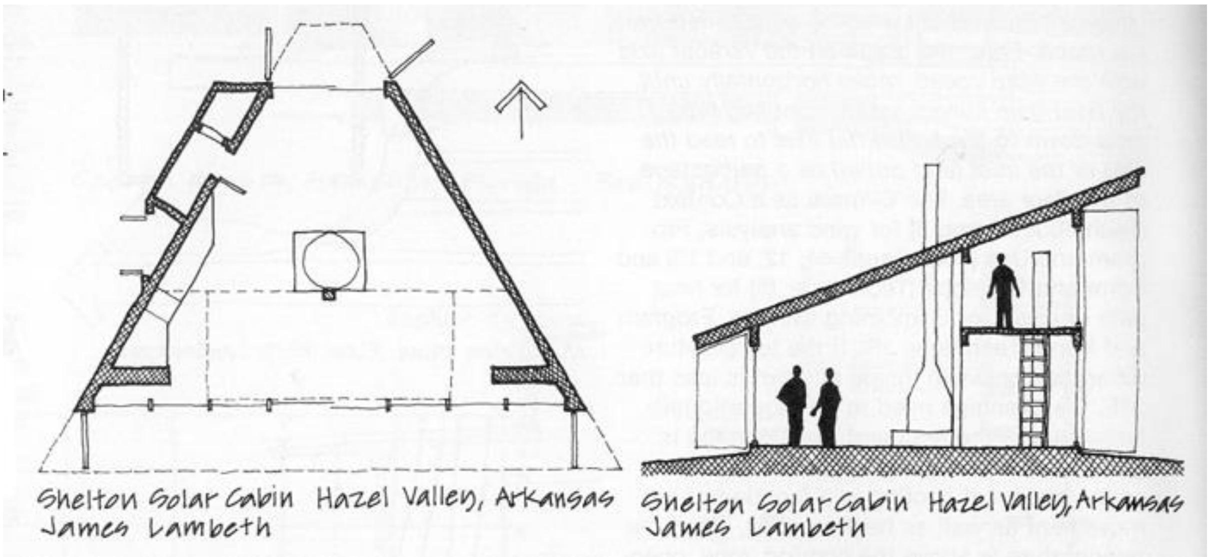



Figure 4-2: Sustainable design of home

2) The National Olympic Stadium, aka the "Bird's Nest" in Beijing shown in Figure 4-3, was designed by Swiss architects Herzog & de Meuron for the 2008 Olympics. Students who are fascinated by energy efficiency might enjoy that the stadium incorporates several green practices covered in sustainable architecture training, such as its use of solar power and rainwater collection. The facilities inside the stadium are all built as self-contained units which allow the exterior façade to remain mostly open, resembling a bird's nest. The exterior façade allows for natural ventilation of the stadium, which is the most important aspect of its sustainable design.

Figure 4-3: The bird's nest – eco-friendly architecture in Beijing

3) The Center for Sustainable Development shown in Figure 4-4 was built to serve as a model for anyone interested in learning more about environmentally conscious construction. The center features artwork on display that's made from 100% organic materials. The building has :
    - A five-story living wall that acts as a natural air filter
    - Countertops that are made from recycled glass
    - Geothermal heating and cooling systems
    - Raised floor heating

Figure 4-4: Center for sustainable development – Montreal

# 5.EXPERIMENTAL DESIGN

The purpose of experiments may be to gather data, to get plots of behaviors of a model, to determine the sensitivity of parameters, etc. A good design of an experiment is a critical for getting meaningful results. An experimental design requires the planning and design of experiments and evaluating the budget requirements before starting the experiments. This chapter will explain how experimental design is carried out.

## 5.1 Experimental Design Procedure

1. Determine what information is required from the experiments. What type of curves have to be plotted. What data have to be gathered.
2. Determine which model is suitable for the prediction of behavior of the design.
3. Determine the parameters involved in the model.
4. Determine the range of each parameter.
5. Determine the increments to be applied to each parameter.
6. Determine the order of varying the parameters.
7. Determine the total number of experiments.
8. Determine the equipment, material and manpower required.
9. Calculate the cost and get the funding.
10. Plan the experiments as a team work.
11. Carry out the experiments as planned.
12. Collect data.
13. Plot the required curves.
14. Write a report of the results of the experiments.

## 5.2 Experimental Variables

- Dependent Variables
    - React to change in independent variable(s)
    - Must be measured
    - Usually plotted on the ordinate
- Independent variables
    - Allowed to vary
    - Controlled or manipulated
    - Usually on abscissa
- Control variables
    - Held constant for a set of experiments
    - Simplify the experiment
    - Make it possible to understand the effect of the other variables

## 5.3 Experimental Design Example

As an example, suppose you are interested in the speed of a ball as it rolls across the floor after rolling down a ramp. In physics, you will learn the equations of motion for bodies moving under the influence of gravity. If you are good, you can use these to examine rolling balls. What you will quickly find, however, is that numerous complicating factors make it difficult to apply the basic equations to obtain an adequate answer. Let us suppose you are interested in smooth balls (such as racquetballs), rough balls (tennis balls), heavy balls (bowling balls), and lightweight balls (ping-pong balls). The simplified equations of motion predict that all these will behave in essentially the same way. You will discover, however, that the drag of the air affects the ping-pong ball, the fuzz affects the tennis ball, and the flexible nature of the racquetball will allow it to bounce at steep ramp angles. It is difficult to predict the behavior analytically. Often, one of the quickest ways to learn about the performance of such complex situations is to conduct experiments.

### 5.3.1 Parameters of interest determined

- Parameter 1 is the ramp angle.
- Parameter 2 is the distance up the ramp that we release the ball.
- Parameter 3 is the type of ball.

### 5.3.2 Establish the range of parameters

- Ramp angle can vary between 0 and 90 degrees in theory, but in reality can only vary between 10 degrees (if too shallow, ball would not move) and 45 degrees (if too steep, ball will bounce).
- The distance we release the ball up the ramp can vary between 0 and 3 feet in theory, assuming that the ramp is 3 feet long. We cannot release the ball too close to the bottom of the ramp or it would not move. In reality, we can only vary between 0.5 and 3 feet.
- We will test as many types of balls as we have interest in.

### 5.3.3 Repetitions

- The ramp angle will be set according to the height of the ramp from the floor, so there is not much room for error in this measurement; only one measurement is needed for such geometry.
- Each placement of the ball before release will vary slightly and may cause the ball to roll slightly differently down the ramp; this is probably the most important factor in determining the speed, so three measurements at each location are needed.
- We will assume that every ball is the same, and the actual ball used will not change the outcome of the experiment; only one ball of each type is needed.

### 5.3.4 Increments in Parameters

- We will test every 10 degrees of ramp angle, starting at 10 degrees and ending at 40 degrees.
- We will release the balls at a height of 0.5, 1, 1.5, 2, 2.5, and 3 feet up the ramp.
- We will test five types of balls: racquetball, baseball, tennis ball, ping-pong ball, and bowling ball.

### 5.3.5 Order to vary the parameters determined

- We will set the ramp angle and then test one ball type by releasing it at each of the four different distances up the ramp.
- We will repeat this process three times for each ball. Cl We will then repeat this process for each type of ball.
- We will then change the ramp angle by 10 degrees and repeat the process. I.'J This process is repeated until all conditions have been tested.

### 5.3.6 Number of measurements

It is always important to determine before you start how many measurements you need to make. Sometimes you can be too ambitious and end up developing an experimental program that will take too much effort or cost too much money. If this is the case, then you need to decide which increments can be relaxed, to reduce the number of overall measurements.

The number of measurements *(N)* you will need to make can be easily calculated by the following equation for a total of *n* parameters:

*N* = (# increments parameter 1 * number of repetitions for parameter 1) *

(# increments parameter 2 " number of repetitions for parameter 2) *…

(# increments parameter *n* * number of repetitions for parameter *n)* *…

Continuing the examples given above, the number of actual measurements that we need to make is calculated as

*N* = (4 angles) * (6 distances * 3 repetitions) * (5 types of balls) = 360 measurements

In this example, 360 measurements may be extreme. If we examine our plan, we can probably make the following changes without losing experimental information:

- We decide to test every 10 degrees of ramp angle, starting at 20 degrees and ending at 40 degrees. This will lower the angle testing from four to three angles.
- We will release the balls at a height of 1, 2, 2.5, and 3 ft up the ramp. This will lower the distances from six to four.
- We will test three types of balls: racquetball, ping-pong ball, and bowling ball. This will lower the type of balls from five to three.

The number of actual measurements that we now need to make is calculated as

*N* = (3 angles) * (4 distances * 3 repetitions) * (3 types of balls) = 1 08 measurements

This result seems much more manageable to complete than 360!

## 5.4 Uncertainty in Experiments

Any measurement acquired in an experiment contains three pieces of information:

1) The value measured from the instrument
2) Predictable error in the measurement (Mostly in numerical experiments)
3) Uncertainty in the values measured due to unknown imperfections human errors, etc. Uncertainty in measurement may be due to the following:
    a. Instrumentation error
    b. Systematic error resulting from human or instrumentation malfunction
    c. Random error caused by the data-collection device.

## 5.5 Instruments in Experiments

- Durometer
- Dynamometer
- Euidometer
- Galvanometer
- Gyroscope
- Manometer
- Opisometer
- Pycnometer
- Tachymeter
- Thiele tube

(Students: Add to this list all instruments you have used in the labs.)

# 6.DIMENSIONAL ANALYSIS

## 6.1 Derived Dimensions and Units

With only the seven base dimensions in the metric system, all measurable things in the known universe can be expressed by various combinations of these concepts. These are called **derived dimensions**. As simple examples, area is length squared, volume is length cubed, and velocity is length divided by time.

As we explore more complex parameters, the dimensions become more complex.

For example, the concept of force is derived from Newton's second law, which states that force is equal to mass times acceleration. Force is then used to define more complex dimensions such as pressure, which is force acting over an area, or work, which is force acting over a distance. As we introduce new concepts, we introduce the dimensions and units for each parameter.

Sometimes, the derived dimensions become quite complicated. For example, electrical resistance is mass times length squared divided by both time cubed and current squared. Particularly in the more complicated cases like this, a **derived unit** is defined to avoid having to say things like "The resistance is 15 kilogram-meters squared divided by second cubed ampere squared." It is much easier to say "The resistance is 15 ohms," where the derived unit "ohm" equals one (kg $m^2$) / ($s^3$ $A^2$).

Within this text, dimensions are presented in exponential notation rather than fractional notation. If a dimension is not present in the quantity, it is noted by a zero exponent.

| Quantity | Fractional Notation | Exponential Notation |
|---|---|---|
| Velocity | $\frac{L}{T}$ | $M^0$ L1 T-1 $\Theta^0$ |
| Acceleration | $\frac{L}{T^2}$ | $M^0$ L1 T-2 $\Theta^0$ |

Currently, there are officially 22 named derived units in the SI system. All are named after famous scientists or engineers who are deceased. Five of the most common derived units can be found in Table 6-1. It is worth noting that numerous common derived dimensions do not have a corresponding named derived SI unit. For example, there is no named derived SI unit for the derived dimension velocity as there is for force (newton) or electrical resistance (ohm).

Table 6-1: Common derived units in the SI system

| Dimension | SI Unit | Base SI Units | Derived From |
|---|---|---|---|
| Force (*F*) | newton [N] | $1\ \mathrm{N} = 1\frac{kg\ m}{s}$ | *F* = ma<br>Force = mass time acceleration |
| Energy *(E)* | Joule [J] | $1\ \mathrm{J} = 1\ \mathrm{Nm} = 1\frac{kg\ m^2}{s^2}$ | E = Fd<br>Energy = Force times distance |
| Power *(P)* | Watt [W] | $1\ \mathrm{W} = 1\frac{J}{s} = 1\frac{kg\ m^2}{s^3}$ | P = E/t<br>Power = energy per time |

| | | | |
|---|---|---|---|
| Pressure *(P)* | pascal [Pa] | $1\ \text{Pa} = 1\ \dfrac{N}{m^2} = 1\dfrac{kg}{m\ s^3}$ | P = F/A<br>Pressure = force per area |
| Voltage (*V*) | Volt [V] | $1\ \text{V} = 1\ \dfrac{J}{A} = 1\dfrac{kg\ m^2}{S^3\ A}$ | V = P/I<br>Voltage = power per current |

- A note of caution: One letter can represent several quantities in various engineering disciplines. For example, the letter *"P"* can indicate pressure, power, or vertical load on a beam. It is important to examine and determine the nomenclature in terms of the context of the problem presented.
- Always remember to use the units and the symbol in calculations.

### 6.1.1 Special Unit: Radian

The derived unit of **radian** is defined as the angle at the center of a circle formed by an arc *(S)* equal in length to the radius *(r)* of that circle. In a complete circle there are $2\pi$ radians. Since by definition a radian is a length *(S)* divided by a length *(r)*, it is a dimensionless ratio.

$$1 \text{ radian [rad]} = S / r$$

Thus, an angle has units, but is dimensionless! In addition to radians, another common unit used for angle is the degree [0]. There are 360° in a complete circle.

$$360^o = 2\pi \text{ radians}$$

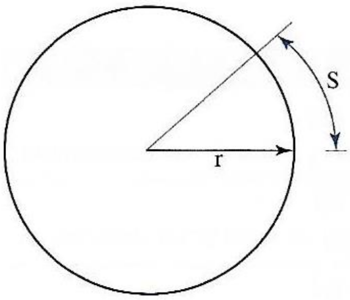



Table 6-2: Dimensions of some common parameters

| Quantity | Units | Common Exponents M | L | T | Ө |
|---|---|---|---|---|---|
| Fundamental Quantities | | | | | |
| Mass | kg | 1 | 0 | 0 | 0 |
| Length | m | 0 | 1 | 0 | 0 |
| Time | s | 0 | 0 | 1 | 0 |
| Temperature | K | 0 | 0 | 0 | 1 |
| Geometric Quantities | | | | | |
| Area | $ft^2$ | | | | |
| | acre | 0 | 2 | 0 | 0 |
| Volume | gal | | | | |
| | L | | | | |

| Rate Quantities | | | | | |
|---|---|---|---|---|---|
| Velocity | mi/h | 0 | 1 | -1 | 0 |
| Acceleration | $ft/s^2$ | 0 | 1 | -2 | 0 |
| Flowrate | gal/min | | | | |
| Evaporation | kg/h | | | | |

## 6.2 Equation Laws

Equations are mathematical "sentences" composed of "words" (terms) that are strung together with "punctuation marks" (mathematical symbols, such as +, -, x, ..;-, and =). Just as there are rules in the English language that govern how a sentence is structured, there exists a set of "rules" for equations.

### 6.2.1 Addition and Subtraction

Suppose we are interested in the manufacture and use of sandpaper for furniture construction. We think for a while and then develop a list of the important quantities that affect the final product, along with their respective units and dimensions:

| W | Wood removed | [in] | L |
|---|---|---|---|
| R | Roughness diameter | [mm] | L |
| D | Density of grains | [$kg/m^3$] | $\frac{M}{L^3}$ |
| A | Adhesive thickness | [mm] | L |
| H | How heavy the paper is | [N] | $\frac{ML}{T^3}$ |
| O | Operation stroke length | [cm] | L |
| K | Kernel (grain) spacing | [mm] | L |

Let us propose a simple equation with only plus and minus signs that could possibly relate several of these parameters. If we are interested in how heavy the product would be, we might assume this would depend on the thickness of the adhesive, the diameter of the roughness, and the grain density. We will try

$$H = A + R + D$$

Each of these terms represents something "real," and consequently we expect that each term can be expressed in terms of fundamental dimensions. Writing the equation in terms of dimensions given:

$$\frac{ML}{T^2} = L + L + \frac{M}{L^3}$$

It is obvious that this is just terrible! We cannot add length and mass or time; as the adage goes, "You can't add apples and oranges!" The same holds true for dimensions. As a result of this observation, we see that this cannot possibly be a valid equation. This gives one important "law" governing equations, the **Plus law**.

Let us try this again with another equation to see if we can determine how effective the sandpaper will be, or how much wood will be removed after each stroke. We might assume this depends on the operation stroke length, the roughness diameter, and the spacing of the grains.

$$W = O + R + K$$

Substituting dimensions,

$$L = L + L + L$$

We see that at least dimensionally, this can be a valid equation, based on the Plus law. Next, units can be inserted to give

inches = centimeters + millimeters + millimeters

Dimensionally, this equation is fine, but from the perspective of units, we cannot carry out the arithmetic above without first converting all the length dimensions into the same units, such as millimeters. We can state an important result from this observation as well, forming the **Unit law**.

It is important to state a corollary to this observation. If two parameters have the same dimensions and units, it is not always meaningful to add or subtract them.

Two examples show this.

1. If Student A has a mass $m_A$ [kilograms] and Student B a mass $m_B$ [kilograms], then the total mass of both students [kilograms] is the sum of the two masses. This is correct and meaningful in both dimensions and units.
2. Suppose we assume that an equation to predict the mass of a car is this: mass of the car in kilograms = mass of an oak tree in kilograms + mass of an opossum in kilograms. This equation has three terms; all with the dimension of mass and units of kilograms; thus, the terms can be added, although the equation itself is nonsense.

Consequently, the requirement that each term must have the same dimensions and units is a necessary, but not a sufficient, condition for a satisfactory equation.

### 6.2.2 Multiplication and Division

There are many ways to express the rate at which things are done. Much of our daily life is conducted on a *"per"* or rate basis. We eat 3 meals *per* day, have 5 fingers *per* hand, there are 11 players *per* team in football, 3 feet *per* yard, 4 tires *per* car, 12 fluid ounces *per* canned drink, and 4 people *per* quartet.

Although it is incorrect to add or subtract parameters with different dimensions, it is perfectly permissible to divide or multiply two or more parameters with different dimensions. This is another law of dimensions, the **Per law**.

When we say 65 miles *per* hour, we mean that we travel 65 miles in 1 hour. We could say we travel at 130 miles per 2 hours, and it would mean the same thing. Either way, this rate is expressed by the *"per"* ratio, distance per time.

One of the most useful applications of your knowledge of dimensions is in helping to determine if an equation is dimensionally correct. This is easy to do and only involves the substitution of the dimensions of every parameter into the equation and simplifying the resulting expressions. A simple application will demonstrate this process.

