Chapter 8

AC Bridges

8- AC Bridges

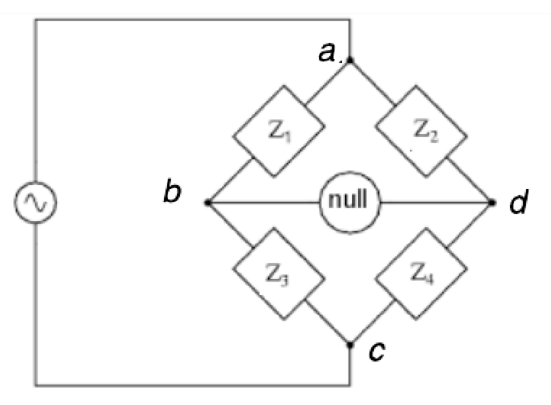
1. AC bridges,
2. Maxwell Bridge,
3. Maxwell-Wien Bridge,
4. Anderson Bridge,
5. Hay’s Bridge,
6. Owen Bridge,
7. De Sauty Bridge
8. Shering bridge,
9. Wien Series Bridge.

1- AC bridges

A.C.Bridges are those circuits which are used to measured the unknown resistances, capacitance, inductance , frequency and mutual inductance.

Any bridge circuit is balanced when the current through branch between two arms is zero as well as phase.

In the simplest form an ac bridge consists of four impedances, an ac source and null detector. In some cases ac amplifier with an output meter is used as null detector.



The null indication is obtained when the voltage at point b is equal to the voltage at point d. These two voltages should be equal in terms of both amplitude and phase. That means to obtain this condition; the voltage drop across ab should be equal to the voltage drop across ad.

In terms of current and impedance we can write,

$I_1Z_1=I_2Z_2$. and $I_3Z_3=I_4Z_4$.

$$\frac{i_1Z_1}{i_1Z_3} = \frac{i_2Z_2}{i_2Z_4}$$

$$\frac{Z_1}{Z_3} = \frac{Z_2}{Z_4} \longleftrightarrow \frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

Example The four impedances of an ac bridge as shown in the figure are $Z_1 = 500 < 40^o\ \Omega$, $Z_2 = 100<90^o\ \Omega$, $Z_3 = 45< 20^o\ \Omega$, $Z_4 = 30 < 30^o\ \Omega$. Find out whether the bridge is balanced or not.

Solution: As we know in polar co-ordinate system representation, the bridge balance conditions are