### 6.2.3 Example

Is the following equation dimensionally correct?

$$t = \sqrt{\frac{d_{final} - d_{initial}}{0.5a}}$$

Where t is time

d is distance

a is acceleration

0.5 is unitless

Determine the dimensions of each parameter:

Acceleration (a) {=} $L^1\ T^{-2}$

Distance (d) {=} L

Time (t) {=} T

Substitute into the equation: $T = \sqrt{\left(\frac{[L-L]}{} \middle| \frac{T^2}{L}\right)}$

Simplifying $T = \sqrt{\left(\frac{L}{} \middle| \frac{T^2}{L}\right)} = \sqrt{T^2} = T$

Yes, the equation is dimensionally correct. Both sides of the equation have the same dimensions

### 6.2.4 Example

We can use dimensional arguments to help remember formulas. We are interested in the acceleration of a body swung in a circle of radius *(r)*, at a constant velocity *(v)*. We remember that acceleration depends on *r* and *v*, and one is divided by the other, but cannot quite remember how. Is the acceleration *(a)* given by one of the following?

$$a = \frac{v}{r}\ or\ \ a = \frac{v}{r^2}\ or\ \ a = \frac{v^2}{r}\ or\ \ a = \frac{r}{v}\ or\ \ a = \frac{r}{v^2}\ or\ \ a = \frac{r^2}{v}$$

Acceleration (a) {=} $L^1\ T^{-2}$

Distance (r) {=} L

Time (v) {=} $L^1\ T^{-1}$

| Original Equation | Substituting into the Equation | Simplify | Correct? |
|---|---|---|---|
| $a = v/r$ | $LT^{-2} = (LT^{-1})\ L^{-1}$ | $LT^{-2} = T^{-1}$ | NO |
| $a = v/t^2$ | $LT^{-2} = (LT^{-1})\ L^{-2}$ | $LT^{-2} = L^{-1}\ T^{-1}$ | NO |
| $a = v^2/r$ | $LT^{-2} = (LT^{-1})2\ L^{-1}$ | $LT^{-2} = LT^{-2}$ | YES |
| $a = r/v$ | $LT^{-2} = L\ (LT^{-1})^1$ | $LT^{-2} = T$ | NO |
| $a = r/v^2$ | $LT^{-2} = L\ (LT^{-1})^2$ | $LT^{-2} = L^{-1}\ T^2$ | NO |
| $a = r^2/v$ | $LT^{-2} = L^2\ (LT^{-1})^{-1}$ | $LT^{-2} = LT$ | NO |

## 6.3 Common Dimensionless Numbers

Sometimes, we form the ratio of two parameters, where each parameter has the same dimensions. Sometimes, we form a ratio with two groups of parameters, where each group has the same dimensions. The final result in both cases is dimensionless.

**Pi *(π):*** One example is the parameter *π,* used in the calculation of a circumference or area of a circle. The reason *π* is dimensionless is that it is actually defined as the ratio of the circumference of a circle to its diameter:

$$\pi = \frac{C}{D} = \frac{circumference}{diameter} \{=\} \frac{length}{length} = \frac{L^1}{L^1} = L^0$$

The ratio of one length to another length yields a dimensionless ratio. We can see this in another way through reversing the process. For the circumference of a circle:

$$C = \pi D$$

and if dimensions are inserted,

$$\{L^1\} = \pi \{L^1\}$$

This equation is dimensionally correct only if *π* has no dimensions. The same result is obtained for the equation of the area of a circle.

$$A = \pi r^2$$

Inserting dimensions:

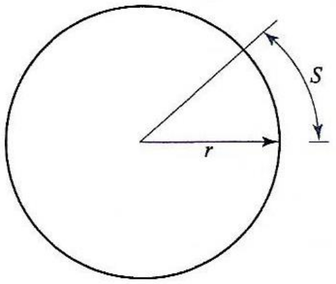
Again, this equation is dimensionally correct only if *π* is dimensionless.

**Specific Gravity (SG):** The specific gravity is the ratio of the density of an object to the density of water.

$$\text{Specific gravity} = \frac{density\ of\ the\ object}{density\ of\ water} = \frac{mass/volume}{mass/volume} \{=\} \frac{\{M/L^3\}}{\{M/L^3\}} = \{M^0 L^0\}$$

**Mach Number (Ma):** We often describe the speed at which an airplane or rocket travels in terms of the Mach number, named after Ernst Mach, an Austrian physicist. This number is the ratio of the speed of the plane compared with the speed of sound in air.

$$\text{Mach number} = \frac{speed\ of\ the\ object}{speed\ of\ sound\ in\ air} \{=\} \frac{\{L/T\}}{\{L/T\}} = \{L^0 T^0\}$$




**Radian [rad]:** The derived unit of a radian is defined as the angle at the center of a circle formed by an arc *(S)* equal in length to the radius *(r)* of that circle. In a complete circle, there are two *π* radians. Since by definition a radian is a length *(S)* divided by a length *(r),* it is a dimensionless ratio. Note, however, that angles are often measured using the radian as a unit, even though it is dimensionless.

$$1\ radian\ [rad] = \frac{S}{r} \{=\} \frac{L}{L} = L^0$$

| Name | Phenomena Ratio | Symbol | Expression |
|---|---|---|---|
| Coefficient of friction | Sideways force (F)/weight of object *(w)* [object static or kinetic (object sliding)] | $\mu_{st}$ and $\mu_k$ | F/w |
| Drag coefficient | Drag force ($F_d$)/inertia force *(ρ,* density; *v*, speed; *A* body area) | $C_d$ | $F_d/^{(1/2\rho v2A)}$ |
| Mach number | Object speed (v)/speed of sound ($v_{sound}$) | Ma | $v/v_{sound}$ |
| Pi | Circle circumference (C)/ circle diameter *(D)* | π | C/D |
| Poisson's ratio | Transverse contraction ($\varepsilon_{trans}$)/ longitudinal extension ($\varepsilon_{long}$) | *ν* | $\varepsilon_{trans}/\ \varepsilon_{long}$ |
| Radian | Arc length (S)/circle radius *(r)* | Rad | S/r |
| Specific gravity | Object density/density of water | SG | $\rho/\ \rho H_20$ |

We must remind ourselves that it is always essential to use the appropriate dimensions and units for every parameter. Suppose that we are interested in computing the sine of an angle. This can be expressed as a dimensionless number by forming the ratio of the length of the opposite side divided by the length of the hypotenuse of a right triangle.

$$\sin(x) = \frac{\text{length opposite side}}{\text{length hypotenuse}} \{=\} \frac{L}{L} = L^0$$

In addition to the ratio of two lengths, you will know from one of your math classes that the sine can be also be expressed as an infinite series given by:

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Let us suppose that the argument *x* had the units of length, say, feet. The units in this series would then read as:

$$\text{ft} - \frac{ft^3}{3!} + \frac{ft}{5!} - \frac{ft}{7!} + \cdots$$

We already know that we cannot add two terms unless they have the same units; recall the Plus law from Chapter 7. This is clearly not the case in the example above. The only way we can add these terms, all with different exponents, is if each term is dimensionless. Consequently, when we calculate sin *(x)*, we see that the *x* must be dimensionless, which is why we use the unit of radians. This conclusion is true for any function that can be computed using a series form, leading to the **Law of Arguments**.

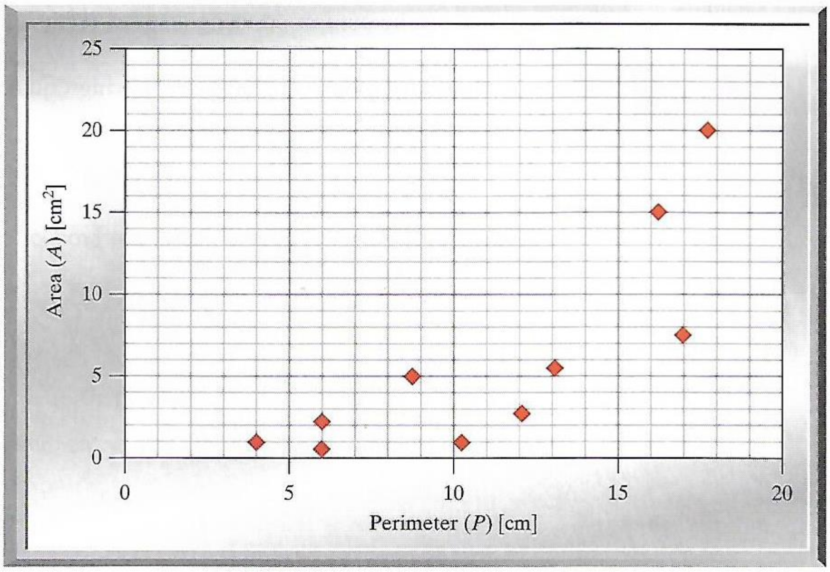
## 6.4 Dimensional Analysis

**Dimensionless quantities** are generated as a result of a process called **dimensional analysis**. As an example, suppose we want to study rectangles, assuming that we know nothing about rectangles. We are interested in the relationship between the area of a rectangle, the width of the rectangle, and the perimeter of the rectangle. We cut out a lot of paper rectangles and ask students in the class to measure the area, the perimeter, and the width (Table 6-3).

Table 6-3: Rectangle measurements

| Perimeter (P) [CM] | Area (A) [$cm^2$] | Width (w) [cm] |
|---|---|---|
| 4.02 | 1.0 | 1 .1 |
| 8.75 | 4.7 | 1.9 |
| 6 | 2.3 | 1.55 |
| 13.1 | 6.0 | 1.1 |
| 17.75 | 19 | 5.25 |
| 10.25 | 1.2 | 0.25 |
| 12.1 | 3.0 | 5.5 |
| 6 | 0.3 | 2.9 |
| 16.25 | 15.4 | 5.1 |
| 17 | 7.8 | 1.05 |

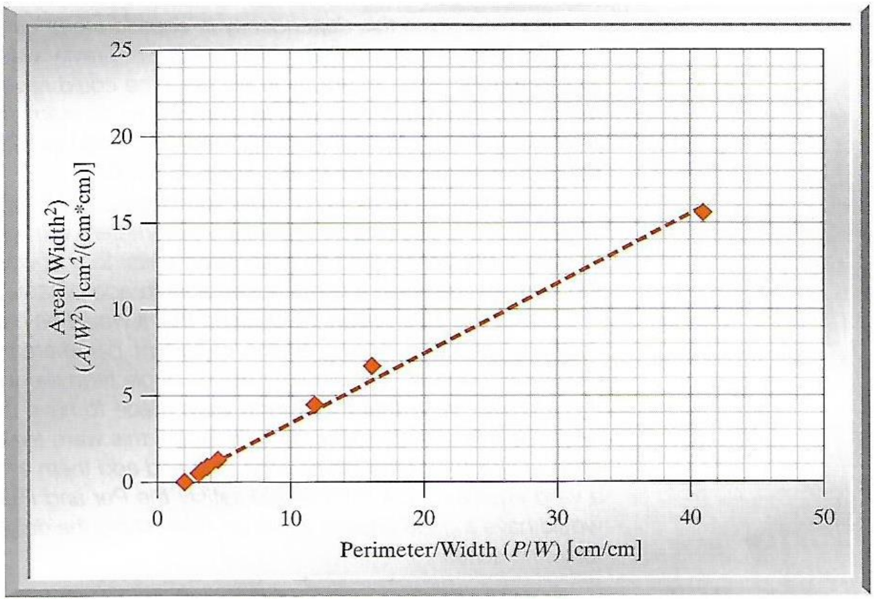
If we graph the area against the perimeter, we obtain the following plot:




From this, we see that the data are scattered. We would not have a great deal of confidence in drawing conclusions about how the area depended on the perimeter of the rectangle (or in trying to draw a line through the data). The best we could do is to make a statement such as, "It seems that the larger the perimeter, the larger the area." However, close examination of the data table shows that as the perimeter increases from 8.75 to 10.25 centimeters and from 16.25 to 17 centimeters, the area actually decreases in each case. One reason for this problem is that our plot has omitted one important parameter: the width.

Analysis shows that one way in which to generalize plots of this type is to create dimensionless parameters from the problem variables. In this case, we have perimeter with dimension of length, width with the dimension of length, and area with the dimension of length squared. A little thought shows that we could use the ratio of *P/W* (or *W/P)* instead of just *P* on the abscissa. The ratio *WIP* has the dimensions of length/ length, so it is dimensionless. It does not matter whether this is mileslmiles, or centimeters/centimeters, but the ratio is dimensionless. Similarly, we could write $A/ (W^2)$, and this would also be dimensionless.

These ratios are plotted and shown below. The scatter of the first plot disappears and all the data appear along a single line.




To understand how to read data from this chart, let us examine the following question. If a rectangle has a perimeter of 20 feet and a width of 2 feet, what is the area?

**Step A:** *P/W* = (20 *ft)/ (2* ft) = 10 (with no units).

**Step B:** From the chart, at a *P/W* value of 10, we read a value from the line of $A/\,(W^2)$ = 3.5.

**Step C:** Calculate *A* from this as *A* = 3.5 * (2 ft * 2 ft) = 14 ft$^2$.

Some of you may be thinking that we made this problem unnecessarily difficult.

After all, anyone who manages to get to college knows that the "sensible" measurements to make are length, width, and area. However, many phenomena are far more complicated than simple rectangles, and it is often not at all obvious what parameters should be measured to characterize the behavior of the system we are studying. In situations of this type, dimensionless analysis can become a powerful tool to help us understand *which* parameters affect the behavior of the system and *how* they affect it. With this in mind, let us look at a slightly more complicated example.

## 6.5 Rayleigh's Method

In this section we introduce a method of dimensional analysis devised by Lord Rayleigh, John William Strutt, the third Baron Rayleigh (1842-1919).

No matter the problem, the way we solve it stays the same:

**Step 1:** Write each variable and raise each to an unknown exponent (use *all* the variables, even the dependent variable). Order and choice of exponent do not matter.

**Step 2:** Substitute dimensions of the variables into Step 1. Be sure to raise each dimension to the proper exponent groups from Step 1.

**Step 3:** Group by dimension.

**Step 4:** Exponents on each dimension must equal zero for dimensionless numbers, so form a set of equations by setting the exponent groups from Step 3 for each dimension equal to zero.

**Step 5:** Solve the simultaneous equations (as best as you can).

*Hint:* Number of unknowns - number of equations = number of groups you will have!

**Step 6:** Substitute results of Step 5 back into Step 1 exponents.

**Step 7:** Group variables by exponent. These resulting groups are your dimensionless numbers.

**Step 8:** Be sure to *check* it out!! Are *all* of the ratios really dimensionless?

*Hint:* If the resulting groups are *not* dimensionless, you most likely goofed in either

To classify the smoothness of flowing fluid, Osborne Reynolds (1842 – 1912, British, ME) developed the now famous dimensionless quantity of Reynolds number. HIS theory stated that the smoothness or roughness (a lot of eddies or swirling) of a fluid depended upon:

| | |
|---|---|
| How fast the fluid was moving (velocity) | $v$ [=] m/s |
| The density of the fluid | $P$ [=] kg/m$^3$ |
| The diameter of the pipe | $D$ [=] m |
| How hard it was to move the fluid (viscosity) | $\mu$ [=] g/(cm s) |

Reynolds knew the smoothness depended upon these quantities:

$$\text{Smoothness of the flow} = f:(v, p, D, \mu)$$

But how did they depend on one another? We could write the four variables above as

$$V^a p^b D^c \mu^d$$

and if this was dimensionless, it would appear as $M^0\ L^0\ T^0\ \Theta^0$.

To make this grouping dimensionless, first we substitute in the dimensions of the four variables to obtain:

$$\left\{\frac{L}{T}\right\}^a \left\{\frac{M}{L^3}\right\}^b L^c \left\{\frac{M}{LT}\right\}^d = M^{b+d} L^{a-3b+c-d} T^{-a-d}$$

Note we generally have dimensions of mass, length, time, and temperature in this problem; there is no temperature dimension. If this is to be dimensionless, then the exponents on all of the dimensions must equal zero, therefore:

M: $b = -d$

T: $a = -d$

L: $c = -a + 3b + d = d - 3d + d = -a$

This gives three equations in four unknowns, so we will have to solve for three of the variables in terms of the fourth. In this example, we solve for the three unknowns a, b, c in terms of d:

M: $b = -d$

T: $a = -d$

L: $c = -a + 3b + d = d - 3d + d = -d$

Substituting these back into the original parameters gives:

$$V^{-d} p^{-d} D^{-d} \mu^d$$

We see that there is one dimensionless group, since all the parameters have an exponent of d. We can write

$$\frac{\mu}{vpD}\{=\}\frac{\frac{M}{LT}}{\frac{L}{T}\frac{M}{L^3}L}$$

Since the variables of diameter and velocity can approach zero, the Reynolds number is commonly written as follows:

$$\text{Re} = \frac{pDv}{\mu}$$

If the Reynolds number has a value less than 2,000, the flow is described as **laminar,** meaning it moves slowly and gently with no mixing or churning. If the Reynolds number has a value greater than 10,000, the flow is described as **turbulent,** meaning it moves quickly with much mixing and churning (lots of eddies) occurring. The region in between 2,000 and 10,000 is called the **transition region.**

The Reynolds number is used to describe fluid flow.

Re < 2,000 = laminar

2,000 < Re < 10,000 = transitional

Re > 10,000 = turbulent

### 6.5.1 Example

Suppose we conduct an experiment with a ball that we throw from the top of a tall tower of height *H.* We throw it directly downward with some initial velocity *v*, and then measure the elapsed time *t* until it hits the ground. We vary the initial height and the initial velocity. The variables of interest in this problem are *H, v,* and *t.* A little thought leads us to include g, since it is the force of gravity that causes the ball to fall in the first place.

Using Rayleigh's method, find a set of dimensionless ratios that can be used to correlate our data.

**Step 1:** Write each variable and raise each to an unknown exponent (use all the variables, even the dependent variable).

$$t^a H^b v^c g^d$$

**Step 2:** Substitute dimensions of the variables into Step 1. Be sure to raise each dimension to the proper exponent from Step 1.

$$t^a \{=\} T^a \quad H^b \{=\} L^b \quad v^c \{=\} L^c T^{-c} \quad g^d \{=\} L^d T^{-2d}$$

**Step 3:** Group by dimension.

$$L^{b+c+d} \; T^{a-c-2d}$$

**Step 4:** Exponents on each dimension must equal zero for dimensionless numbers! Form a set of equations by setting the exponents for each dimension equal to zero.

$$B + c + d = 0 \qquad a - c - 2d = 0$$

**Step 5:** Solve the simultaneous equations (as best as you can).

$$b = -c - d \qquad a = c + 2d$$

**Step 6:** Substitute results of Step 5 back into Step 1 exponents.

$$T^{c+2d} \; H^{-c-d} \, v^c \, g^d$$

**Step 7:** Group variables by exponent. These resulting groups are your dimensionless numbers.

$$[\frac{vt}{H}]^{c}\ [\frac{g\,t^{2}}{H}]^{d}$$

**Step 8:** Be sure to check it out!! Are all of the ratios really dimensionless?

Always remember that we initiate this procedure simply by providing a list of parameters we think are important to the situation at hand. If we omit an important parameter, our final result will not be physically correct, even if it is dimensionally correct. Consequently, if we select an improper parameter, then when tests are conducted, we will discover that it was not important to the problem and we can drop it from further consideration. *We cannot decide whether any variable is important until we conduct some experiments.*

Consequently, if we are *sure* that a parameter is important, then we know it should *not* drop from the analysis. The only way it can be retained is if at least one other parameter contains the missing dimension. In this case, we need to ask ourselves what other parameters might be important, add them to our list, and rework the analysis.

Dimensional analysis helps us organize data by allowing us to plot one-dimensionless parameter against another, resulting in one line on a single plot. This is a powerful result, and reduces a problem of multiple initial parameters to one containing only two. This discussion leads to the **Problem Simplification Rule**:

By performing dimensional analysis of the parameters, we can generally find dimensionless groupings to effectively reduce the number of parameters, facilitating the presentation of interdependencies and often simplifying the problem.

We are not sure mat the results of this technique are *physically correct,* only that they are *dimensionally correct.*

At the beginning of the analysis, when in doubt about the importance of a parameter, put it in the list of important parameters.

# 7.DESIGN CALCULATIONS

## 7.1 Conversion Procedure for Units

We use conversion factors to translate from one set of units to another. This must be done correctly and consistently to obtain the right answers. Some common conversion factors can be found inside the cover of this book, categorized by dimension. Although many more conversions are available, all the work for a typical engineering class can be accomplished using the conversions found in this table.