$$Z_1 Z_4 = Z_2 Z_3 \quad \text{and}$$

$$\angle\theta_1 + \angle\theta_4 = \angle\theta_2 + \angle\theta_3$$

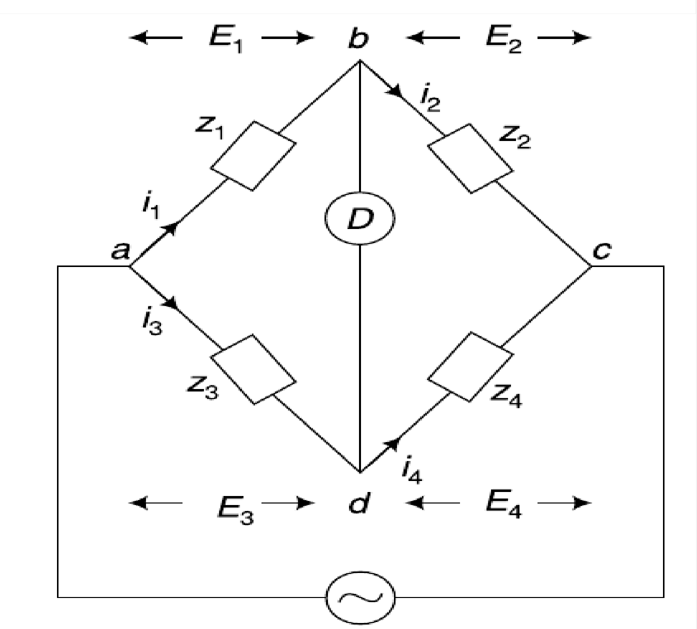
Thus,

$$Z_1 Z_4 = 500 \times 30 = 15 \times 10^3\ \Omega$$

$$Z_2 Z_3 = 100 \times 45 = 4.5 \times 10^3\ \Omega$$

$$\angle\theta_1 + \angle\theta_4 = 40^o + 30^o = 70^o$$

$$\angle\theta_2 + \angle\theta_3 = 90^o + 20^o = 110°$$



bridge is unbalanced.

2- Maxwell's Bridge

Maxwell Inductance Bridge

In this bridge arrangement the value of unknown inductance is measured by comparison with a variable standard self-inductance. Figure shows the circuit arrangement for Maxwell's inductance bridge under balance condition. Two branches bc and cd consist of non-inductive resistance R_3 and R_4. One of the arms ad consists of variable inductance L_2 connected in series with variable resistance R_2. The remaining arm ab consists of unknown inductance L_1 and resistance R_1. A source of current is applied to two opposite junctions across ac and a null detector is connected to the other two junctions b and d.

$$I_1(R_1+j\omega L_1) = I_2(R_2+j\omega L_2)$$

$$I_1R_3 = I_2R_4$$

$$\therefore \frac{R_1 + j\omega L_1}{R_3} = \frac{R_2 + j\omega L_2}{R_4}$$

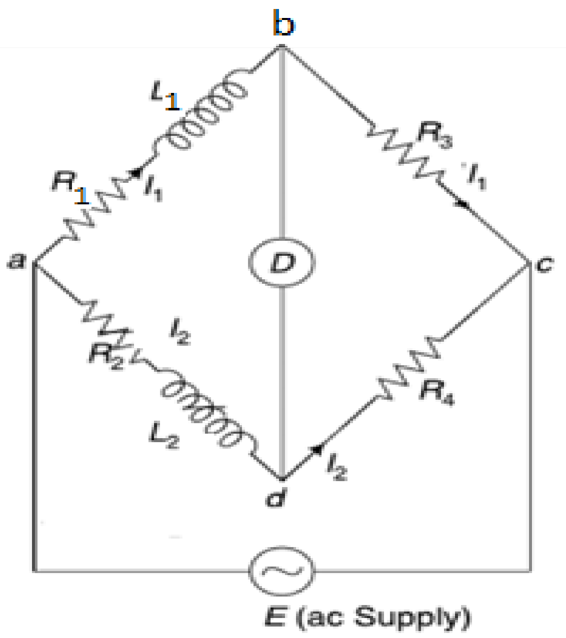
$$\therefore R_1R_4 + j\omega L_1R_4 = R_2R_3 + j\omega L_2R_3$$

$$R_1R_4 = R_2R_3$$

$$\therefore \frac{R_1}{R_2} = \frac{R_3}{R_4}$$

$$L_1R_4 = L_2R_3$$

$$\frac{L_1}{L_2} = \frac{R_3}{R_4}$$

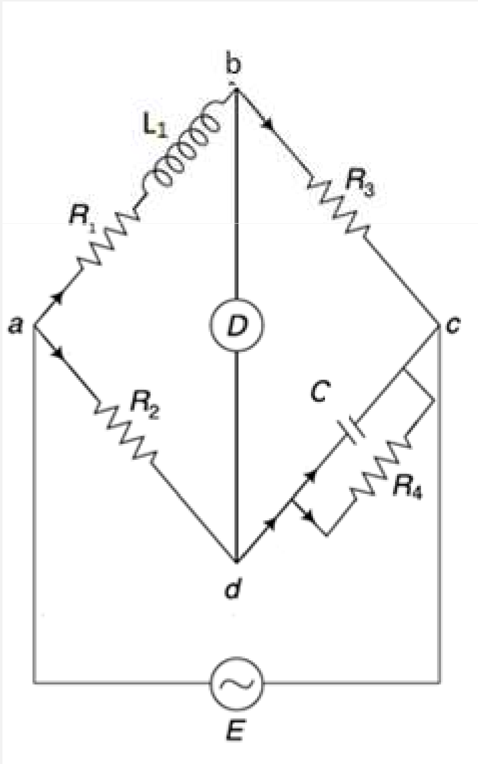


2- Maxwell-Wien Bridge

Maxwell Inductance Capacitance Bridge

In this bridge arrangement, the value of unknown inductance is measured by comparison with a variable standard capacitor. Figure shows its circuit arrangement. Two arms bc and ad consist of non-inductive resistance R_2 and R_3.

One of the arms ac consists of variable standard capacitor C connected in parallel to a non-inductive resistance R4.



The remaining arm ab consists of unknown inductance L_1 of effective resistance R_1. A source of current is applied to two opposite junctions across ac and a null detector is connected to the other two junctions b and d. Hence, the impedances of four arms are

At balance we get

$$\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4}$$

$$Z_1 = R_1 + j\omega L_1 \qquad Z_2 = R_2 \qquad Z_3 = R_3$$

$$\frac{1}{Z_4} = \frac{1}{R_4} + \frac{1}{-J/\omega C} = \frac{1}{R_4} + J\omega C \Rightarrow Z_4 = \frac{R_4}{1 + J\omega C R_4}$$

$$Z_1 Z_4 = Z_2 Z_3$$

$$(R_1 + j\omega L_1)\frac{R_4}{1 + jwC\ R_4} = R_2 R_3$$

$$= R_1 R_4 + jwL_1 R_4 = R_2 R_3 + jwC\ R_2 R_3 R_4$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

$$L_1 = R_2 R_3 C$$

Hence, from the above condition for ***<u>bridge balance</u>*** the unknown inductance value can be determined by comparison with variable standard capacitor.

Advantages

- The balance equation is independent of frequency.
- It is useful for measurement of wide range of inductance at power and audio frequency.
- The scale of resistance can be calibrated to read inductance directly.

Disadvantages

- It cannot be used for measurement of high Q values ($Q \geq 10$).
- It cannot be used for measurement of very low Q values, because of balance converge problem.

Example

A Maxqell-Wien bridge uses a standard capacitor of C = 0. I µF and operates at a supply frequency of 100Hz. Balance is achieved when R_2 =1.26kΩ ,R_3= 500Ω, and R_4 =410 Ω. Calculate the inductance and resistance of the measured inductor,

$$L_1 = R_2R_3C$$

=1.26kΩ x500Ω x 0. I µF

= 63mH

$$R_1 = \frac{R_2R_3}{R_4} = \frac{1.26\ \text{k}\Omega \times 500\ \Omega}{470\ \Omega} = 1.34\ \text{k}\Omega$$

4- Anderson Bridge

- This bridge, in fact, is a modification of the Maxwell-Wien bridge.
- In this method, the self-inductance is measured in terms of a standard capacitor.
- Figure shows the connections of the bridge for balanced conditions.