Let us examine the conversions found for the dimension of length, as shown in the box, beginning with the conversion: 1 meter [m] = 3.28 feet [ft]. Therefore, we can write $\frac{1m}{3.28\,ft}$ = 1, noting that the numerator and denominator are equal so the result of dividing must be equal to 1. Similarly, we can write $\frac{3.28\,ft}{1m}$ =1. Again the numerator is equal to the denominator giving the answer = 1.

For example, on a trip we note that the distance to Atlanta is 123 miles [mi]. How many kilometers [km] is it to Atlanta? From the conversion table, we can find that 1 kilometer [km] = 0.621 miles [mi], or

$$\frac{1\,km}{0.621\,mi} = 1$$

By multiplying the original quantity of 123 miles by 1, we can say

$$(123 \text{ mi})\,(1) = (123 \text{ mi})\left(\frac{1\,km}{0.621\,mi}\right) = 198 \text{ km}$$

Note that we could have multiplied by the following relationship:

$$1 = \frac{0.621\,mi}{1\,km}$$

We would still have multiplied the original answer by 1, but the units would not cancel and we would be left with an awkward, meaningless answer.

$$(123 \text{ mi})(1) = (123 \text{ mi})\left(\frac{0.621\,mi}{1\,km}\right) = 76\,\frac{mi^2}{1\,km}$$

As a second example, we are designing a reactor system using 2-inch [in] diameter plastic pipe. The design office in Germany would like the pipe specifications in units of centimeters [cm]. From the conversion table, we find that 1 inch [in] = 2.54 centimeters [em], or

$$1 = \frac{1\,in}{2.54\,cm}$$

By multiplying the original quantity of 2 inches by 1, we can say

$$(2 \text{ in})(1) = (2 \text{ in})\,\frac{2.54\,cm}{1\,in} = 5 \text{ cm}$$

In a final example, suppose a car travels at 40 miles per hour (abbreviated mph). Stated in words, "a car traveling at a rate of 40 mph will take 1 hour to travel 40 miles if the velocity remains constant." By simple arithmetic this means that the car will travel 80 miles in 2 hours or 120 miles in 3 hours. In general,

$$\text{Distance} = (\text{velocity})\ (\text{time elapsed at that velocity})$$

### 7.1.1 Length

Suppose the car is traveling at 40 mph for 6 minutes. How far does the car travel?

Simple calculation shows

Distance = (40) (6) = 240

Without considering units, the preceding example implies that if we drive our car at 40 mph, we can cover the distance from Charlotte, North Carolina, to Atlanta, Georgia, in 6 minutes! What is wrong? Note that the velocity is given in miles per hour, and the time is given in minutes. If the equation is written with consistent units attached, we get

Distance = $\left(\frac{40\ mi}{h}\right)\left(\frac{6\ min}{} \middle| \frac{1\ h}{60\ min}\right)$ = 4 mi

It seems more reasonable to say "traveling at a rate of 40 miles per hour for a time period of 6 minutes will allow us to go 4 miles."

### 7.1.2 Example

Convert the length 40 yards [yd] into units of feet [ft].

| Method | Steps |
|---|---|
| (1) Term to be converted | 40 yd |
| (2) Conversion formula | 1 yd = 3 ft |
| (3) Make a fraction (equal to one) | $\frac{3\ ft}{1\ yd}$ |
| (4) Multiply | $\left(\frac{40\ yd}{} \middle| \frac{3\ ft}{1\ yd}\right)$ |
| (5) Cancel, calculate, be reasonable | 120 ft |

## 7.2 Conversions Involving Multiple Steps

Sometimes, more than one conversion factor is needed. We can multiply by several conversion factors, each one of which is the same as multiplying by 1, as many times as needed to reach the desired result. For example, suppose we determined that the distance to Atlanta is 123 miles [mi]. How many yards [yd] is it to Atlanta? From the conversion table, we do not have a direct conversion between miles and yards, but we see that both can be related to feet. We can find that 1 mile [mi] = 5,280 feet [ft], or

$$1 = \frac{5{,}280\ ft}{1\ mi}$$

We can also find that 1 yard [yd] = 3 feet [ft], or

$$1 = \frac{1\ yd}{3\ ft}$$

By multiplying the original quantity of 123 miles by 1 using the first set of conversion factors, we can say:

$$(123\ \text{mi})\ (1) = (123\ \text{mi})\left(\frac{5{,}280\, ft}{1\ mi}\right) = 649{,}449\ \text{ft}$$

If we multiply by 1 again, using the second set of conversion factors and applying reasonableness:

$$(649{,}440 \text{ ft})\ (1) = (649{,}440 \text{ ft}) \left(\frac{1\ yd}{3\ ft}\right) = 216{,}000 \text{ yd}$$

This is usually shown as a single step:

$$(123 \text{ mi}) \left(\frac{5{,}280\ ft}{1\ mi}\right)\left(\frac{1\ yd}{3\ ft}\right) = 216{,}000 \text{ yd}$$

### 7.2.1 Unit Conversion Procedure

1. Write the value and unit to be converted.
2. Write the conversion formula between the given unit and the desired unit.
3. Make a fraction, equal to 1, of the conversion formula in Step 2, such that the original unit in Step 1 is located either in the denominator or in the numerator, depending on where it must reside so that the original unit will cancel.
4. Multiply the term from Step 1 by the fractions developed in Step 3.
5. Cancel units, perform mathematical calculations, and express the answer in "reasonable" terms (i.e., not too many decimal places).

### 7.2.2 Example

Convert the length 40 yards [yd] into units of millimeters [mm].

| Method | Steps |
|---|---|
| (1) Term to be converted | 40 yd |
| (2) Conversion formula | 1 yd = 3 ft    1 ft = 12 in<br>1 in = 2.54 cm    1 cm = 10 mm |
| (3) Make a fraction (equal to one) | $\frac{3\ ft}{1\ yd}$   $\frac{12\ in}{1\ ft}$   $\frac{2.45\ cm}{1\ in}$   $\frac{10\ cm}{1\ cm}$ |
| (4) Multiply | |
| (5) Cancel, calculate, be reasonable | 37,000 mm |

### 7.2.3 Example

Convert 55 miles per hour [mph] to units of meters per second [m/s].

Note that we have two units to convert here, miles to meters, and hours to seconds.

| Method | Steps |
|---|---|
| (1) Term to be converted | 55 mph |
| (2) Conversion formula | 1 km = 0.621 mi    1 km = 1,000 m |

| | | |
|---|---|---|
| | 1 h = 60 min | 1 min = 60 s |
| (3) Make a fraction (equal to one)<br>(4) Multiply | | |
| (5) Cancel, calculate, be reasonable | 24.6 m/s | |

TIME

1 d = 24 h

1 h = 60 min

1 min = 60 s

1 yr = 365 d

### 7.2.4 Example

Convert the volume of 40 gallons [gal] into units of cubic feet [$ft^3$].

By examining the "Volume" box in the conversion table, we see that the following facts are available for use:

1 L = 0.264 gal and 1 L = 0.0353 $ft^3$

By the transitive property, if a = b and a = c, then b = c. Therefore, we can directly write

0.264 gal = 0.0353 $ft^3$

| Method | Steps |
|---|---|
| (1) Term to be converted | 40 gal |
| (2) Conversion formula | 0.264 gal= 0.0353 $ft^3$ |
| (3) Make a fraction (equal to one) | $\frac{0.0353\ ft^3}{0.264\ gal}$ |
| (4) Multiply | $\left(\frac{40\ gal}{}\middle\vert\frac{0.0353^3}{0.264\ gal}\right)$ |
| (5) Cancel, calculate, be reasonable | 5.3 $ft^3$ |

VOLUME

1 L = 0.264 gal

1 L = 0.0353 ft3

1 L = 33.8 fl oz

1 mL = 1 $cm^3$

One frequently needs to convert a value that has some unit or units raised to a power, for example, converting a volume given in cubic feet to cubic meters. It is critical in this case that the power involved be applied to the *entire* conversion factor, both the numerical values and the units.

### 7.2.5 Example

| Method | Steps |
|---|---|
| (1) Term to be converted | 35 $in^3$ |
| (2) Conversion formula | 1 in = 2.54 cm |
| (3) Make a fraction (equal to one) | $\left(\frac{35\ in^3}{} \middle\vert \frac{(2.54\ cm)^3}{(1\ in)^3}\right)$ |
| (4) Multiply | $\left(\frac{35\ in^3}{} \middle\vert \frac{(2.54)^3(cm)^3}{1\ in^3}\right)$ |
| (5) Cancel, calculate, be reasonable | 574 $cm^3$ |

### 7.2.6 Example

Convert a velocity of 250 kilometers per second [km/s] to units of millimeters per picosecond [mm/ps].

| Method | Steps |
|---|---|
| (1) Term to be converted | 250km/s |
| (2) Conversion formula | 1 k = $10^3$ m<br>1 mm = $10^{-3}$ m<br>1 ps = $10^{-12}$ s |
| (3) Make a fraction (equal to one) | |
| (4) Multiply | |
| (5) Cancel, calculate, be reasonable | 250 X $10^{-6}$ mm/ps |

- Comment: Following the rules for use of prefixes given earlier would indicate that the term in the denominator should not have a prefix at all, and it should be transferred to the numerator, giving 250 x $10^6$ mm/s. Beyond that, it was earlier stated that in general, the prefix should be adjusted to give one, two, or three digits to the left of the decimal place (with no power of 10). This would yield 250 km/s, right back where we started.
- Moral: **specific instructions** (such as "convert to mm/ps") **usually override the general rules.** In this case, perhaps we are studying the velocity of protons in a particle accelerator, and for comparison with other experiments, we need to know how many millimeters the particles go in one picosecond, rather than ending up with the units the general rules would dictate.

## 7.3 Conversions Involving "New" Units

In the past, many units were derived from common physical objects. The "inch" was the width of man's thumb, and the "foot" was the heel-to-toe length of a king's shoe. Obviously, when one king died or was deposed and another took over, the unit of "foot" changed, too. Over time, these units were standardized and have become common terminology.

New units are added as technology evolves; for example, in 1999 the unit of katal was added as an SI derived unit of catalytic activity used in biochemistry. As you proceed in your engineering field, you will be introduced to many "new" units. The procedures discussed here apply to *any* unit in *any* engineering field.

### 7.3.1 Example

According to the U.S. Food and Drug Administration (21CFR101.9), the following definition applies for nutritional labeling:

1 fluid ounce means 30 milliliters

Using this definition, how many fluid ounces [fl oz] are in a "U.S. standard" beverage can of 355 milliliters [mL]?

| Method | Steps |
| --- | --- |
| (1) Term to be converted | 355 mL |
| (2) Conversion formula | 1 fl oz = 30 mL |
| (3) Make a fraction (equal to one)<br>(4) Multiply | $\left(\frac{355\ mL}{} \middle\vert \frac{1\ fl\ oz}{30\ mL}\right)$ |
| (5) Cancel, calculate, be reasonable | 11.8 fl oz |

### 7.3.2 Example

The volume of water in a reservoir or aquifer is often expressed using the unit of acrefoot. A volume of 1 acre-foot is the amount of water covering an area of 1 acre to a depth of 1 foot.

Lake Mead, located 30 miles southeast of Las Vegas, Nevada, is the largest manmade lake in the United States. It holds approximately 28.5 million acre-feet of water behind the Hoover Dam. Convert this volume to units of gallons.

| Method | Steps |
| --- | --- |
| (1) Term to be converted | 28.5 X $10^6$ acre feet |
| (2) Conversion formula | 1 acre = 4,047 $m^2$ 1m= 3.28ft<br>1 $m^3$ = $100^3$ $cm^3$ 1,000 $cm^3$ =0.264 gal |
| (3) Make a fraction (equal to one)<br>(4) Multiply | |
| (5) Cancel, calculate, be reasonable | 9.3 X $10^{12}$ gal |

## 7.4 Force

When you push a grocery cart, it moves. If you keep pushing, it keeps moving. The longer you push, the faster it goes; the velocity increases over time, meaning that it accelerates. If you push a full grocery cart that has a high mass, it does not speed up as much, meaning it accelerates less than a cart with low mass. Simply put, the acceleration *(a)* of a body depends on the force *(F)* exerted on it and its mass *(m).* This is a simple form of "Newton's second law of motion" and is usually written as $F = ma$.

Table 7-1: Dimensions of force

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Ө |
| Force | N | 1 | 1 | -2 | 0 |

The SI unit of force, the newton [N], is defined as the force required to accelerate a mass of one kilogram at a rate of one meter per second squared (see Table 8-2). It is named for Sir Isaac Newton (1643-1727), Newton's *Principia* is considered one of the world's greatest scientific writings, explaining the law of universal gravitation and the three laws of motion. Newton also developed the law of conservation of momentum, the law of cooling, and the reflecting telescope. He shares credit for the development of calculus with Gottfried Leibniz.

In the SI system, mass, length, and time are base units and force is a derived unit; force is found from combining mass, length, and time using Newton's second law. The SI system is called "coherent," because the derived unit is set at one by combing base units. The AES system is considered non-coherent as it uses units that do not work together in the same fashion as the SI units do. There are two uses of the term "pound" in the AES system, which occurred in common usage long before Newton discovered gravity. To distinguish mass in pounds and force in pounds, the unit of mass is given as pound-mass ($Ib_m$) and the unit of force is given as pound-force ($lb_f$). One pound-force is the amount of force needed to accelerate one pound-mass at a rate of 32.2 feet per second squared. Since this relationship is not easy to remember or use in conversions, we will stick with SI units for problem solving, following the procedure discussed in Chapter 7.

### 7.4.1 Example

A professional archer is designing a new longbow with a full draw weight of 63 pounds-force [$Ib_f$]. The draw weight is the amount of force needed to hold the bowstring at a given amount of draw, or the distance the string has been pulled back from the rest position. What is the full draw weight of this bow in units of newtons [N]?

| Method | Steps |
|---|---|
| (1) Convert term | 63 $Ib_f$ |
| (2) Apply conversion formula | 1 N = 0.225 $Ib_f$ |
| (3) Make a fraction<br>(4) Multiply | $\left(63\ Ib_f \middle\| \frac{1\ N}{0.225\ Ib_f}\right)$ |
| (5) Cancel, calculate, be reasonable | 280 N |

## 7.5 Weight

The mass of an object is a fundamental dimension. Mass is a quantitative measure of how much of an object there is, or in other words, how much matter it contains. The **weight** *(w)* of an object is a force equal to the mass of the object *(m)* times the acceleration of **gravity** *(g).*

While mass is independent of location in the universe, weight is dependent upon both mass and gravity (Table 7-2).

*On the Earth, the pull of gravity is approximately* 9.8 *meters per second squared [m/s$^2$].* On the moon, gravity is approximately one-sixth this value, or 1.6 m/s$^2$. A one kilogram [kg] object acted on by Earth's gravity would have a weight of 9.8 N, but on the moon it would have a weight of 1.6 N. Unless otherwise stated, assume all examples take place on the Earth.

Table 7-2: Dimensions of weight

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Weight | N | 1 | 1 | -2 | 0 |

### 7.5.1 Example

What is the weight of a 225-kilogram [kg] bag of birdseed in units of newtons [N]?

| Step One: Convert to Base SI Units | |
|---|---|
| No conversion necessary | |
| **Step Two: Calculate** | |
| **Method** | **Steps** |
| (1) Determine appropriate equation | w = mg |
| (2) Insert known quantities | $w = \left(225kg \middle\vert \frac{9.8\ m}{s^2}\right)$ |
| (3) Calculate, be reasonable | $w = 2{,}205\ \frac{kg\ m}{s^2}$ |

This is apparently our final answer, but the units are puzzling. If the unit of force is the newton, and if this is a valid equation, then our final result for force should be newtons. If we consider the dimensions of force:

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Force | N | 1 | 1 | -2 | 0 |

A unit of force has dimensions F {=} ML/T$^2$, which in terms of base SI units would be F [=] kg m/s$^2$. As this term occurs so frequently it is given the special name "newton" (see Table 8-1). Anytime we see the term [kg m/s$^2$], we know we are dealing with a force equal to a newton.

| | |
|---|---|
| (3) Calculate, be reasonable | $w = 2{,}205 \left( \frac{kg\ m}{s^2} \middle| \frac{1\ N}{\frac{kg\ m}{s^2}} \right)$ = 2,205 N |
| Step Three: Convert from Base SI Units to Desire Units | |
| No conversion necessary | |

## 7.6 Density

**Density** *(p,* Greek letter rho) is the mass of an object *(m)* divided by the volume the object occupies *(V).* Density should not be confused with weight-think of the old riddle: which weighs more, a pound of feathers or a pound of bricks? The answer is they both weigh the same amount, one pound, but the density of each is different. The bricks have a higher density than the feathers, since the same mass takes up less space.

**Specific weight** *($\gamma$,* Greek letter gamma) is the weight of an object *(w)* divided by the volume the object occupies (V) (Table 7-3).

Table 7-3: Dimensions of density and specific weight

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Density | Kg/m$^3$ | 1 | -3 | 0 | 0 |
| Specific weight | N/m$^3$ | 1 | 2 | 2 | 0 |

### 7.6.1 Example

The density of sugar is 1.61 grams per cubic centimeter [g/cm$^3$]. What is the density of sugar in units of pound-mass per cubic foot [lb$_m$/ft$^3$]?

| Method | Steps |
|---|---|
| (1) Term to be convert | 0.72 g/cm$^3$ |
| (2) Conversion formula<br>(3) Make fractions<br>(4) Multiply | $\left\langle 1.61\,\frac{g}{cm^3} \middle| \frac{2.205 Ib_m}{1{,}000 g} \middle| \frac{1{,}000\ cm^3}{0.0353\ ft^3} \right\rangle$ |
| (5) Cancel, calculate, be reasonable | 720 kg/m$^3$ |

### 7.6.2 Example

The density of a biofuel blend is 0.72 grams per cubic centimeter [g/cm$^3$]. What is the density of the biofuel in units of kilograms per cubic meter [kg/m$^3$]?

| Method | Steps |
|---|---|
| (1) Term to be convert | 0.72 g/cm$^3$ |
| (2) Conversion formula<br>(3) Make fractions<br>(4) Multiply | $\left\langle 0.72 \frac{g}{cm^3} \middle| \frac{1\ kg}{1{,}000 g} \middle| \frac{(100\ cm)^3}{1\ m^3} \right\rangle$ |
| (5) Cancel, calculate, be reasonable | 720 kg/m$^3$ |

### 7.6.3 Example

What is the weight of water, in units of pounds-force [$lb_f$], in a 55-gallon drum completely full? Assume the density of water to be 1 gram per cubic centimeter. Ignore the weight of the drum.

| Step One: Convert to Base SI Units | | |
|---|---|---|
| Method | Steps | |
| (1) Term to be convert | 55 gal | 1 g/cm$^3$ |
| (2) Conversion formula<br>(3) Make fractions<br>(4) Multiply | $\left\langle \frac{55\ gal}{} \middle| \frac{1{,}000\ cm^3}{0.264\ gal} \middle| \frac{1\ m^3}{100^3\ cm^3} \right\rangle$ | $\left\langle \frac{1\ g}{cm^3} \middle| \frac{1\ kg}{1{,}000\ g} \middle| \frac{100^3 cm^3}{1\ m^3} \right\rangle$ |
| (5) Cancel, calculate, be reasonable | 0.208 m$^3$ | 1,000 kg/m$^3$ |

| Step Two: Calculate | |
|---|---|
| Method | Steps |
| (1) Determine appropriate equation | m = $pV$ |
| (2) Insert known quantities | $w = \left( \frac{m}{} \middle| \frac{9.8\ m}{S^2} \right)$ |
| For Unknown Quantities, Repeat the Process | |
| Method | Steps |
| (1) Determine appropriate equation | M = $pV$ |
| (2) Insert known quantities | $m = \left( \frac{1{,}000 kg}{m^3} \middle| \frac{0.208\ m^3}{} \right)$ |
| (3) Calculate, be reasonable | m = 208 kg |
| | |
| (2) Insert known quantities | $w = \left( \frac{208\ kg}{} \middle| \frac{9.8\ m}{S^2} \right)$ |

| | |
|---|---|
| (3) Calculate, be reasonable | $w = \left(2{,}038\,\frac{kg\ m}{S^2}\middle\vert\frac{1\ N}{\frac{1\ kg\ m}{S^2}} = 2{,}038\ N\right)$ |
| **Step Three: Convert from Base SI Units to Desire Units** | |
| Method | Steps |
| (1) Term to be converted | 2,038 N |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left(\frac{2{,}038\ N}{}\middle\vert\frac{0.225\ Ib_f}{1\ N}\right)$ |
| (5) Cancel, calculate, be reasonable | 460 $Ib_f$ |

### 7.6.4 Specific Gravity

In technical literature, density is rarely given; instead, the **specific gravity** is reported. The specific gravity (SG) of an object is a dimensionless ratio of the density of the object to the density of water (see Table 7-4). It is convenient to list density in this fashion so *any* unit system may be applied by our choice of the units of the density of water. The specific gravities of several common substances are listed in Table 7-5.