From loop abd $$I_1(R_1+J\omega L) = I_3R_3+I_r r$$

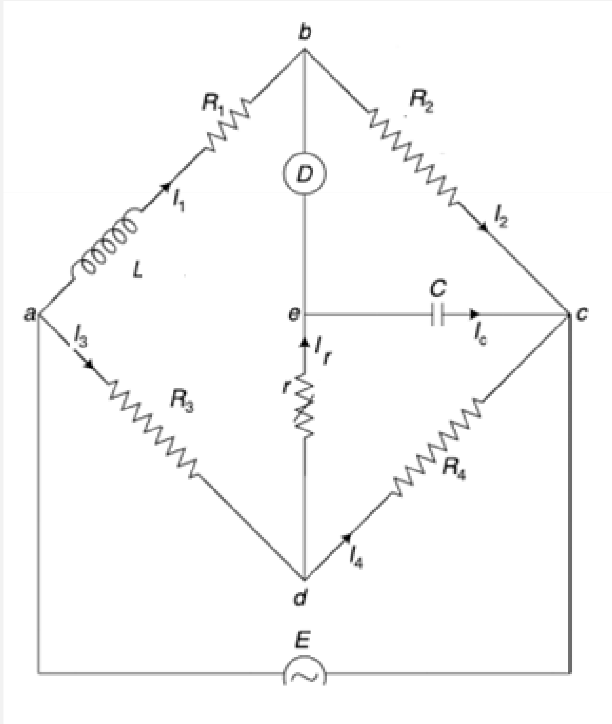
From loop bce $$I_2R_2=I_C\left(\frac{-J}{\omega C}\right)$$

From loop dce $$I_2R_4 = I_r r + I_C\left(\frac{-J}{\omega C}\right)$$

At balance between b and d $I_1 = I_2$ $I_c = I_r$

from the the rule at any junction the input current equal the output

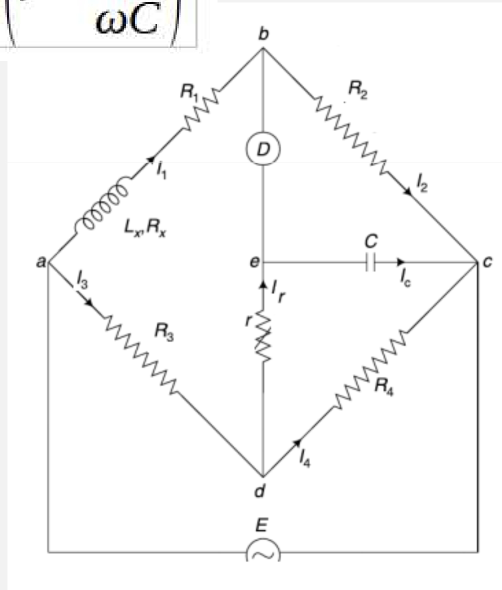
$$I_3= I_4+I_r = I_C + I_4$$



$$\frac{I_1(R_1+J\omega L)}{I_2R_2} = \frac{I_3R_3+I_r r}{-JI_C/\omega C} = \frac{I_3R_3+I_C r}{-JI_C/\omega C} = \frac{I_C[(I_3R_3/I_C)+r]}{-JI_C/\omega C} = \frac{(I_3R_3/I_C)+r}{-J/\omega C}$$

from $\dfrac{I_3}{I_C} = 1+\dfrac{I_4}{I_C}$ $\Rightarrow$ $I_4R_4 = I_C r - J\dfrac{I_C}{\omega C} = I_C\left(r-\dfrac{J}{\omega C}\right)$

$$\therefore \frac{I_4}{I_C} = \frac{r - J/\omega C}{R_4}$$



$$-J\frac{R_1}{\omega C} - \frac{J^2\omega L}{\omega C} = R_2R_3 + \frac{R_2R_3r}{R_4} - J\frac{R_2R_3}{\omega C R_4} + rR_2$$

$$\frac{L}{C} = \frac{rR_2R_3+R_2R_3R_4+rR_2R_4}{R_4}$$ and $$\frac{R_1}{R_2} = \frac{R_3}{R_4}$$

the equations of balance

Advantages

- Anderson's bridge balance is easily obtained for low Q coils.
- The bridge can be used for accurate determination of capacitance in terms of inductance.

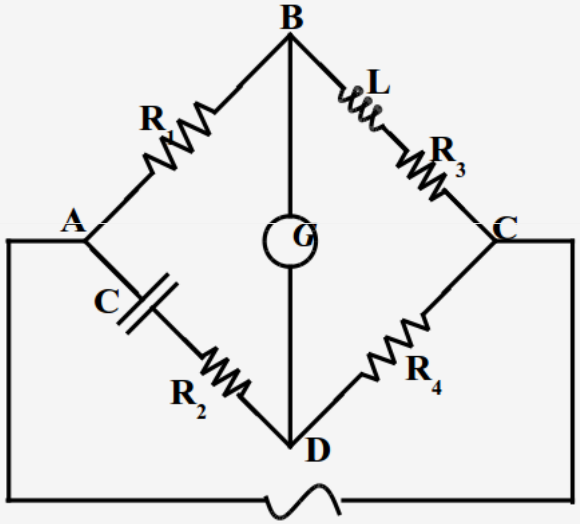
Disadvantages

- It is complex.
- The bridge balance equations are not simple. They are rather more tadious.

Hay's Bridge

It is also a modification of the Maxwell's Wien Bridge and is particularly useful if the phase angle of the inductive impedance is large.
The limitation of Maxwell's bridge is that it can be used for high Q values.The Hay's bridge is suitable for the coils having high Q values.

The different in Maxwell's Wien bridge and Hay's bridge is that the Hay's bridge consists of resistance R_2 in series with the standard capacitor C in one of the ratio arms. Hence for larger phase angles R_2 needed is very low, which is practicable.

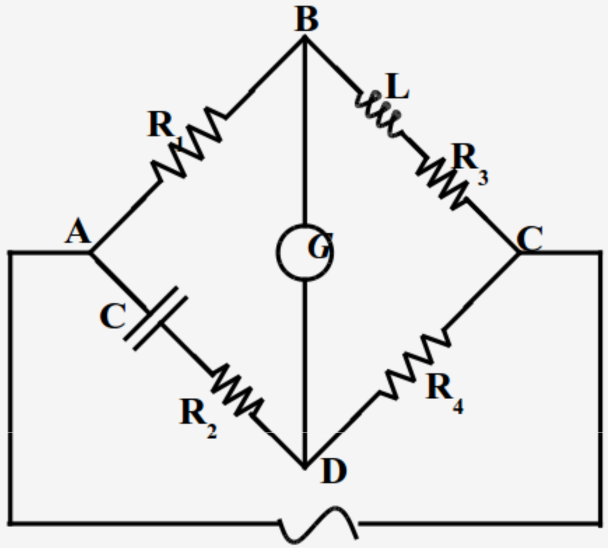


$Z_1 = R_1$ $\quad$ $Z_2 = R_2 - \frac{J}{\omega C}$ $\quad$ $Z_3 = R_3 + J\omega L$

$Z_4 = R_4$ $\quad$ $\frac{Z_1}{Z_2} = \frac{Z_3}{Z_4} \Rightarrow Z_1 Z_4 = Z_2 Z_3$ $\quad$ $R_1 R_4 = (R_3 + J\omega L) \times \left(R_2 - \frac{J}{\omega C}\right)$

$$R_1 R_4 = R_2 R_3 + \frac{L}{C} - J\frac{R_3}{\omega C} + J\omega L R_2$$

$$R_1 R_4 = R_2 R_3 + \frac{L}{C} \qquad \frac{R_3}{\omega C} = \omega L R_2$$



$$L = \frac{C R_1 R_4}{1 + \omega^2 C^2 R_2^2} \qquad R_3 = \frac{\omega^4 C^2 R_1 R_2 R_4}{1 + \omega^2 C^2 R_2^2}$$

Advantages

- It is best suitable for the measurement of inductance with high Q, typically greater than 10.
- It gives very simple expression for Q factor in terms of elements in the bridge.
- It requires very low value resister R_1 to measure high Q inductance.