Table 7-4: Dimensions of specific gravity

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Specific gravity | - | 0 | 0 | 0 | 0 |

Table 7-5: Specific gravity values for common substances

| Liquids | SG |
|---|---|
| Acetone | 0.785 |
| Benzene | 0.876 |
| Citric acid | 1.67 |
| Gasoline | 0.739 |
| Glycerin | 1.26 |
| Iodine | 4.93 |
| Mercury | 13.6 |
| Mineral oil | 0.900 |
| Olive oil | 0.703 |

| Solids | SG |
|---|---|
| Aluminum | 2.70 |
| Baking soda | 0.689 |
| Brass | 8.40-8.75 |
| Concrete | 2.30 |
| Copper | 8.96 |
| Gallium | 5.91 |
| Gold | 19.3 |
| Graphite | 2.20 |
| Iron | 7.87 |

| | | | |
|---|---|---|---|
| Propane | 0.806 | Lead | 11.4 |
| Sea water | 1.03 | Polyvinyl chloride (PVC) | 1.38 |
| Toluene | 0.865 | Silicon | 2.33 |
| Water | 1.00 | Zinc oxide | 5.60 |

When calculating or considering specific gravities, it is helpful to keep in mind the range of values that you are likely to have.

The densest naturally occurring elements at normal temperature and pressure are osmium and iridium, both with a specific gravity close to 22.6. The *densest substances that a normal person is likely to encounter are platinum (SG* = 21.5) *and gold (SG* = 193). Thus, if you calculate a specific gravity to be higher than about 23, you have almost certainly made an error.

*Most liquids are similar to water, with a specific gravity around* 1. One notable exception is mercury, with a specific gravity of 13.

On the lower end of the scale, the *specific gravity of air is about 0.001*, whereas hydrogen has a specific gravity of slightly less than 0.0001.

Therefore, if you get a specific gravity value less than about $10^{-4}$, you need to check your work very carefully.

Note: Density of water

$= 1 \text{ g/cm}^3$

$= 1 \text{ kg/L}$

$= 1{,}000 \text{ kg/m}^3$

$= 62.4 \text{ Ib}_m/\text{ft}^3$

$= 1.94 \text{ slug/ft}^3$

### 7.6.5 Example

The specific gravity of butane is 0.599. What is the density of butane in units of kilograms per cubic meter?

| Steps One: Convert to Base SI Units | |
|---|---|
| No conversion needed | |
| **Step Two: Calculate** | |
| **Method** | **Steps** |
| (1) Determine appropriate equation | $\rho_{object} = (SG)(\rho_{water})$ |
| (2) Insert known quantities | $\rho_{object} = (0.599)(1{,}000\ \frac{kg}{m^3})$ |
| (3) Calculate, be reasonable | $\rho_{object} = 599\frac{kg}{m^3}$ |
| **Step Three: Convert from Base SI Units to Desire Units** | |
| No conversion needed | |

### 7.6.6 Example

Mercury has a specific gravity of 13.6. What is the density of mercury in units of slugs per cubic foot?

| Steps One: Convert to Base SI Units | |
|---|---|
| No conversion needed | |
| **Step Two: Calculate** | |
| Method | Steps |
| (1) Determine appropriate equation | $\rho_{object} = (SG)(\rho_{water})$ |
| (2) Insert known quantities | $\rho_{object} = (13.6)(62.4\ \frac{lbm}{ft^3})$ |
| (3) Calculate, be reasonable | $\rho_{object} = 848.64\ \frac{lbm}{ft^3}$ |
| **Step Three: Convert from Base SI Units to Desire Units** | |
| Method | Steps |
| (1) Term to be converted | $848.64\ lb_m/ft^3$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left(\frac{848.64\ lb_m}{ft^3}\middle|\frac{1\ slug}{32.2\ lb_m}\right)$ |
| (5) Cancel, calculate, be reasonable | $26.4\ slug/ft^3$ |

## 7.7 Amount

Some things are really very large and some are very small. Stellar distances are so large that it becomes inconvenient to report values such as 235 trillion miles, or 6.4 X $10^{21}$ feet when we are interested in the distance between two stars or two galaxies. To make things better, we use a new unit of length that itself is large-the distance that light goes in a year; this is a very long way, 3.1 X $10^{16}$ feet. As a result, we do not have to say that the distance between two stars is 620,000,000,000,000,000 feet, we can just say that they are 2 light-years apart.

This same logic holds when we want to discuss very small things such as molecules or atoms. Most often we use a constant that has been named after Amedeo Avogadro, an Italian scientist (1777-1856) who first proposed the idea of a fixed ratio between the amount of substance and the number of elementary particles. The Avogadro constant has a value of 6.022 X $10^{23}$ particles per mole. If we have 12 of something, we call it a dozen. If we have 20, it is a score. If we have 6.022 x $10^{23}$ of anything, we have a mole. If we have 6.022 X $10^{23}$ baseballs, we have a mole of baseballs. If we have 6.022 X $10^{23}$ elephants, we have a mole of elephants, and if we have 6.022 X $10^{23}$ molecules, we have a mole of molecules. Of course, the mole is never used to define amounts of macroscopic things like elephants or baseballs, being relegated to the realm of the extremely tiny. In the paragraphs below we will see how this rather odd value originated and how this concept simplifies our calculations.

The mass of a nucleon (neutron or proton) is about 1.66 X $10^{-24}$ grams. To avoid having to use such tiny numeric values when dealing with nucleons, physicists defined the **atomic mass unit** [amu] to be approximately the mass of one nucleon. Technically, it is defined as one-twelfth of the mass of a carbon twelve atom. In other words, 1 amu = 1.66 X $10^{-24}$ g. The symbol "u" is often used for amu, which is also known as a **Dalton** [Da].

If there is (1.66 X $10^{-24}$ *g)/(1* amu), then there is (1 *amu)/(I.66* X $10^{-24}$ g). Dividing this out gives 6.022 X $10^{23}$ amu/g. This numeric value is used to define the **mole** [mol]. One mole of a substance (usually an element or compound) contains exactly 6.022 X $10^{23}$ fundamental units (atoms or molecules) of that substance. In other words, there are 6.022 X $10^{23}$ fundamental units per mole. This is often written as

$$N_A = 6.022 \text{ X } 10^{23} \text{ mol}^{-1}$$

As mentioned above, this is called Avogadro's constant or **Avogadro's number**, symbolized by $N^A$. So why is this important? Consider combining hydrogen and oxygen to get water ($H_2O$). We need twice as many atoms of hydrogen as atoms of oxygen for this reaction; thus, for every mole of oxygen, we need two moles of hydrogen, since one mole of anything contains the same number of fundamental units, atoms in this case.

The problem is that it is difficult to measure a substance directly in moles, but it is easy to measure its mass. *Avogadro's number affords a conversion path between moles and mass.* Consider hydrogen and oxygen in the above. The atomic mass of an atom in **atomic mass units** [amu] is approximately equal to the number of nucleons it contains. Hydrogen contains one proton, and thus has an atomic mass of 1 amu. We can also say that there is 1 amu per hydrogen atom. Oxygen has an atomic mass of 16; thus, there are 16 amu per oxygen atom. Since atomic mass refers to an individual specific atom, the term **atomic weight** is used, representing the average value of all isotopes of the element. This is the value commonly listed on periodic tables.

Let us use this information, along with Avogadro's number, to determine the mass of one mole of each of these two elements.

$$Hydrogen\colon \left\langle \frac{1\ amu}{H\ atom} \middle| \frac{1\ g}{6.022\ X\ 10^{23}\ amu} \middle| \frac{6.022\ X\ 10^{23}\ amu}{1\ mol} = \frac{1\ g}{1\ mol\ H} \right\rangle$$

$$Oxygen\colon \left\langle \frac{16\ amu}{O\ atom} \middle| \frac{1\ g}{6.022\ X\ 10^{23}\ amu} \middle| \frac{6.022\ X\ 10^{23}\ amu}{1\ mol} = \frac{16\ g}{1\ mol\ O} \right\rangle$$

The numerical value for the atomic mass of a substance is the same as the number of grams in one mole of that substance, often called the **molar mass**.

Atomic weight = molar mass

Avogadro's number is the link between the two. Hydrogen has a molar mass of 1 gram per mole; oxygen has a molar mass of 16 grams per mole.

When groups of atoms react together, they form molecules. Consider combining hydrogen and oxygen to get water ($H_2O$). Two atoms of hydrogen combine with one atom of oxygen, so 2 * 1 amu H + 16 amu O = 18 amu $H_2O$. The **molecular mass** of water is 18 amu. By an extension of the example above, we can also state that one mole of water has a mass of 18 grams, called the **formula weight**.

molecular weight = formula weight

The difference between these ideas is summarized in Table 7-6.

This text assumes that you have been exposed to these ideas in an introductory chemistry class and so does not cover them in any detail in all problems presented, you will be given the atomic weight of the elements or the formula weight of the molecule, depending on the question asked.

This topic is briefly introduced because Avogadro's number ($N_A$) is important in the relationship between several constants, including the following:

- The gas constant ($R$ [=] J/(mol K)) and the Boltzmann constant ($k$ [=] J/K), which relates energy to temperature: $R = kN_A$.
- The elementary charge ($e$ [=] C) and the Faraday constant ($F$ [=] C/mol) , which is the electric charge contained in one mole of electrons: $F = eN_A$.
- An electron volt [e V] is a unit of energy describing the amount of energy gained by one electron accelerating through an electrostatic potential difference of one volt: 1 eV = 1.602 X $10^{-19}$ J.

Table 7-6: Definitions of "amount" of substance

| the quantity … | measure the … | in units of … | and is found by … |
|---|---|---|---|
| Atomic mass | Mass of one atom of an individual isotope of an element | [amu] | Direct laboratory measurement |
| Atomic weight | Average mass of all isotopes of an element | [amu] | Listed on Periodic Table |
| Molar mass | Mass of one mole of the atom | [g/mol] | Listed on Periodic Table |
| Molecular mass or molecular weight | Sum of average weight of isotopes in molecule | [amu] | Combining atomic weights of individual atoms represented in the molecule |
| Formula weight | Mass of one mole of the molecule | [g/mol] | Combining molar mass of individual atoms represented in the molecule |

### 7.7.1 Example

Let us return to the problem of combining hydrogen and oxygen to get water. Assume you have 50 grams of oxygen with which you want to combine the proper mass of hydrogen to convert it completely to water. The atomic weight of hydrogen is 1 and the atomic weight of oxygen is 16.

First determine how many moles of oxygen are present.

$$\left(\frac{50\ g\ O}{}\middle|\frac{1\ mol\ O}{16\ g\ O} = 3.125\ mol\ O\right)$$

We need twice as many moles of hydrogen as oxygen ($H_2O$), so we need 6.25 moles of hydrogen. Converting to mass gives

$$\left(\frac{6.25\ mol\ H}{}\middle|\frac{1\ g\ H}{1\ mol\ H} = 6.25\ gH\right)$$

### 7.7.2 Example

Acetylsalicylic acid (aspirin) has the chemical formula $C_9H_8O_4$. How many moles of aspirin are in a 1-gram dose? Use the following facts:

- Atomic weight of carbon = 12
- Atomic weight of hydrogen = 1
- Atomic weight of oxygen = 16

First, determine how many grams are in 1 mole of aspirin (determine formula weight).

$$FW \text{ of aspirin} = \left[\left(\frac{12\frac{g}{mole}}{1\ molecule\ C}\middle|\frac{9\ C\ molecules}{}\right)\right] + \left[\left(\frac{1\frac{g}{mole}}{1\ molecule\ H}\middle|\frac{8\ H\ molecules}{}\right)\right] + \left[\left(\frac{16\frac{g}{mole}}{1\ molecule\ O}\middle|\frac{4\ O\ molecules}{}\right)\right] = 180\ \frac{g}{mole}$$

Finally, convert to moles per dose.

$$\left(\frac{1\ g\ aspirin}{dose}\middle|\frac{1\ mol\ aspirin}{180\ g\ aspirin}\right) = 5.56\ X\ 10^{-3}\ \frac{mol\ aspirin}{dose}$$

### 7.7.3 Example

Many gases exist as diatomic compounds in nature, meaning two of the atoms are attached to form a molecule. Hydrogen, oxygen, and nitrogen all exist in a gaseous diatomic state under standard conditions.

Assume there are 100 grams of nitrogen gas in a container. How many moles of nitrogen ($N_2$) are in the container? Atomic weight of nitrogen = 14.

First, determine how many grams are in 1 mole of nitrogen (determine the formula weight).

$$FWof\ N_2 = \left(\frac{14\frac{g}{mol}}{1\ mol\ N}\middle|\frac{2\ mol\ N}{}\right) = 28\ \frac{g}{mol}$$

Next, convert mass to moles.

$$\left(\frac{10\ grams\ of\ N_2}{}\middle|\frac{mol}{28\ gram}\right) = 3.57\ mol\ N_2$$

## 7.8 Temperature

Temperature was originally conceived as a description of energy: heat (thermal energy) flows spontaneously from "hot" to "cold." But how hot is "hot"? The thermometer was devised as a way to measure the "hotness" of an object. As an object gets warmer, it usually expands. In a thermometer, a temperature is a level of hotness that corresponds to the length of the liquid in the tube. As the liquid gets warmer, it expands and moves up the tube. To give temperature a quantitative meaning, numerous temperature scales have been developed.

Many scientists, including Isaac Newton, have proposed temperature scales. Two scales were originally developed about the same time – **Fahrenheit** [$^{o}$F] and **Celsius** [$^{o}$C] - and have become widely accepted in laymen use. These are the most frequently used temperature scales by the general public. Gabriel Fahrenheit (1686-1736), a German physicist and engineer, developed the Fahrenheit scale in 1708. Anders Celsius (1701-1744), a Swedish astronomer, developed the Celsius scale in 1742. The properties of each scale are in Table 7-7.

You may wonder why the Celsius scale seems so reasonable, and the Fahrenheit scale so random. Actually, Mr. Fahrenheit was just as reasonable as Mr. Celsius. Mr. Celsius set the freezing point of water to be 0 and the boiling point to be 100. Mr. Fahrenheit took as 0 a freezing mixture of salt and ice, and as 100 body temperature. With this scale, it just so happens that the freezing and boiling points of water work out to be odd numbers. Here are some numbers to remember:

- Human body -37$^{o}$C = 98°F
- Room temperature -21°C = 70°F
- Melting point of mercury -39°C = -38°F
- Melting temperature of lead -330°C = 620°F

Table 7-7: Properties of water

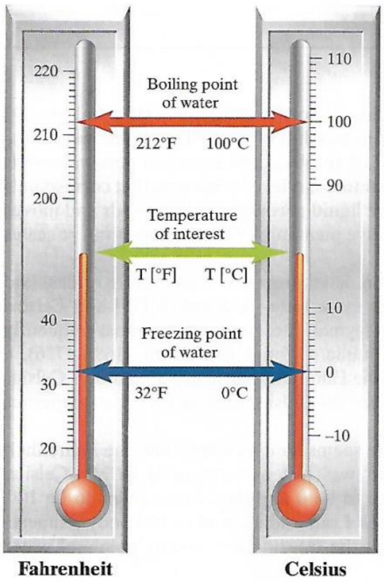
| Scale | Freezing Point | Boiling Point | Divisions Between Freezing and Boiling |
|---|---|---|---|
| Fahrenheit [$^o$F] | 32 | 212 | 180 |
| Celsius [$^o$C] | 0 | 100 | 100 |
| Kelvin [K] | 273 | 373 | 100 |
| Rankine [$^o$R] | 492 | 672 | 180 |

Some units can cause confusion in conversion. One of those is temperature. One reason for this is that we use temperature in two different ways: (1) reporting an actual temperature value and (2) discussing the way a change in temperature affects a material property. To clarify, we resort to examples.

### 7.8.1 Conversion Between Temperature Values

When an actual temperature reading is reported, such as "the temperature in this room is *70°F,"* how do we convert from one temperature unit to another? The scales have different zero points, so they cannot be converted with a single conversion factor as done previously but require a conversion formula. Most of you are familiar with the conversion between Fahrenheit and Celsius, but this equation is cumbersome to remember.

$$T[^oF] = \frac{9}{5}T[^oC] + 32$$




Let us imagine we have two thermometers, one with the Fahrenheit scale and the other with the Celsius scale. We set two thermometers side by side so that the freezing point and the boiling point of water are at the same location on both thermometers. We are interested in the relationship between these two scales. From this figure we see that the fraction of the distance from the freezing point to the boiling point in both scales is the same. This means that we can write

$$\frac{T[°F] - 32}{212 - 32} = \frac{T[°C] - 0}{100 - 0}$$

This relationship is really all we need to know to relate a temperature in degrees Fahrenheit to one in degrees Celsius. You can easily do the algebra to convert from Fahrenheit to Celsius, or vice versa. By remembering this form, you do not have to remember if the conversion is 9/5 or 5/9, or to add or subtract 32. This formula is determined by the method of interpolation.

There are numerous other temperature scales, but two are worth mentioning: **kelvin** [K] and **degrees rankine** [$^o$R]. The kelvin scale is named for First Baron William Thomson Kelvin (1824-1907), an English mathematician and physicist. Kelvin first proposed the idea of "infinite cold," or absolute zero, in 1848, using the Celsius scale for comparison. The Rankine scale is named for William J. M. Rankine (1820-1872), a Scottish engineer and physicist, who proposed an analogy to the kelvin scale, using the Fahrenheit scale. Both men made significant contributions to the field of thermodynamics.

The kelvin and Rankine scales are "absolute," which means that at absolute zero, the temperature at which molecules have minimum possible motion, the temperature is zero. Absolute temperature scales therefore have no negative values. In the kelvin scale, the degree sign is not used; it is simply referred to as "kelvin," not "degrees kelvin." It is the base SI unit for temperature and the most frequently used temperature unit in the scientific community.

### 7.8.2 Example

The hottest temperature in the United States ever recorded by the National Weather Service, 56.7 degrees Celsius [°C], occurred in Death Valley, California, on July 10,1913. State this value in units of degrees Fahrenheit [°F].

| Steps One: Convert to Base SI Units | |
|---|---|
| No conversion needed | |
| Step Two: Calculate | |
| Method | Steps |
| (1) Determine appropriate equation | $\frac{T[°\mathrm{F}] - 32}{212 - 32} = \frac{T[°\mathrm{C}] - 0}{100 - 0}$ |
| (2) Insert known quantities | $\frac{T[°\mathrm{F}] - 32}{180} = \frac{56.7}{100}$ |
| (3) Calculate, be reasonable | T = 134 °F |
| Step Three: Convert from Base SI Units to Desire Units | |
| No conversion needed | |

### 7.8.3 Conversions Involving Temperature Within a Material Property

When considering how a change in temperature affects a material property, we use a scalar conversion factor. In general, we encounter this in sets of units relating to the property of the material; for example, the units of the thermal conductivity are given by *W/m* K, which is read as "watts per meter kelvin." When this is the case, we are referring to the size of the degree, not the actual temperature.

To find this relationship, remember that between the freezing point and the boiling point of pure water, the Celsius scale contains 100 divisions, whereas the Fahrenheit scale contains 180 divisions. The conversion factor between Celsius and Fahrenheit is 100°C ≡ 180°F, or 1°C ≡ *1.8°F.*

### 7.8.4 Example

The specific heat ($C_p$) is the ability of an object to store heat. Specific heat is a material property, and values are available in technical literature. The specific heat of copper is 0.385 *J/ (g* °C), which is read as "joules per gram degree Celsius." Convert this to units of J/ ($lb_m$ OF), which reads "joules per pound-mass degree Fahrenheit."

| Method | Steps |
|---|---|
| (1) Term to be converted | $0.385\ \frac{J}{g\ °\mathrm{C}}$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left\langle \frac{0.385\ J}{g\ °\mathrm{C}} \middle\vert \frac{1{,}000\ g}{2{,}205\ lb_m} \middle\vert \frac{1\ °\mathrm{C}}{1.8\ °\mathrm{F}} \right\rangle$ |

| | |
|---|---|
| (5) Cancel, calculate, be reasonable | $97\ \frac{J}{lb_m °F}$ |

## 7.9 Pressure

**Pressure** is defined as force acting over an area, where the force is perpendicular to the area. In SI units, a **pascal** [Pa] is the unit of pressure, defined as one newton of force acting on an area of one square meter (Table 7-8). The unit pascal is named after Blaise Pascal (1623-1662), a French mathematician and physicist who made great contributions to the study of fluids, pressure, and vacuums. His contributions with Pierre de Fermat on the theory of probability were the groundwork for calculus.

Table 7-8: Dimensions of pressure

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Pressure | Pa | 1 | -1 | -2 | 0 |

### 7.9.1 Units of Pressure

1 atm = 1.01325 bar

= 33.9 ft $H_20$

= 29.92 in Hg

= 760 mm Hg

= 101,325 Pa

= 14.7 psi

In this chapter, we consider four forms of pressure, all involving fluids. The general term **fluid** applies to a gas, such as helium or air, or a liquid, such as water or honey.

- **Atmospheric pressure** – the pressure created by the weight of air above us.
- **Hydrostatic pressure** – the pressure exerted on a submerged object by the fluid in which it is immersed.
- **Total pressure** – the combination of atmospheric and hydrostatic pressure.
- **Gas pressure** – the pressure created by a gas inside a closed container.

### 7.9.2 Pressure

1 atm ~ 14.7 psi ~ 101 kPa

### 7.9.3 Atmospheric Pressure

Atmospheric pressure results from the weight of the air above us, which varies with both altitude and weather patterns. Standard atmospheric pressure is an average air pressure at sea level, defined as one atmosphere [atm], and is approximately equal to 14.7 pound-force per square inch [psi].

### 7.9.4 Pressure Measurement

When referring to the measurement of pressure, two types of reference points are commonly used.

**Absolute pressure** uses a perfect vacuum as a reference point. Most meteorological readings are given as absolute pressure, using units of atmospheres or bars.

**Gauge pressure** uses the local atmospheric pressure as a reference point. Note that local atmospheric pressure is generally *not* standard atmospheric pressure at sea level. Measurements such as tire pressure and blood pressure are given as gauge pressure.

Absolute pressures are distinguished by an "a" after the pressure unit, such as "psia" to signify "pound-force per square inch absolute." Gauge pressure readings are distinguished by a "g" after the pressure unit, such as "psig" to signify "pound-force per square inch gauge." When using instrumentation to determine the pressure, be sure to note whether the device reads absolute or gauge pressure.

Gauge pressure, absolute pressure, and atmospheric pressure are related by

$$P_{\text{absolute}} = P_{\text{gauge}} + P_{\text{atmospheric}}$$

A few notes on absolute and gauge pressure. Except as otherwise noted, assume that local atmospheric pressure is 14.7 psi.

- 35 psig = 49.7 psia (35 psig + 14.7 psi)
- Using gauge pressure, local atmospheric pressure would be 0 psig, although this would seldom be used.
- If a gauge pressure being measured is less than the local atmospheric pressure, this is usually referred to as **vacuum pressure**, and the negative sign is dropped. Thus, a perfect vacuum created at sea level on the Earth would read about 14.7 psig vacuum pressure. (A perfect vacuum is 0 psia, and thus is about 14.7 psi less than atmospheric pressure.)
- A vacuum pressure of 10 psig is an absolute pressure of 4.7 psia (14.7 psi – 10 psi).
- To illustrate the effect of local atmospheric pressure, consider the following scenario. You fill your automobile's tires to 35 psig on the shore of the Pacific Ocean in Peru, and then drive to Lake Titicaca on the Bolivian border at about 12,500 feet above sea level. Your tire pressure now reads about 40 psig due to the decreased atmospheric pressure (9.5 psia at 12,500 feet altitude versus 14.7 psi a at sea level).

Occasionally in industry, it may be helpful to use a point of reference other than atmospheric pressure. For these specific applications, pressure may be discussed in terms of **differential pressure**, distinguished by a "d" after the pressure unit, such as "psid."

### 7.9.5 Hydrostatic Pressure

Hydrostatic pressure ($P_{\text{hydro}}$) results from the weight of a liquid or gas pushing on an object. *Remember, weight is a force!* A simple way to determine this is to consider a cylinder with a cross-sectional area *(A)* filled with a liquid of density *p.*

The pressure *(P)* at the bottom of the container can be found by **Pascal's law**, named after (once again) Blaise Pascal. Pascal's law states the hydrostatic pressure of a fluid is equal to the force of the fluid acting over an area.

Pascal's Law

$$P_{\text{hydro}} = pgH$$

### 7.9.6 Example

We want to know the hydrostatic pressure in a lake at a depth of 20 feet in units of pascals.

For hydrostatic pressure, we need to know the density of the fluid in the lake. Since a density is not specified, we assume the density to be the standard density of water. We want a/l quantities in units of kilograms, meters, and seconds, so we use a density of 1, 000 kilograms per cubic meter for water.

| Step One: Convert to Base SI Units | |
|---|---|
| Method | Steps |
| (1) Term to be converted | $20\ ft$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left(\frac{20\ ft}{} \middle| \frac{1\ m}{3.28\ ft}\right)$ |
| (5) Cancel, calculate, be reasonable | 6.1 m |

| Step Two: Calculate | |
|---|---|
| Method | Steps |
| (1) Determine appropriate equation | $P_{hydro} = pgH$ |
| (2) Insert known quantities | $p_{hydro} \left(\frac{1{,}000\ kg}{m^3} \middle| \frac{9.8\ m}{S^2} \middle| \frac{6.1\ m}{}\right)$ |
| (3) Calculate, be reasonable | $p_{hydro} = 59{,}760 \frac{kg}{m\ s^2}$ |

This is apparently our final answer, but the units are puzzling. If the units of pressure are pascals and if this is a valid equation, then our final result for pressure should be pascals. If we consider the dimensions of pressure:

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Ɵ |
| Pressure | Pa | 1 | -1 | -2 | 0 |

A unit of pressure has dimensions, P {=}M/(LT$^2$), which in terms of base SI units would be P [ = ] kg/m S$^2$. As this term occurs so frequently it is given the special name "Pascal." When we see this term, we know we are dealing with a pressure equal to a pascal.

| | |
|---|---|
| (3) Calculate, be reasonable | $p_{hydro} = \left(59{,}760\ \frac{kg}{m\ s^2} \middle| \frac{1\ Pa}{\frac{kg}{m\ s^2}}\right)$<br>$= 59{,}760\ Pa$ |

| Step Three: Convert from Base SI Units to Desire Units |
|---|
| No conversion needed |

### 7.9.7 Total Pressure

We need to realize that Pascal's law is only a part of the story. Suppose we dive to a depth of 5 feet in a swimming pool and measure the pressure. Now we construct an enclosure over the pool and pressurize the air above the water surface to 3 atmospheres. When we dive back to the 5-foot depth, the pressure will have increased by 2 atmospheres.

Consequently, we conclude that total pressure at any depth in a fluid is the sum of hydrostatic pressure and surface pressure.

### 7.9.8 Example

When you dive to the bottom of a pool, at 12 feet under water, how much total pressure do you feel in units of atmospheres?

| Step One: Convert to Base SI Units | |
|---|---|
| Method | Steps |
| (1) Term to be converted | $12\ ft$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left(\frac{12\ ft}{}\middle|\frac{1\ m}{3.28\ ft}\right)$ |
| (5) Cancel, calculate, be reasonable | 3.66 m |

For hydrostatic pressure, we need to know the density of the fluid in the pool. Since a density is not specified, we assume the density to be the standard density of water: We want all quantities in units of kilograms, meters, and seconds, so we use a density of 1, 000 kilograms per cubic meter for water:

For total pressure, we need to know the surface pressure on top of the pool. Since a surface pressure is not specified, we assume the pressure to be 1 atmosphere. We want all quantities in units of kilograms, meters, and seconds, so we use a pressure of 101,325 pascals, or 101,325 kilograms per meter second squared.

| Step Two: Calculate | |
|---|---|
| Method | Steps |
| (1) Determine appropriate equation | $P_{total} = P_{surface} + pgH$ |
| (2) Insert known quantities | $p_{total} = \left(\frac{101{,}325\ kg}{m\ s^2} + \frac{1{,}000\ kg}{m^3}\middle|\frac{9.8\ m}{s}\middle|\frac{3.66\ m}{}\right)$ |

| | |
|---|---|
| (3) Calculate, be reasonable | $p_{total} = \left( 137{,}193 \frac{kg}{m\ s^2} \middle\vert \frac{Pa}{\frac{kg}{m\ s^2}} \right) = 137{,}193\ Pa$ |

| Step Three: Convert to Base SI Units to Desire Units | |
|---|---|
| Method | Steps |
| (1) Term to be converted | $137{,}193\ Pa$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left( \frac{137{,}193\ Pa}{} \middle\vert \frac{1\ atm}{101{,}325\ Pa} \right)$ |
| (5) Cancel, calculate, be reasonable | 1.35 atm |

### 7.9.9 Gas Pressure

Gas pressure results when gas molecules impact the inner walls of a sealed container. The **ideal gas law** relates the quantities of pressure *(P)*, volume *(V)*, temperature (1), and amount *(n)* of gas in a closed container:

$$PV = nRT$$

In this equation, *R* is a fundamental constant called the **gas constant.** It can have many different numerical values, depending on the units chosen for pressure, volume, temperature, and amount, just as a length has different numerical values, depending on whether feet or meters or miles is the unit being used. Scientists have defined an "ideal" gas as one where one mole [mol] of gas at a temperature of 273 kelvin [K] and a pressure of one atmosphere [atm] will occupy a volume of 22.4 liters [L]. Using these values to solve for the constant *R* yields

$$R = \frac{PV}{nT} = \frac{1\,[atm]22.4\,[L]}{1\,[mol]273\,[K]} = 0.08206\ \frac{atm\ L}{mol\ K}$$

*Note that we* must *use* absolute *temperature units in the ideal gas equation.* We cannot begin with relative temperature units and then convert the final answer. Also, all pressure readings must be in absolute, not gauge, units.

In previous chapters, we have suggested a procedure for solving problems involving equations and unit conversions. For ideal gas law problems, we suggest a slightly different procedure.

### 7.9.10 Ideal Gas Law Procedure

1. Examine the units given in the problem statement. Choose a gas constant *(R)* that contains as many of the units given in the problem as possible.
2. If necessary, convert all parameters into units found in the gas constant *(R)* that you choose.
3. Solve the ideal gas law for the variable of interest.
4. Substitute values and perform all necessary calculations.
5. If necessary, convert your final answer to the required units and apply reasonableness.

Ideal Gas Law: PV = nRT

Only absolute temperature units (K or $^{o}$R) can be used in the ideal gas equation.

R = 8314 $\frac{Pa\ L}{mol\ K}$

= 0.08206 $\frac{atm\ L}{mol\ K}$

### 7.9.11 Example

A container holds 1.43 moles of nitrogen (formula: $N_2$) at a pressure of 3.4 atmospheres and a temperature of *500* degrees Fahrenheit. What is the volume of the container in liters?

| Method | Steps |
|---|---|
| (1) Choose ideal gas constant | Given units: mol, atm, $^{o}$F, L<br>Select R: 0.08206 $\frac{atm\ L}{mol\ K}$ |
| (2) Convert to units of chosen *R* | 500°F = 533 K |
| (3) Solve for variable of interest | $V = \frac{nRT}{P}$ |
| (4) Calculate | $V = \left\langle \frac{1.43\ mol}{} \middle\vert \frac{0.08206\ atm\ L}{mol\ K} \middle\vert \frac{533\ K}{} \middle\vert \frac{}{3.4 atm} \right\rangle$ |
| (5) Cancel, be reasonable | V = 18.4 L |

### 7.9.12 Example

A container holds 1.25 moles of nitrogen (formula: $N_2$) at a pressure of 350 kilopascals and a temperature of 160 degrees Celsius. What is the volume of the container in liters?

| Method | Steps |
|---|---|
| (1) Choose ideal gas constant | Given units: mol, Pa, $^{o}$C, L<br>Select R: 8,314 $\frac{Pa\ L}{mol\ K}$ |
| (2) Convert to units of chosen *R* | 160°C = 433 K |
| (3) Solve for variable of interest | $V = \frac{nRT}{P}$ |
| (4) Calculate | $V = \left\langle \frac{1.25\ mol}{} \middle\vert \frac{8{,}314\ Pa\ L}{mol\ K} \middle\vert \frac{433\ K}{} \middle\vert \frac{}{350{,}000 Pa} \right\rangle$ |
| (5) Cancel, be reasonable | V = 13 L |

### 7.9.13 Example

A gas originally at a temperature of 300 kelvin and 3 atmospheres pressure in a 3.9-liter flask is cooled until the temperature reaches 284 kelvin. What is the new pressure of gas in atmospheres?

| Method | Steps |
|---|---|
| (1) Choose ideal gas constant | Given units: mol, K, L<br>Select R: 0.08206 $\frac{atm\ L}{mol\ K}$ |
| (2) Convert to units of chosen *R* | None needed |
| (3) Solve for variable of interest, eliminating any variables that remain constant between the initial and final state | $\frac{P_1V_1}{P_2V_2} = \frac{n_1RT_1}{n_2RT_2}$<br>$\frac{P_1}{P_2} = \frac{T_1}{T_2}$ |
| (4) Calculate | $\frac{3\ atm}{P_2} = \frac{300K}{284\ K}$ |
| (5) Cancel, be reasonable | $P$ = 2.8 atm |

## 7.10 Energy

**Energy** is an abstract quantity with several definitions, depending on the form of energy being discussed. You may be familiar with some of the following types of energy.

Types of Energy

- **Work** (W) is energy expended by exertion of a force *(F)* over a distance *(d)*. As an example, if you exert a force on (push) a heavy desk so that it slides across the floor, which will make you more tired: pushing it 5 feet or pushing it 50 feet? The farther you push it, the more work you do.
- **Potential energy** (PE) is a form of work done by moving a weight (w) – which is a force – a vertical distance *(H)*. Recall that weight is mass *(m)* times gravity *(g)*. Note that this is a special case of the work equation, where force is weight and distance is height.
- **Kinetic energy** (KE) is a form of energy possessed by an object in motion. If a constant force is exerted on a body, then by $F = ma$, we see that the body experiences a constant acceleration, meaning the velocity increases linearly with time. Since the velocity increases as long as the force is maintained, work is being done on the object. Another way of saying this is that the object upon which the force is applied acquires kinetic energy, also called **energy of translational motion.** For a nonrotating body moving with some velocity *(v)* the kinetic energy can be calculated by $KE_T = (1/2)mv^2$.

  This, however, is not the entire story. A rotating object has energy whether it is translating (moving along a path) or not. If you have ever turned a bicycle upside down, spun one of the wheels fairly fast, then tried to stop it with your hand, you understand that it has energy. This is **rotational kinetic energy,** and for an object spinning in place (but not going anywhere), it is calculated by $KE_T = (1/2)I\omega^2$.

The Greek letter omega *(ω)* symbolizes angular velocity or the object's rotational speed, typically given in units of radians per second. The moment of inertia *(l)* depends on the mass and the geometry of the spinning object. The table shown lists the moments of inertia for a few common objects.

- **Thermal Energy** or heat *(Q)* is energy associated with a change in temperature ($\Delta T$).

It is a function of the mass of the object *(m)* and the specific heat ($C_p$), which is a property of the material being heated:

$$Q = mC_p\Delta T$$

| | |
|---|---|
| Work | $W = FAX$ |
| Potential Energy | $PE = mg\Delta H$ |
| Kinetic Energy, translational | $KE_T = 1/2m(v_f^2 - v_i^2)$ |
| Kinetic Energy, rotational | $KE_R = 1/2I(\omega_f^2 - \omega_i^2)$ |
| Kinetic Energy, total | $K_E = KE_T + KE_R$ |
| Thermal Energy | $Q = mC_p\Delta T$ |

### 7.10.1 Calories and BTUs and Joules

The SI unit of work is **joule,** defined as one newton of force acting over a distance of one meter (Table 7-9). The unit is named after James Joule (1818-1889), an English physicist responsible for several theories involving energy, including the definition of the mechanical equivalent of heat and Joule's law, which describes the amount of electrical energy converted to heat by a resistor (an electrical component) when an electric current flows through it. In some mechanical systems, work is described in units of foot pound-force [ft $lb_f$],

For energy in the form of heat, units are typically reported as British thermal units and calories instead of joules. A **British thermal unit** [BTU] is the amount of heat required to raise the temperature of one pound-mass of water by one degree Fahrenheit. A **calorie** [cal] is amount of heat required to raise the temperature of one gram of water by one degree Celsius.

Table 7-9: Dimensions of energy

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Work | J | 1 | 2 | -2 | 0 |
| Thermal energy | BTU | 1 | 2 | -2 | 0 |
| | Cal | 1 | 2 | -2 | 0 |

### 7.10.2 Example

A 50-kilogram load is raised vertically a distance of 5 meters by an electric motor. How much work in units of joules was done on the load?