Disadvantages

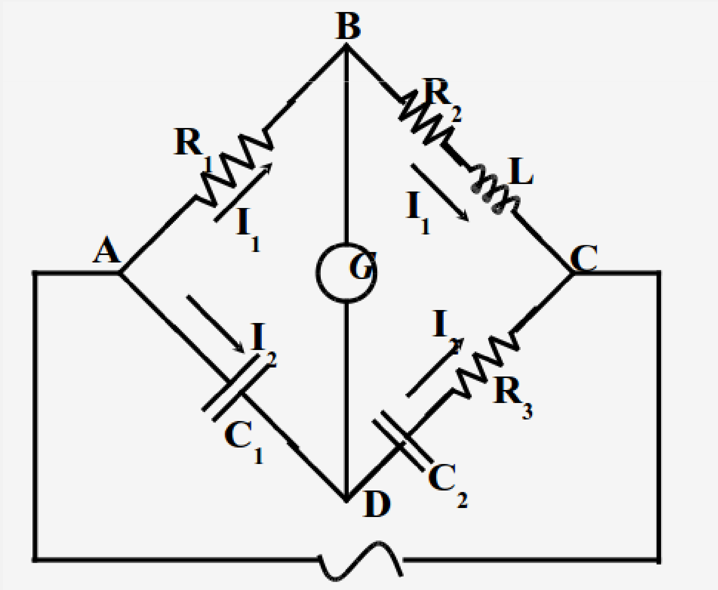
- It is only suitable for measurement of high Q inductance.

6- Owen Bridge,

This bridge used to measure the self-induction L of the coil with capacitor and two variable resistors

$$\frac{Z_1}{Z_2} = \frac{Z3}{Z_4} \qquad \frac{R_1}{R_2 + J\omega L} = \frac{-\frac{J}{\omega C_1}}{R_3 - \frac{J}{\omega C_2}}$$

$$\therefore R_1 R_2 - J\frac{R_1}{\omega C_2} = -J\frac{R_2}{\omega C_1} + \frac{L}{C_1}$$



$$L = C_1 R_1 R_3$$

from this equation measure L

$$R_1 C_1 = R_2 C_2$$

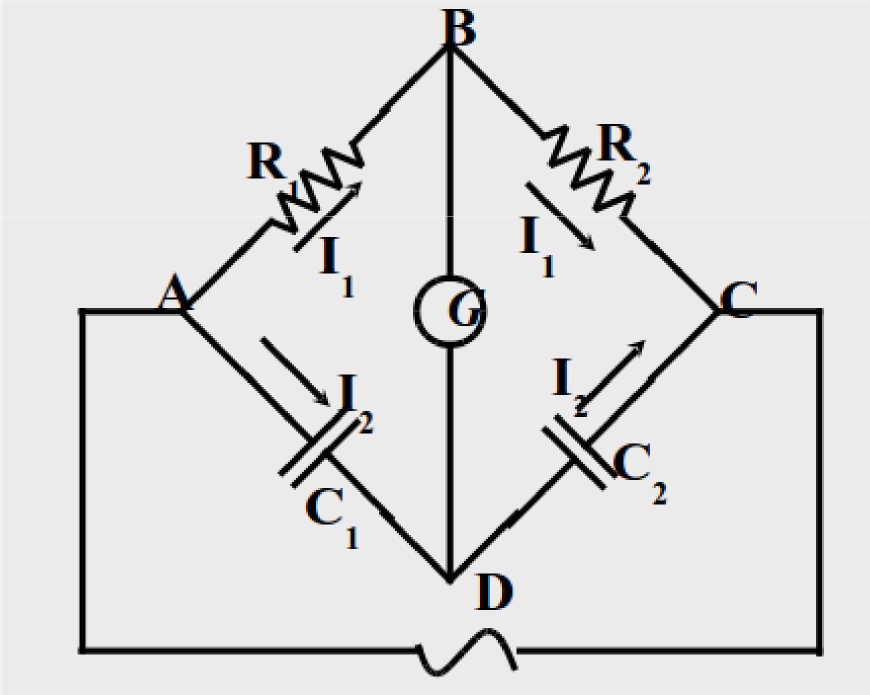
from this equation measure C_2

7- De Sauty Bridge

This is the simplest method for capacitance measurement fig. shows the basic circuit arrangement.