First, we must determine the type of energy. The parameters we are discussing include mass (kilograms) and height (meters). Examining the energy formulas given above, the equation for potential energy fits. Also, the words "load is raised vertically a distance" fits with our understanding of potential energy.

| Step One: Convert to Base SI Units |
|---|
| No conversion needed |
| **Step Two: Calculate** |

| Method | Steps |
|---|---|
| (1) Determine appropriate equation | PE = mgΔH |
| (2) Insert known quantities | $PE = \left\langle \frac{50\ kg}{} \middle| \frac{9.8\ m}{s^2} \middle| \frac{m}{} \right\rangle$ |
| (3) Calculate, be reasonable | $PE = 2{,}450\ \frac{kg\ m^2}{s^2}$ |

This is apparently our final answer, but the units are puzzling. If the units of energy are joules and if this is a valid equation, then our final result for energy should be joules. If we consider the dimensions of energy:

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Energy | J | 1 | 2 | -2 | 0 |

A unit of energy has dimensions E {=} $ML^2/T^2$, which in terms of base SI units would be E [=] kg $m^2$/ $S^2$. As this term occurs so frequently it is given the special name 'Joule." Anytime we see this term (kg $m^2$/ $S^2$), we know we are dealing with an energy, equal to a joule.

| | |
|---|---|
| (3) Calculate, be reasonable | $PE = \left( 2{,}450\ \frac{kgm^2}{s^2} \middle| \frac{1\ J}{1\ \frac{kgm^2}{s^2}} = 2.450\ J \right)$ |

| Step Three: Convert from Base SI Units to Desire Units |
|---|
| No conversion needed |

### 7.10.3 Example

In the morning, you like to drink your coffee at a temperature of exactly 70 degrees Celsius [ºC]. The mass of the coffee in your mug is 470 grams. To make your coffee, you had to raise the temperature of the water by 30 degrees Celsius. How much energy in units of British thermal units [BTU] did it take to heat your coffee? The specific heat of water is 4.18 joules per gram degree Celsius [J/ (g ºC)].

First, you must determine the type of energy we are using. The parameters discussed include mass, temperature, and specific heat. Examining the energy formulas given above, the equation for thermal energy fits. Also, the words "How much energy ... did it take to heat your coffee" fits with an understanding of thermal energy.

| Step One: Convert to Base SI Units |
|---|
| No conversion needed |

| Step Two: Calculate | |
|---|---|
| Method | Steps |

| | |
|---|---|
| (1) Determine appropriate equation | $Q = mC_p\Delta H$ |
| (2) Insert known quantities | $Q = \left(\frac{40\ g}{}\middle\vert\frac{4.18\ J}{g°C}\middle\vert\frac{30°C}{}\right)$ |
| (3) Calculate, be reasonable | $Q = 59{,}370\ J$ |
| **Step Three: Convert from Base SI Units to Desire Units** | |
| Method | Steps |
| (1) Term to be converted | $59{,}370\ J$ |
| (2) Conversion formula<br>(3) Make a fraction<br>(4) Multiply | $\left(\frac{59370\ J}{}\middle\vert\frac{9.8\ X\ 10^{-4}\ BTU}{1\ J}\right)$ |
| (5) Cancel, calculate, be reasonable | 56 BTU |

### 7.10.4 Power

**Power** is defined as energy per time (Table 8-11). The SI unit of power is **watt,** named after James Watt (1736-1819), a Scottish mathematician and engineer whose improvements to the steam engine were important to the Industrial Revolution. He is responsible for the definition of **horsepower** [hp], a unit of power originally used to quantify how the steam engine could replace the work done by a horse.

Table 7-10: Dimensions of power

| Quantity | Common Units | Exponents | | | |
|---|---|---|---|---|---|
| | | M | L | T | Θ |
| Power | W | 1 | 2 | -3 | 0 |

To help understand the relationship between energy and power imagine the following. Your 1,000 – kilogram car has run out of gas on a level road. There is a gas station not far ahead, so you decide to push the car to the gas station. Assume that you intend to accelerate the car up to a speed of one meter per second (about 2.2 miles per hour), and then continue pushing at that speed until you reach the station. Ask yourself the following questions:

- Can I accelerate the car to one meter per second in one minute?
- On the other hand, can I accelerate it to one meter per second in one second?

Most of you would probably answer "yes" to the first and "no" to the second, but why? Well, personal experience! But that is not really an explanation. Since the change in kinetic energy is the same in each case, to accelerate the car in one second, your body would have to generate energy at a rate 60 times greater than the rate required if you accelerated it in one minute. The key word is rate, or how much energy your body can produce per second. If you do the calculations, you will find that for the one – minute scenario, your body would have to produce about 1/90 horsepower, which seems quite reasonable. On the other hand, if you try to accomplish the same acceleration in one second, you would need to generate 2/3 horsepower. Are you two-thirds as powerful as a horse?

As another example, assume that you attend a class on the third floor of the engineering building. When you are on time, you take 2 minutes to climb to the third floor. On the other hand, when you are late for class, you run up the three flights in 30 seconds.

- In which case do you do the most work (expend the most energy)?
- In which case do you generate the most power?

### 7.10.5 Example

A 50-kilogram load is raised vertically a distance of 5 meters by an electric motor in 60 seconds. How much power in units of watts does the motor use, assuming no energy is lost in the process?

The energy used by the system was found to be 2,450 joules, the analysis of which is not repeated here.

| Step One: Convert to Base SI Units | |
|---|---|
| No conversion needed | |
| **Step Two: Calculate** | |
| **Method** | **Steps** |
| (1) Determine appropriate equation | $\text{Power} = \dfrac{energy}{Time}$ |
| (2) Insert known quantities | $Power = \left(\dfrac{2.450\ J}{} \middle\vert \dfrac{}{60\ s}\right)$ |
| (3) Calculate, be reasonable | $Power = \left(41\,\dfrac{J}{s} \middle\vert \dfrac{1\ W}{1\,\dfrac{J}{s}}\right) = 41\ W$ |
| **Step Three: Convert from Base SI Units to Desire Units** | |
| No conversion needed | |

Note that since power = energy/time, energy = power x time. We pay the electric company for energy calculated this way as kilowatt-hours. If power is constant, we can obtain the total energy involved simply by multiplying the power by the length of time that power is applied. If power is *not* constant, we would usually use calculus to determine the total energy, but that solution is beyond the scope of this book.

## 7.11 Efficiency

**Efficiency** (η, Greek letter eta) is a measure of how much of a quantity, typically energy or power, is lost in a process. In a perfect world, efficiency would always be 100%. All energy put into a process would be recovered and used to accomplish the desired task. We know that this can never happen, so *efficiency is always less than 100%*. If a machine operates at 75% efficiency, 25% of the energy is lost. This means you have to put in "extra" energy to complete the work.

The use of the terms "input" and "output" require some explanation. The **input** is the quantity of energy or power or whatever required by the mechanism from some source to operate and accomplish its task. The **output** is the amount of energy or power or whatever is actually applied to the task itself by the mechanism. Note that the rated power of a device, whether a light bulb, a motor, or an electric heater, refers to the input power – the power needed to operate the device – not the output power. In an ideal, 100% efficient system, the input and output would be equivalent. In an inefficient system (the real world), the input is equivalent to the sum of the output and the power or energy lost. This is perhaps best explained by way of examples.

Efficiency ($\eta$) = output/input

Efficiency ($\eta$) = output/ (output + loss)

Input = quantity required by mechanism to operate

Output = quantity actually applied to task

Loss = quantity wasted during the application

Efficiency is always less than 100%.

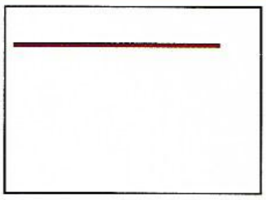
# 8.GRAPH INTEPRETATION

Technical data are presented properly in a graph. Engineers very frequently deal with the graphs and they must be able to interpret the graph, obtain useful results from them. This chapter gives some tips about it.

## 8.1 Straight Lines

### 8.1.1 Horizontal Line

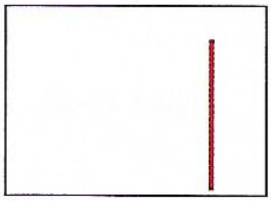
A horizontal line in a graph shows the following:



a) Dependent variable is not changing.
b) Derivative of the dependent variable is zero.
c) Integral of the dependent variable is changing at a constant rate.

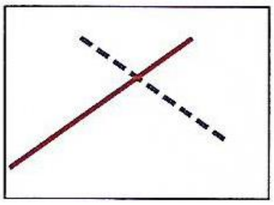
### 8.1.2 Vertical Line

A horizontal line in a graph shows the following:



a) Dependent variable changes instantaneously.
b) Derivative of the dependent variable is infinity.
c) Integral of the dependent variable is zero.

### 8.1.3 Other Straight Lines



A horizontal line in a graph shows the following:

a) Dependent variable changes at a constant rate.
b) Derivative of the dependent variable is a constant value.
c) (may be positive or negative)
d) Integral of the dependent variable is changing not at a constant rate but at a variable rate.

## 8.2 Examples of Straight Line Graph Interpretation

### 8.2.1 Example 1 of graph interpretation

In Figure 8-1, the voltage is constant, as indicated by the horizontal line at 23 volts, from time = 0 to 8 seconds. At time = 8 seconds, the voltage changes instantly to 15 volts, as indicated by the vertical line. Between time = 8 seconds and 20 seconds, the voltage decreases at a constant rate, as indicated by the straight line, and reaches 0 volts at time = 20 seconds, where it remains constant.

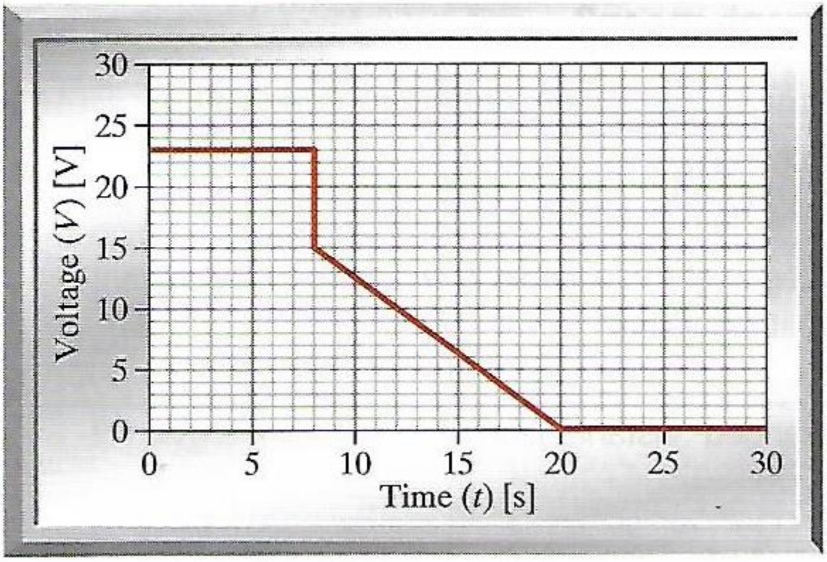



Figure 8-1: Example 1 of graph interpretation – Voltage Drop

### 8.2.2 Example 2 of graph interpretation

In Figure 8-2, the force on the spring increases at an increasing rate from time = 0 until time = 2 minutes, then remains constant for 1 minute, after which it increases at a decreasing rate until time = 5 minutes, after which it remains constant at about 6.8 newtons.

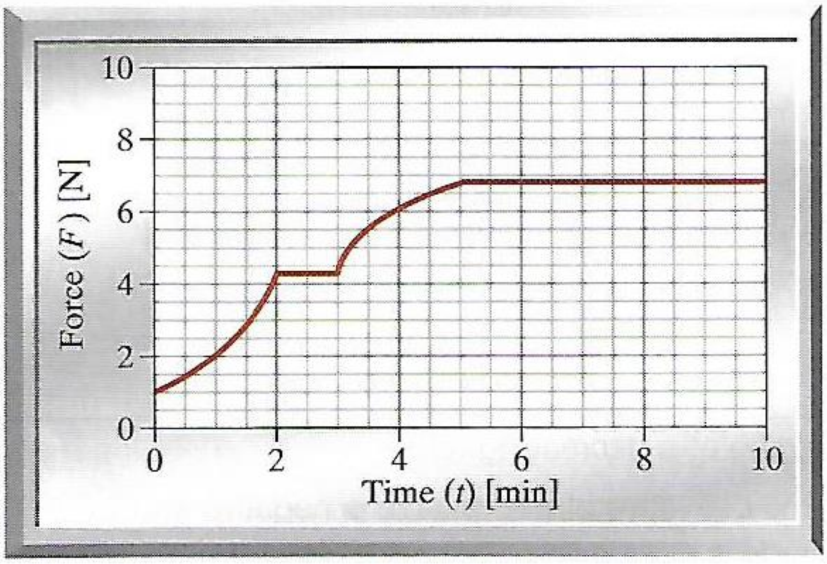



Figure 8-2: Example 2 of graph interpretation – Spring Force

### 8.2.3 Example 3 of graph interpretation

The height of a blimp is shown in Figure 8-3. The height decreases at an increasing rate for 5 minutes, then remains constant for 2 minutes, after which its height decreases at a decreasing rate until $t$ = 10 minutes, after which its height remains constant at 10 meters.

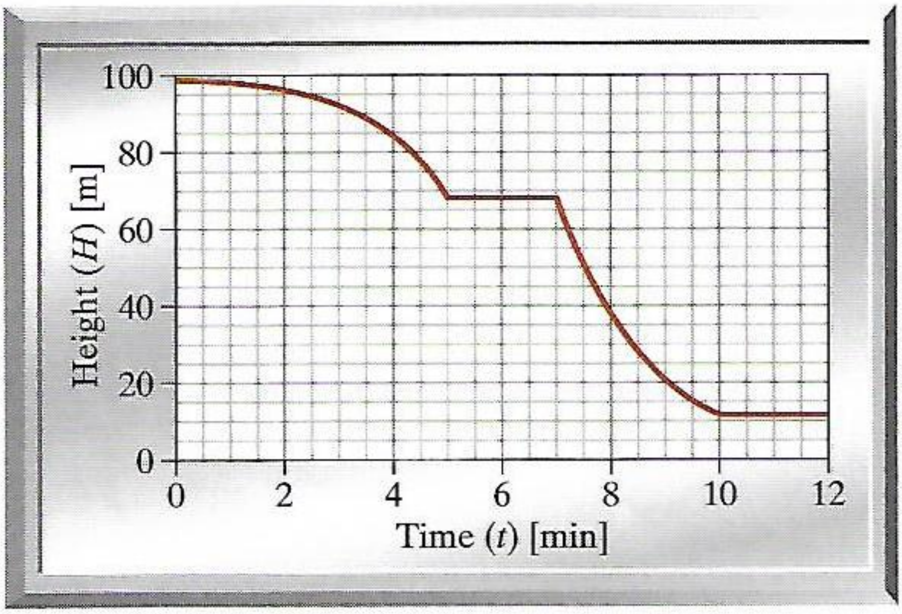



Figure 8-3: Example 3 of graph interpretation – Blimp Height

### 8.2.4 Example 4 of graph interpretation

Use the graph of Figure 8-4 to answer the following questions. Choose from the following answers:

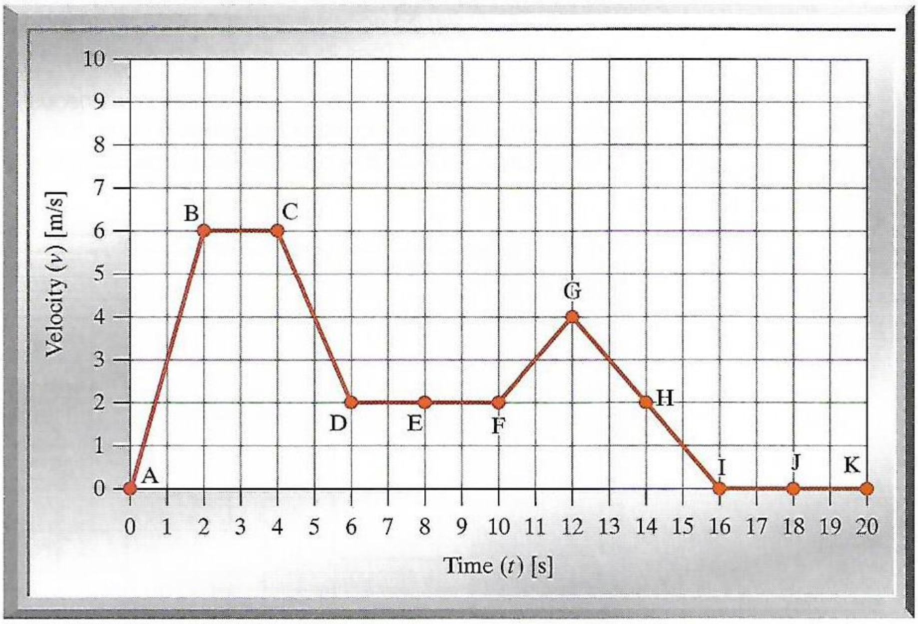



Figure 8-4: Example 4 of graph interpretation – Spring Force

- Equal to a constant, positive value
- Equal to zero
- Equal to a constant, negative value
- Decreasing at a constant rate

- Increasing at a constant rate
- Increasing at a decreasing rate
- Increasing at an increasing rate
- Decreasing at a decreasing rate
- Decreasing at an increasing rate

**a)** Between points (A) and (B), the acceleration is _
**b)** Between points (B) and (C), the acceleration is_
**c)** Between points (C) and (D), the acceleration is_
**d)** Between points (D) and (E), the distance is_
**e)** Between points (F) and (G), the distance is_
**f)** Between points (G) and (H), the distance is_

### 8.2.5 Example 5 of graph interpretation

As a club effort to raise money for charity, we are going to push a university fire truck along a flat section of highway. The fire truck weighs 29,400 newtons. Using teams, we will exert a force of 10 newtons for the first 2 minutes, 20 newtons for the next 3 minutes, and 5 newtons for the next 4 minutes. We notice a billboard nearby that reads, "Newton says that 'FORCE = MASS X ACCELERATION'!"

Graph the acceleration and speed versus time on separate graphs, using the same timescale for each. In your analysis, ignore the effect of friction and assume the acceleration changes instantaneously.

**Step 1:** Calculate acceleration based on Newton's law. Plot acceleration versus time; the results are shown in Figure 8-5. The details of this calculation are left for the reader, but as a hint, for the first 2 minutes, a force of 10 newtons is applied to the 3,000 kilogram mass truck, giving an acceleration of O. 0033 meters per second squared. Note that in the graph, the horizontal lines indicate that acceleration is constant.

The vertical lines imply that the acceleration instantaneously increased in value. While this is not exactly accurate, no information was given on how the acceleration changes between values. In such cases, an instantaneous change is often assumed.

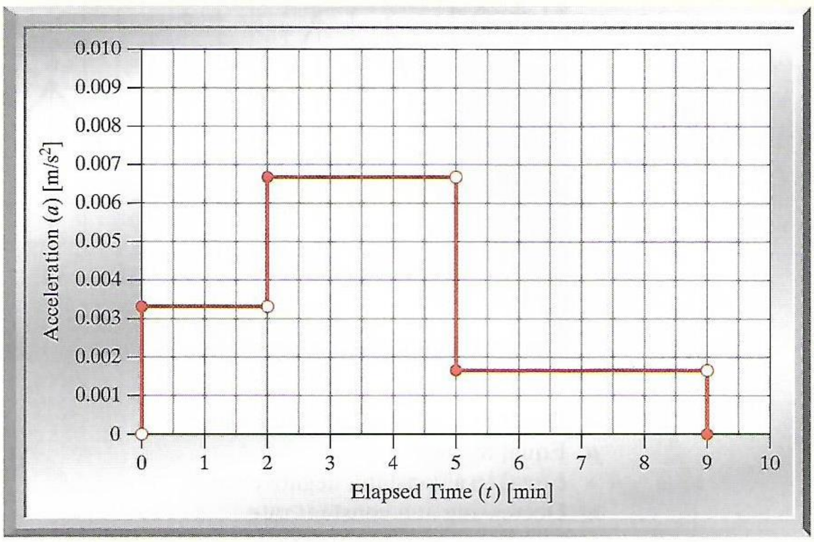



Figure 8-5: Example 5 of graph interpretation – Vehicle Acceleration

**Step 2:** Calculate speed based on the area under the curve generated in Step 1. Plot speed versus time. The details are left to the reader. Results are shown in Figure 8-6.

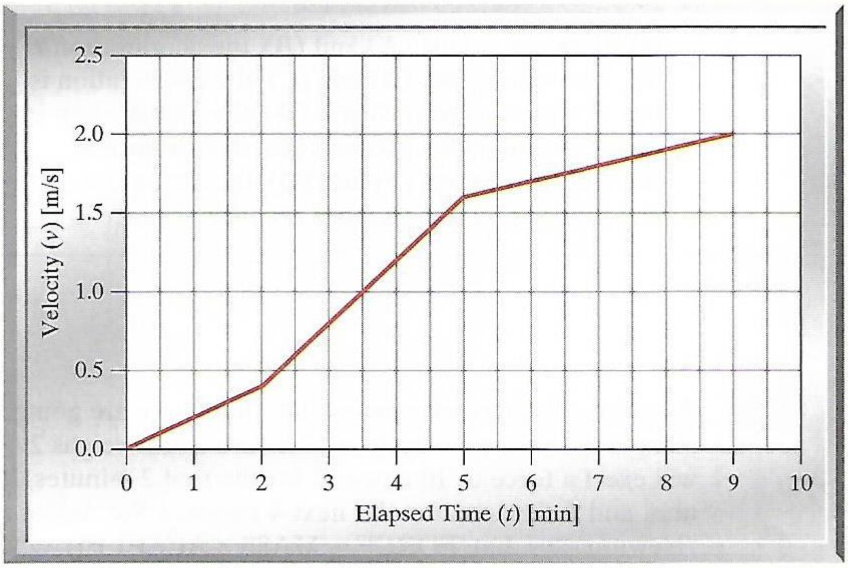
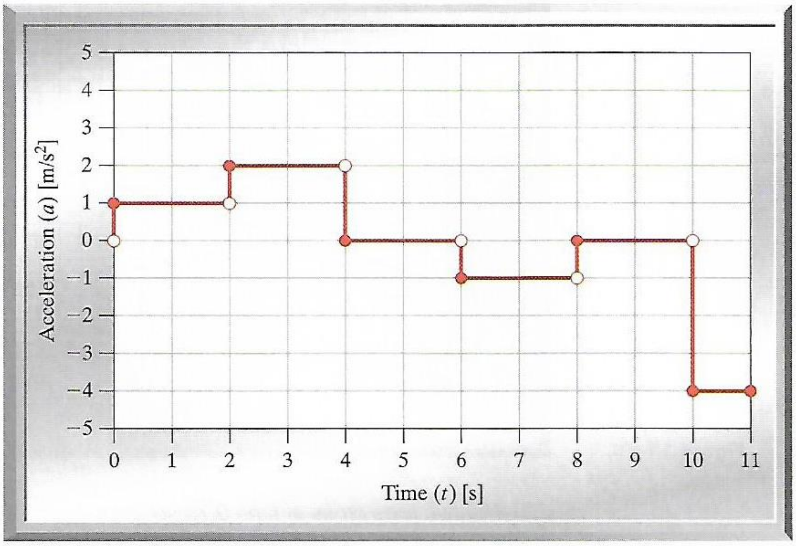



Figure 8-6: Example 5 velocity from area under the acceleration curve.

The graph gives an acceleration profile for a car that begins to move from an initial speed of zero. Draw the corresponding velocity and the corresponding distance profile.




## 8.3 Graphical Solutions

When you have two equations containing the same two variables, it is sometimes desirable to find values of the variables that satisfy both equations. Most of you have studied methods for solving simultaneous linear equations (there are many techniques); however, most of these methods apply only to linear equations and do not work if one or both of the equations is nonlinear. It also becomes problematic if you are working with experimental data.

For systems of two equations (or data sets in two variables), you can use a graphical method to determine the value or values that satisfy both. Essentially, graph the two equations and visually determine where the curves intersect. This may be nowhere, at one point, or at several points.

### 8.3.1 Example 11-8

We assume that the current through two electromagnets is given by the following equations

Electromagnet A: $I = 5t + 6$

Electromagnet B: $I = -3t + 12$

We want to determine when the value of the current through the electromagnets is equal

Graphing both equations gives Figure 8-7. We know not to show the points when plotting data derived from equations.

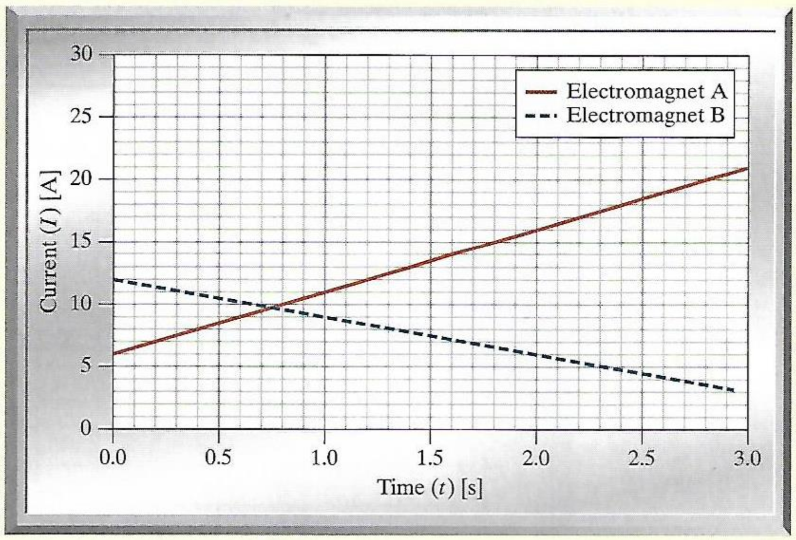



Figure 8-7: Example 11-8 - Current through two electromagnets

The two lines cross at time 0.75 seconds (approximately), and the current at this time is approximately 9.7 amperes. The larger we make this graph and the more gridlines we include, the more accurately we can determine the solution.

Solution: t = 0.75 seconds, 1= 9.7 amperes.

### 8.3.2 Economic Analysis

**Breakeven analysis** determines the quantity of product a company must make before they begin to earn a profit. Two types of costs are associated with manufacturing: fixed and variable. **Fixed costs** include equipment purchases, nonhourly employee salaries, insurance, mortgage or rent on the building, etc., or "money we must spend just to open the doors." **Variable costs** depend on the production volume, such as material costs, hourly employee salaries, and utility costs. The more product produced, the higher the variable costs become.

Total cost = Fixed cost + Variable cost * Amount produced

The product is sold at a **selling price**, creating **revenue**.

Revenue = Selling price * Amount sold

Any excess revenue remaining after all production costs have been paid is **profit**. Until the company reaches the breakeven point, they are operating at a **loss** (negative profit), where the money they are bringing in from sales does not cover their expenses.

Profit = Revenue - Total Cost

The **breakeven point** occurs when the revenue and total cost lines cross, or the point where profit is zero (not negative or positive). These concepts are perhaps best illustrated through an example.

### 8.3.3 Example 11-9

Let the amount of product we produce be G [gallons per year]. Consider the following costs:

- Fixed cost: $1 million
- Variable cost: 10 cents/gallon of G
- Selling price: 25 cents/gallon of G

Plot the total cost and the revenue versus the quantity produced. Determine the amount of G that must be produced to breakeven. Assume we sell everything we make.

The plot of these two functions is shown in Figure 8-8. The breakeven point occurs when the two graphs cross, at a production capacity of 6.7 million gallons of G.

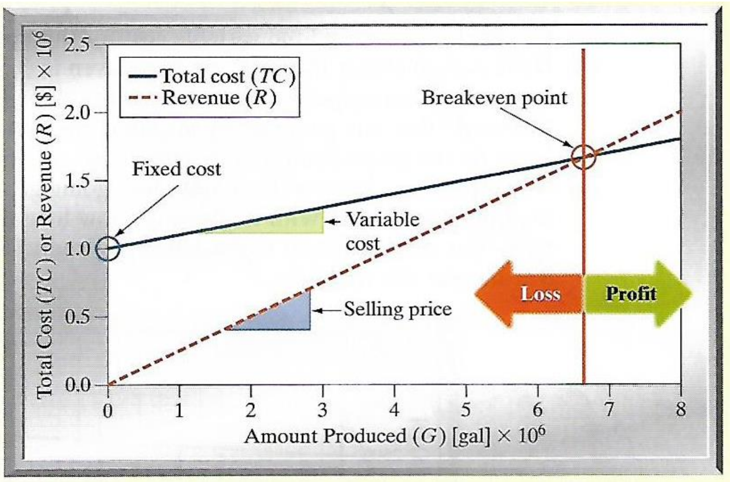
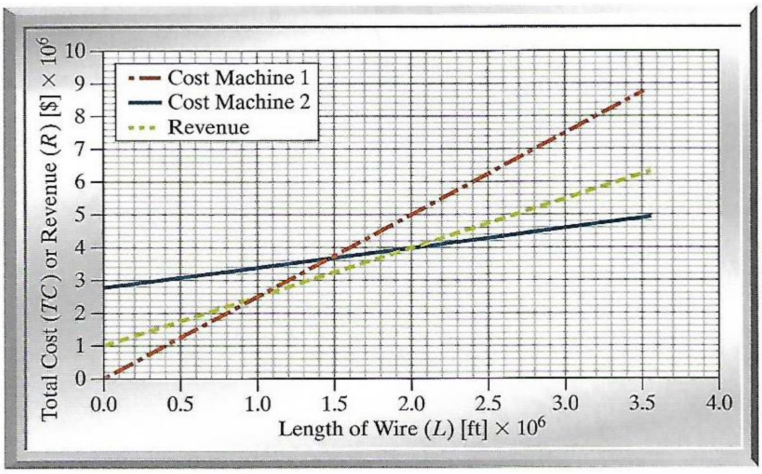



Figure 8-8: Breakeven analysis definitions

### 8.3.4 Example

You are working for a tire manufacturer, producing wire to be used in the tire as a strengthening agent. You are considering implementing a new machining system, and you must present a breakeven analysis to your boss. You develop the graph, showing two possible machines that you can buy.



# 9.MODELING

A **model** is an abstract description of the relationship between variables in a system. A model allows the categorization of different types of mathematical phenomena so that general observations about the variables can be made for use in any number of applications.

For example, if we know that $t = v + 5$ and $M = z + 5$, any observations we make about $v$ with respect to $t$ also apply to $z$ with respect to $M$. A specific model describes a *system* or *function* that has the same *trend* or *behavior* as a generalized model. In engineering, many specific models within different subdisciplines behave according to the same generalized model.

This section covers three general models of importance to engineers: **linear, power,** and **exponential. It** is worth noting that many applications of models within these three categories contain identical math but apply to significantly different disciplines.

Linear models occur when the dependent variable changes in direct relationship to changes in the independent variable. We discuss such systems, including springs, resistive circuits, fluid flow, and elastic materials, in this chapter by relating each model to Newton's generalized law of motion.

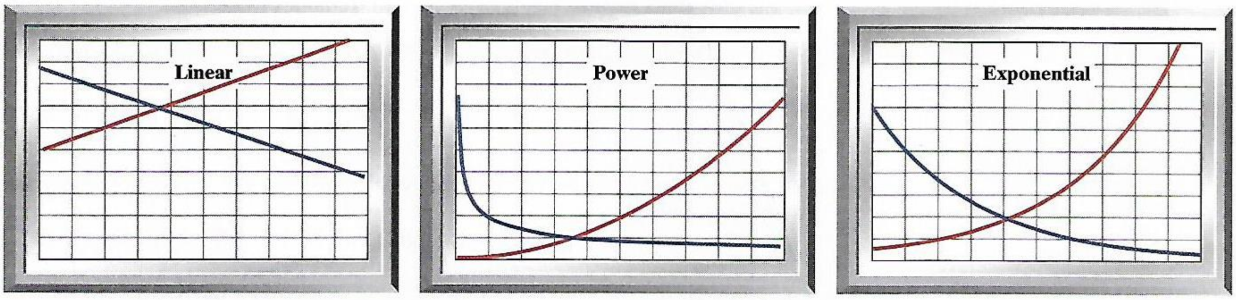
Power law systems occur when the independent variable has an exponent not equal to 1 or 0. We discuss these models by addressing integer and rational real exponents.

Exponential models are used in all engineering disciplines in a variety of applications.

As we have already seen, a large number of phenomena in the physical world obey one of the three basic mathematical models.

- Linear: $y = mx + b$
- Power: $y = bx^m + \mathrm{c}$
- Exponential: $y = be^{mx} + \mathrm{c}$

Here, we consider how to determine the best model type for a specific data set, as well as learning methods of dealing with data that fit a power or exponential model best but have a nonzero value of c.




Except as otherwise noted, the entire discussion in this chapter assumes that the data fits one of the three trendlines models: linear, power, or exponential. You should always keep this in mind when using the techniques discussed here.

## 9.1 Selecting A Trendline Type

When you determine a trendline to fit a set of data, in general you want the line, which may be straight or curved, to be as close as is reasonable to most of the data points.

The objective is not to ensure that the curve passes through every point.

To determine an appropriate model for a given situation, we use five guidelines, presented in general order of importance:

1. Do we already know the model type that the data will fit?
2. What do we know about the behavior of the process under consideration, including initial and final conditions?
3. What do the data look like when plotted on graphs with logarithmic scales?
4. How well does the model fit the data?
5. Can we consider other model types?

### 9.1.1 Guideline 1: Determine if the Model Type Is Known

If you are investigating a phenomenon that has already been studied by others, you may already know which model is correct or perhaps you can learn how the system behaves by looking in appropriate technical literature. In this case, all you need are the specific values for the model parameters since you already know the form of the equation. As we have seen, Excel is quite adept at churning out the numerical values for trendline equations.

If you are certain you know the proper model type, you can probably skip guidelines 2 and 3, although it might be a good idea to quantify how well the model fits the data as discussed in guideline 4. For example, at this point you should know that the extension of simple springs has a linear relationship to the force applied. As another example, from your study of the ideal gas law, you should know that pressure is related to volume by a power law model (exponent = -1).

At other times, you may be investigating situations for which the correct model type is unknown. If you cannot determine the model type from experience or references, continue to Guideline 2.

### 9.1.2 Guideline 2: Evaluate What Is Known About the System Behavior

The most important thing to consider when selecting a model type is whether the model makes sense in light of your understanding of the physical system being investigated. Since there may still be innumerable things with which you are unfamiliar, this may seem like an unreasonable expectation. However, by applying what you *do* know to the problem at hand, you can often make an appropriate choice without difficulty.

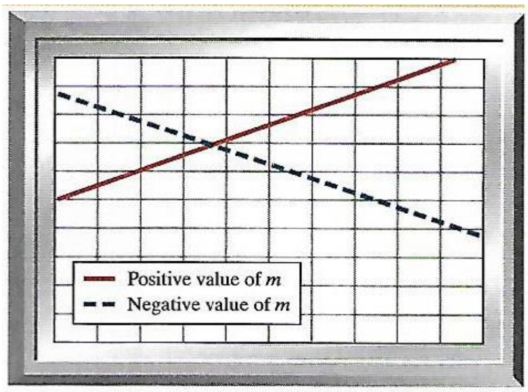
When investigating an unknown phenomenon, we typically know the answer to at least one of three questions:

1. How does the process behave in the initial state?

2. How does the process behave in the final state?

3. What happens to the process between the initial and the final states-if we sketch the process, what does it look like? Does the parameter of interest increase or decrease? Is the parameter asymptotic to some value horizontally or vertically?

## 9.2 Linear Functions

| Trend | Equation | Data Form | Graphical Example |
|---|---|---|---|
| Linear | $y = mx + c$ | Defined value *(e)* *at x = 0*<br>Data appears as a linear (straight) line |  |

One of the most common models is **linear**, taking the form $y = mx + c$, where the ordinate value *(y)* is a function of the abscissa value *(x)* and a constant factor called the **slope** *(m)*. At an initial value of the abscissa ($x = 0$), the ordinate value is equal to the **intercept** *(c)*. Examples include

- Distance *(d)* traveled at constant velocity *(v)* over time *(t)* from initial position *($d_0$)*:
  d = vt + do
- Rate of rotation *(ω)* as a function of time *(t)* and angular acceleration *(α)* from initial rotational rate *($\omega_i$)*:

$\omega = \alpha t + \omega_i$

- Total pressure ($P_{total}$), relating density *(p)*, gravity *(g)*, liquid height *HI)*, and the pressure above the surface ($P_{surface}$):

$P_{total} = pgH + P_{surface}$

• Newton's second law, relating force *(E)*, mass *(m)*, and acceleration *(a)*:

F = ma

Note that the intercept value (c) is zero in the last example.

### 9.2.1 General Model Rules

Given a linear system of the form $y = mx + c$ and assuming $x \geq 0$:

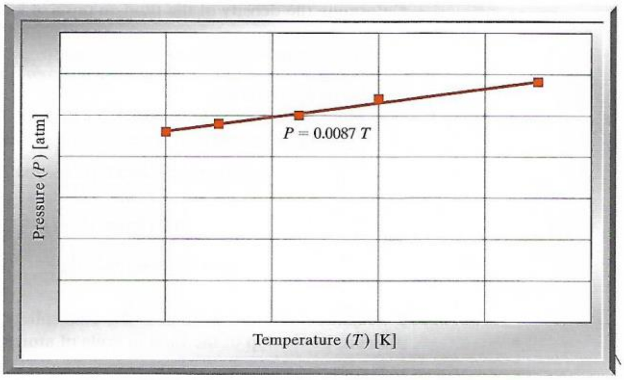
- When $m = 1$, the function is equal to $x + c$.
- When $m = 0$, $y = c$, regardless of the value of *x (y* never changes).
- When $m > 0$, as *x* increases, *y* increases, regardless of the value of c.
- When $m < 0$, as *x* increases, *y* decreases, regardless of the value of c.

### 9.2.2 Exercise

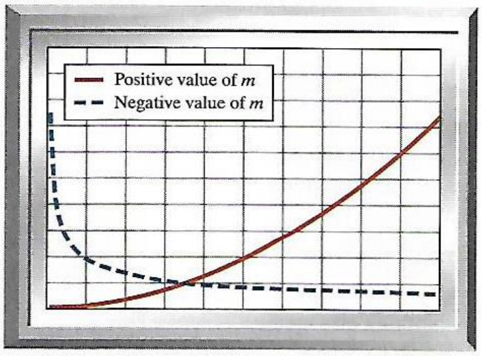
The graph shows the ideal gas law relationship *(PV = nRT)* between pressure *(P)* and temperature *(T)*.

**a)** What are the units of the slope (0.0087)?
**b)** If the tank has a volume of 12 liters and is filled with nitrogen (formula, $N_2$; molecular weight, 28 grams per mole), what is the amount of gas in the tank *(n)* in units of grams?