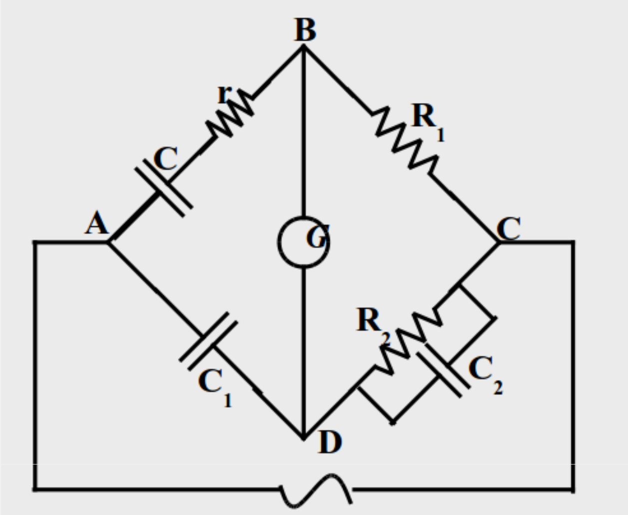
From the balance

$$\frac{R_1}{R_2} = \frac{-\dfrac{J}{\omega C_1}}{-\dfrac{J}{\omega C_2}} = \frac{C_2}{C_1}$$



8-Schering Bridge

Schering bridge used for the measurement of capacitance and dielectric loss of a capacitor.
It is a device for comparing an imperfect capacitor C_1 in terms of a loss-free standard capacitor C_2. As shown in fig.



$$\frac{Z_1}{Z_2} = \frac{Z3}{Z_4}$$

$$\frac{\frac{1}{J\omega C} + r}{R_1} = \frac{\frac{1}{J\omega C_1}}{Z_4}$$

$$\frac{\frac{1}{J\omega C} + r}{R_1} = \frac{\frac{1}{J\omega C_1}\left(R_2 + \frac{1}{J\omega C_2}\right)}{R_2 / J\omega C_2}$$

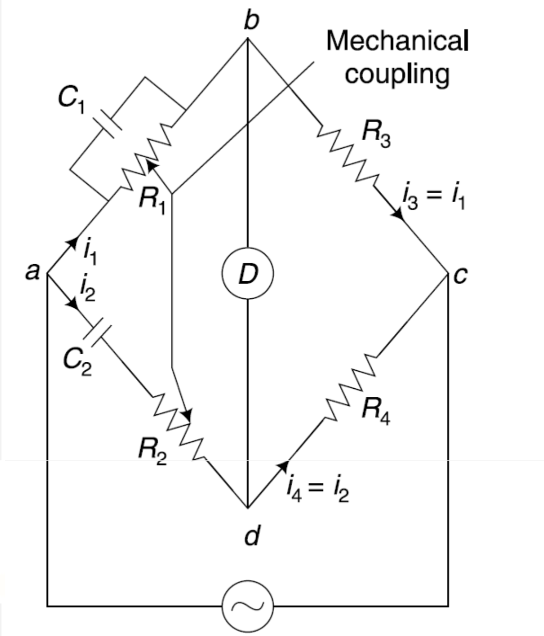
$$\frac{\frac{1}{J\omega C} + r}{R_1} = \frac{C_2}{R_2 C_1}\left(R_2 + \frac{1}{J\omega C_2}\right)$$

$$\therefore \frac{1}{J\omega C} + r = \frac{R_1 C_2}{R_2 C_1}\left(R_2 + \frac{1}{J\omega C_2}\right)$$

$$r = \frac{C_2}{C_1} R_1 \qquad C = \frac{R_2}{R_1} C_1$$

9-Wien Series Bridge

It is a bridge used for the measurement of frequency.
Apart from this it has a variety of applications in the harmonic distortion analyzer where it is used as a notch filter,discriminating against a specific frequency in the audio and HF oscillators as frequency determining against a specific frequency in the audio and HF oscillators as frequency determining element.