**c)** If the tank is filled with 48 grams of oxygen (formula, $O_2$; molecular weight, 32 grams per mole), what is the volume of the tank (V) in units of liters?




## 9.3 Power Functions

| Trend | Equation | Data Form | Graphical Example |
|---|---|---|---|
| Power | $Y = bx^m$ | Positive m<br>Value of zero at $x = 0$<br><br>Negative m<br>Value of infinity at $x = 0$ |  |

Power models take the form $y = bx^m$, Examples include

- Many geometric formulae involving areas, volumes, etc., such as the volume of a sphere *(V)* as a function of radius *(r):*

V = 4 / 3πr^3

- Distance *(d)* traveled by a body undergoing constant acceleration *(a)* over time *(t),* starting from rest:

$d = at^2$

- Energy calculations in a variety of contexts, both mechanical and electrical, such as the kinetic energy *(KE)* of an object as a function of the object's velocity *(v),* where the constant *(k)* depends upon the object shape and type of motion:

$KE = kmv^2$

- Ideal gas law relationships, such as Boyle's law, relating volume *(V)* and pressure *(P)* of an ideal gas, holding temperature (1) and quantity of gas *(n)* constant:

$V = (nRT)P^{-1}$

### 9.3.1 General Model Rules

Given a power system of the form $y = bx^m + c$, assuming $x \geq 0$:

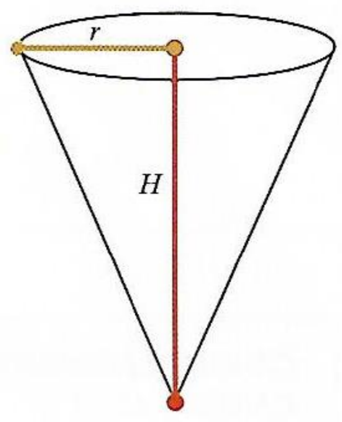
- When $m = 1$, the model is a linear function.

- When $m = 0$, $y = b + c$, regardless of the value of $x$.
- When $m$ is rational, the function will contain a rational exponent or may be described with a radical symbol ($\sqrt{\ }$). Certain rational exponents have special names (1/2 is "square root," 1/3 is "cube root").
- When $m$ is an integer, the function will contain an integer exponent on the independent variable. Certain exponents have special names (2 is "squared," 3 is "cubed").
- When $0 < |m| < 1$ and $x < 0$, the function may contain complex values. In this chapter, we will only consider power law models, where c is zero. In the next chapter we will discuss ways of dealing with data when the value of c is non-zero.

### 9.3.2 Example 11-5

The volume (V) of a cone is calculated in terms of the radius *(r)* and height *(H)* of the cone. The relationship is described by the following equation:

$$V = \frac{\pi r^2 H}{3}$$




Given a height of 10 centimeters, calculate the volume of the cone when the radius is 3 centimeters.

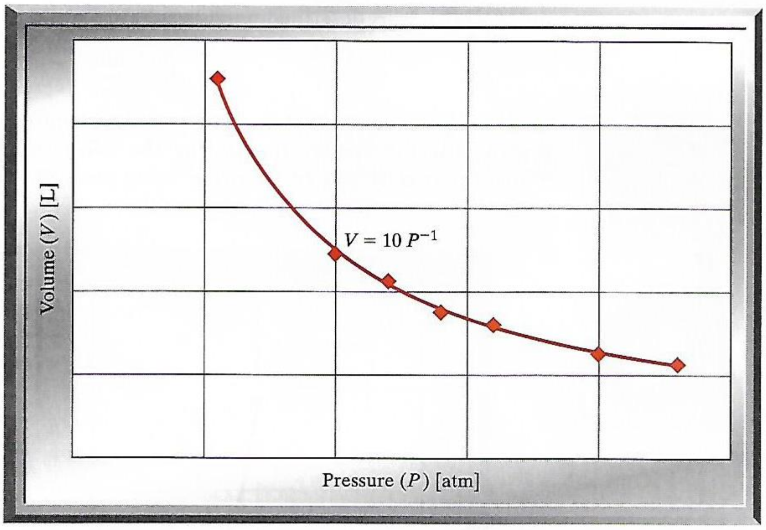
$$V = \frac{\pi (3cm)^2 (10\ cm)}{3} \approx 94.2\ cm^3$$

What is the volume of the cone when the radius is 8 centimeters?

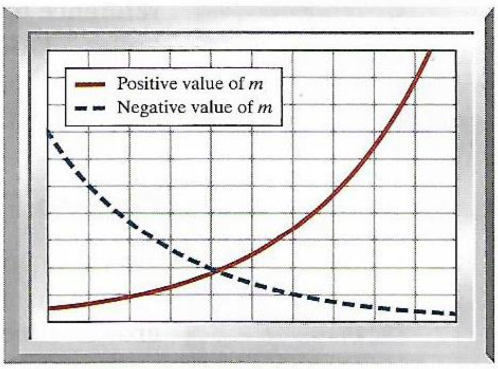
$$V = \frac{\pi (8\ cm)^2 (10\ cm)}{3} \approx 670\ cm^3$$

### 9.3.3 Comprehension Check 11-7

The graph shows the ideal gas law relationship *(PV= nRT)* between pressure *(P)* and volume *(V)*. If the tank is at a temperature of 300 kelvin and is filled with nitrogen (formula, $N_2$; molecular weight, 28 grams per mole), what is the amount of gas in the tank *(n)* in units of grams?




## 9.4 Exponential Functions

| Trend | Equation | Data Form | Graphical Example |
|---|---|---|---|
| Exponential | $Y = be^{mx}$ | Defined value b ($b \neq 0$) at $x = 0$<br>Positive m: asymptotic to 0 for large negative values of *x*<br>Negative m: asymptotic to 0 at large positive values of *x* |  |

Exponential models take the form $y = be^{mx} + c$. Examples include

- The voltage *(V)* across a capacitor (C) as a function of time *(t)*, with initial voltage $(V_0)$ discharging its stored charge through resistance *(R):*

$$V = V_0 e^{-t/(RC)}$$

- The number *(N)* of people infected with a virus such as smallpox or HINI flu as a function of time *(t)*, given the following: an initial number of infected individuals $(N_0)$, no artificial immunization available and dependence on contact conditions between species (C):

$$N = N_0 e^{Ct}$$

- The transmissivity *(T)* of light through a gas as a function of path length *(L)*, given an absorption cross-section *(s)* and density of absorbers *(N):*

$$T = e^{-sNL}$$

- The growth of bacteria (C) as a function of time *(t)*, given an initial concentration of bacteria $(C_0)$ and depending on growth conditions *(g):*

$$C = C_0 e^{gt}$$

Note that all exponents must be dimensionless, and thus unitless. For example, in the first equation, the quantity RC must have units of time.

Note that the intercept value (c) is zero in all of the above examples.

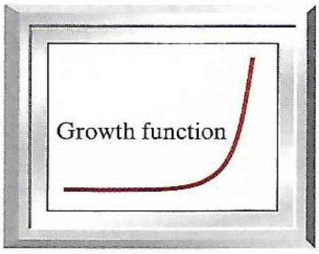
### 9.4.1 General Model Rules

Given an exponential system of the form $y = be^{mx} + c$:

- When $m = 0$, $y = b + c$ regardless of the value of $x$.
- When $m > 0$, the model is a **growth function**. The minimum value of the growth model for $x \geq 0$ is $b + c$. As $x$ approaches infinity, $y$ approaches infinity.
- When $m < 0$, the model is a **decay function**. The value of the decay model approaches c as $x$ approaches infinity. When $x = 0$, $y = b + c$.

### 9.4.2 What is "e"?

The **exponential constant** "e" is a transcendental number, thus also an irrational number, that can be rounded to 2.71828. It is defined as the base of the natural logarithm function. Sometimes, e is referred to as **Euler's number** or the **Napier constant.** The reference to Euler comes from the Swiss mathematician Leonhard Euler (pronounced "oiler," 1707-1783), who made vast contributions to calculus, including the notation and terminology used today. John Napier (1550-1617) was a Scottish mathematician credited with inventing logarithms and popularizing the use of the decimal point.




### 9.4.3 Growth Functions

An exponential *growth function* is a type of function that increases without bound with respect to an independent variable. For a system to be considered an exponential growth function, the exponential growth model ($y = be^{mx} + c$) requires that $m$ be greater than zero.

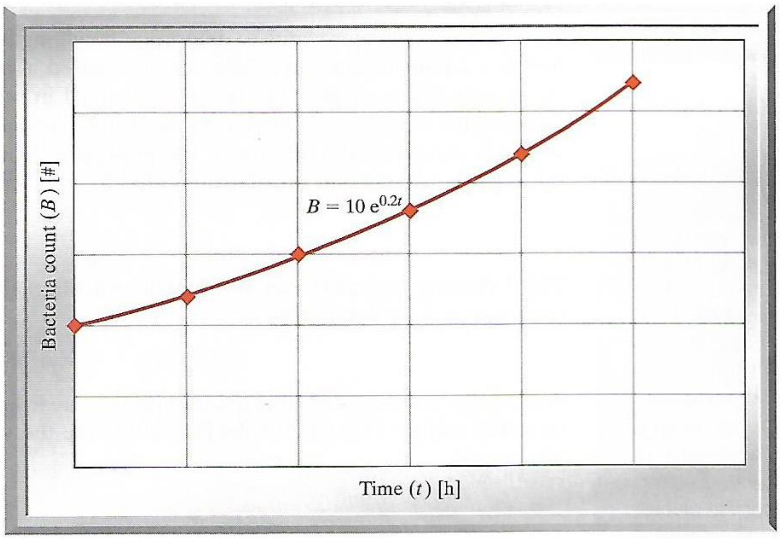
A more general exponential growth function can be formed by replacing the Napier constant with an arbitrary constant, or $y = ba^{mx} + c$. In the general growth function, $a$ must be greater than 1 for the system to be a growth function. The value of $a$ is referred to as the *base*, $m$ is the *growth rate*, $b$ is the *initial value*, and c is a *vertical shift.* Note that when $a = 1$ or $m = 0$, the system is reduced to $y = b + c$, which is a constant.

NOTE

An irrational number is a real number that cannot be expressed as the ratio of two integers. Pi $(\pi)$ is an example.

### 9.4.4 Example 11-8

An environmental engineer has obtained a bacteria culture from a municipal water sample and allowed the bacteria to grow. After several hours of data collection, the following graph is created.




What was the initial concentration of bacteria?

In theory: $B = B_0e^{gt}$ and from graph: $B = 10e^{0.2t}$

By comparison: $B_0 = 10$ bacteria

What was the growth constant *(g)* of this bacteria strain?

In theory: $B = B_{0\tau}\, e^{gt}$ and from graph: $B = 10e^{0.2t}$

By comparison: g = 0.2 per hour. Recall that exponents must be unit Jess, so the quantity of (g t) must be a unitless group. To be unitless, g must have units of inverse time.

The engineer wants to know how long it will take for the bacteria culture population to grow to 30,000.

To calculate the amount of time, plug in 30,000 for B and solve for t:

$30{,}000 = 10e^{0.2t}$

$3{,}000 = e^{0.2t}$

$In\ (3{,}000) = In\ (e^{0.2t}) = 0.2t$

$$t = \frac{In\ (3{,}000)}{0.2[\frac{1}{2}]} = 40\ h$$

# 10. APPENDIX

## 10.1 Fundamental Dimensions and Units

There are three primary unit systems in use today:

1. **SI (**SI units, from Le Systeme International d'Unites, also called **metric units**)
2. ***USCS***: US Customary Units
3. British Gravitational System of Units (BG)
4. English Engineering System of Units (commonly called English units)
5. ***AES***: American Engineering System

Table 10.1: Fundamental dimensions and units

| | **Primary Dimension** | **Symbol** | **SI unit (MKS)** | **BG unit (USCS)** | **English unit(AES)** |
|---|---|---|---|---|---|
| 1 | Mass | M | kg (kilogram) | slug | $lb_m$ (pound-mass) |
| 2 | Length | L | m (meter) | ft (foot) | ft (foot) |
| 3 | Time | T | s (second) | s (second) | s (second) |
| 4 | Temperature | θ | K (Kelvin) | $^oR$ (degree Rankine) | R (Rankine) |
| 5 | Electric current | I | A (ampere) | A (ampere) | A (ampere) |
| 6 | Light intensity | J | c (candela) | c (candela) | c (candela) |
| 7 | Amount of matter | N | mol (mole) | mol (mole) | mol (mole) |

## 10.2 SI Prefixes

Table 10-2: SI prefixes

| Numbers Less than One | | | Numbers Greater than One | | |
|---|---|---|---|---|---|
| Power of 10 | Prefix | Abbreviation | Power of 10 | Prefix | Abbreviation |
| $10^{-1}$ | deci- | d | $10^{1}$ | deca- | da |
| $10^{-2}$ | centi- | c | $10^{2}$ | hecto- | h |
| $10^{-3}$ | milii- | m | $10^{3}$ | kilo- | k |
| $10^{-6}$ | micro- | π | $10^{6}$ | Mega- | M |
| $10^{-9}$ | nano- | n | $10^{9}$ | Giga- | G |
| $10^{-12}$ | pico- | p | $10^{12}$ | Tera- | T |
| $10^{-15}$ | femto- | f | $10^{15}$ | Peta- | P |
| $10^{-18}$ | atto- | a | $10^{18}$ | Exa- | E |
| $10^{-21}$ | zepto- | z | $10^{21}$ | Zetta- | Z |
| $10^{-24}$ | yocto- | y | $10^{24}$ | Yotta- | Y |

## 10.3 Common derived units in the SI system

Table 10-3: Common derived units in the SI system

| Dimension | SI Unit | Base SI Units | Derived from |
|---|---|---|---|
| Force ($F$) | newton[N] | $1\ \mathrm{N} = 1\frac{kg\ m}{s^2}$ | F = ma<br>Force = mass times acceleration |
| Energy ($E$) | joule [J] | $1\ \mathrm{J} = 1\ \mathrm{N\ m} = 1\ \frac{kg\ m^2}{s^2}$ | E = Fd<br>Energy = force times distance |
| Power ($P$) | watt [W] | $1\ \mathrm{W} = 1\ \frac{J}{s} = 1\ \frac{kg\ m^2}{s^3}$ | P = E/t<br>Power = energy per time |
| Pressure ($P$) | pascal [Pa] | $1\ \mathrm{Pa} = \mathrm{a}\ \frac{N}{m^2} = 1\ \frac{kg}{m\ s^2}$ | P = F/A<br>Pressure = force per area |
| Voltage ($V$) | volt [V] | $1\ \mathrm{V} = \mathrm{a}\ \frac{W}{A} =1\ \frac{kg\ m^2}{s^2\ A}$ | V = P/I<br>Voltage = power per current |

## 10.4 Notations

**Scientific notation** is typically expressed in the form #.### x $10^N$, where the digit to the left of the decimal point is the most significant nonzero digit of the value being represented. Sometimes, the digit to the right of the decimal point is the most significant digit instead. The number of decimal places can vary, but is usually two to four. $N$ is an integer, and multiplying by $10^N$ serves to locate the true position of the decimal point.

**Engineering notation** is expressed in the form ###.### x $10^M$, where $M$ is an integer multiple of 3, and the number of digits to the left of the decimal point is 1, 2, or 3 as needed to yield a power of 10 that is indeed a multiple of 3. The number of digits to the right of the decimal point is typically between two and four.

Table 10-4: Common derived units in the SI system

| Standard | Scientific | Engineering |
|---:|:---:|:---:|
| 43,480,000 | $4.348 \times 10^{7}$ | $43.48 \times 10^{6}$ |
| 0.0000003060 | $3.060 \times 10^{-7}$ | $306.0 \times 10^{-9}$ |
| 9,860,000,000 | $9.86 \times 10^{9}$ | $9.86 \times 10^{9}$ |
| 0.0351 | $3.51 \times 10^{-2}$ | $35.1 \times 10^{-3}$ |
| 0.0000000522 | $5.22 \times 10^{-14}$ | $52.2 \times 10^{-15}$ |
| 456200 | $4.562 \times 10^{8}$ | $456.2 \times 10^{6}$ |

## 10.5 Numbers to Remember

1 quart ≈ 1 liter

1 cubic meter ≈ 250 gallons

1 cup ≈ 250 milliliters

1 m = 3.28 ft

1 km = 0.621 mi 1 in = 2.54 cm

1 mi = 5,280 ft

1 yd = 3 ft

1 cubic foot ≈ 7.5 gallons

1 cubic meter ≈ 5, 55-gallon drums

1 golf ball ≈ 1 cubic inch

## 10.6 Common Conversions

**Angle**

| | |
|---|---|
| 1 rad | = 57.3 deg |
| **$\pi$ rad** | **= 180 deg** |

**Area**

| | |
|---|---|
| 1 acre | = 4047 $m^2$ |
| | = 0.00156 $mi^2$ |

**Energy**

| | |
|---|---|
| 1 J | = 0.239 cal |
| | = $9.48 \times 10^{-4}$ BTU |
| | = 0.7376 ft $lb_f$ |
| **1 kW h** | **= 3,600,000 J** |

**Force**

| | |
|---|---|
| 1 N | = 0.225 $lb_f$ |
| | **= 1 E 5 dyne** |
| **1 kip** | **= 1,000 $lb_f$** |

**Length**

| | |
|---|---|
| 1 m | = 3.28 ft |
| 1 km | = 0.621 mi |
| **1 in** | **= 2.54 cm** |
| **1 mi** | **= 5280 ft** |
| **1 yd** | **= 3 ft** |

**Mass**

| | |
|---|---|
| 1 kg | = 2.205 $lb_m$ |
| 1 slug | = 32.2 $lb_m$ |
| **1 ton** | **= 2,000 $lb_m$** |

**Power**

| | |
|---|---|
| 1 W | = 3.412 BTU/h |
| | = 0.00134 hp |
| | = 14.34 cal/min |
| | = 0.7376 ft $lb_f$/s |

**Pressure**

| | |
|---|---|
| 1 atm | = 1.01325 bar |
| | = 33.9 ft $H_2O$ |
| | = 29.92 in Hg |
| | = 760 mm Hg |
| | = 101,325 Pa |
| | = 14.7 psi |

**Time**

| | |
|---|---|
| **1 d** | **= 24 h** |
| **1 h** | **= 60 min** |
| **1 min** | **= 60 s** |
| 1 yr | = 365 d |

**Temperature**

| | |
|---|---|
| **1 K** | **= 1 °C** |
| | **= 1.8 °F** |
| | **= 1.8 °R** |

**Volume**

| | |
|---|---|
| 1 L | = 0.264 gal |
| | = 0.0353 $ft^3$ |
| | = 33.8 fl oz |
| **1 mL** | **= 1 $cm^3$ = 1 cc** |

**Named Units**

| | |
|---|---|
| 1 F | = 1 A s/V |
| 1 H | = 1 V s/A |
| 1 Hz | = 1 $s^{-1}$ |
| 1 J | = 1 N m |

| | |
|---|---|
| 1 N | = 1 kg $m/s^2$ |
| 1 P | = g/(cm s) |
| 1 Pa | = 1 N/$m^2$ |
| 1 St | = 1 $cm^2$/s |

| | |
|---|---|
| 1 V | = 1 W/A |
| 1 W | = 1 J/s |
| 1 Ω | = 1 V/A |

***Conversions shown in bold text above indicate exact conversions***