Hence, the impedances of four arms are

$$Z_1 = \left(R_1 \parallel \frac{1}{j\omega C_1}\right) = \frac{R_1}{1 + j\omega C_1 R_1}$$

$$Z_2 = R_2 + \frac{1}{j\omega C_2}$$

$Z_3 = R_3$ $\qquad$ $Z_4 = R_4$

Hence, from balance

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2}$$

$$f = \frac{1}{2\pi\sqrt{R_1 R_2 C_1 C_2}}$$

Example

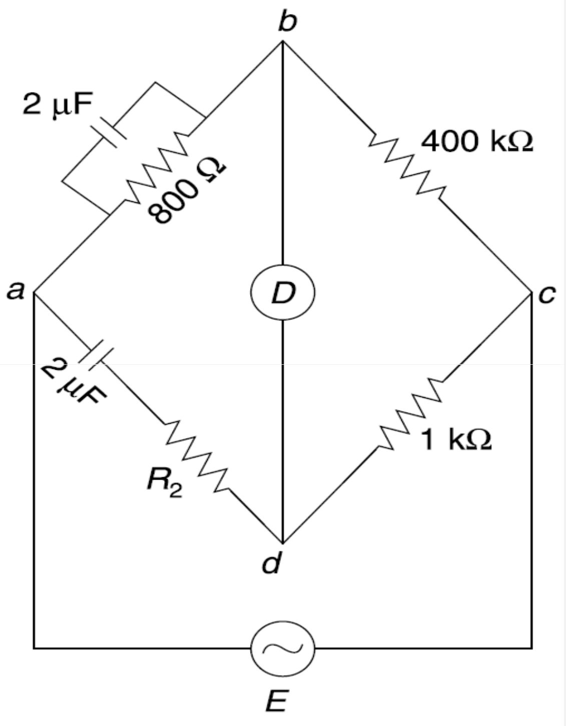
The arms of a four-arm bridge *a*, *b*, *c* and *d* supplied with sinusoidal voltage have the following values.

arm *ab*: A resistance of 800 Ω in parallel with a capacitance of 2 μF

arm *bc*: 400 Ω resistance

arm *cd*: 1 kΩ resistance

arm *da*: A resistance R_2 in series with 2 μF capacitance



Determine the value of R_2 and frequency at which the bridge will balance?

Solution from balance equation

$$\frac{R_4}{R_3} = \frac{R_2}{R_1} + \frac{C_1}{C_2} \implies R_2 = \left(\frac{R_4}{R_3} - \frac{C_1}{C_2}\right) R_1$$

$$= \left(\frac{1000}{400} - \frac{2 \times 10^{-6}}{2 \times 10^{-6}}\right) \times 800 \quad = 1.2\ \text{k}\Omega$$

$$f = \frac{1}{2\pi\sqrt{R_1 R_2\, C_1 C_2}} = \frac{1}{2\pi\sqrt{1200 \times 1000 \times 2 \times 10^{-6} \times 2 \times 10^{-6}}} \quad = 72.6\ \text{Hz}$$

Multiple Choice Questions

1. Maxwell inductance capacitance bridge is used for measurement of inductance of
(a) low *Q* coils (b) medium *Q* coils
(c) high *Q* coils (d) low and medium *Q* coils

2. Frequency can be measured by using
(a) Wien bridge (b) Schering bridge
(c) Maxwell bridge (d) Heaviside Campbell bridge

3. The effective reactance of an inductive coil
(a) increases because of stray capacitance as frequency increases
(b) remains unchanged
(c) decreases because of stray capacitance as frequency increases

4. Inductance can be measured by
(a) Maxwell bridge b) Hay bridge
(c) Schering bridge (d) Wien bridge