Umm Al-Qura University

College of Engineering & Islamic Architecture

Mechanical Engineering Department

Course # 804314-2 : Heat Transfer (1)

Second Semester 1435/1436 H (2014/2015 G)

Solution of Tutorial 1

1. Define:

- **Heat transfer**

Heat transfer (or heat) is thermal energy in transit due to a spatial temperature difference.

- **Conduction:**

It is the heat transfer that occurs across a stationary medium (a solid or a fluid) when a temperature gradient exists in the medium.

- **Thermal conductivity:**

The thermal conductivity is the property of a material to conduct heat

- **Fourier's law:**

$$q''_x = -k\frac{dT}{dx} \quad \text{or} \quad q_x = -kA\frac{dT}{dx}$$

where q''_x (W/m^2) ≡ heat flux = heat transfer in the x-direction per unit area perpendicular to the direction of transfer and k (W/m.k) ≡ thermal conductivity of the medium. q_x (W) is the heat transfer rate.

- **Conductor:**

It is a material which can transfer heat through conduction.

- **Insulation:**

It is material used to reduce the rate of heat transfer (with low thermal conductivity).

- **Convection:**

It is is the transfer of heat from one place to another by the movement of fluids.

- **Newton law of cooling:**

$$q'' = h(T_s - T_\infty) \text{ or } q = hA(T_s - T_\infty)$$

where q'' is the heat flux, T_s is the surface temperature, T_∞ is the fluid temperature and h (W/m^2.K) is termed the *average convection heat transfer coefficient.*

- **Radiation:**

The net heat transfer that occurs between two surfaces at different temperatures as a result of energy emitted in the form of electromagnetic waves.

- **Stefan-Boltzmann law:**

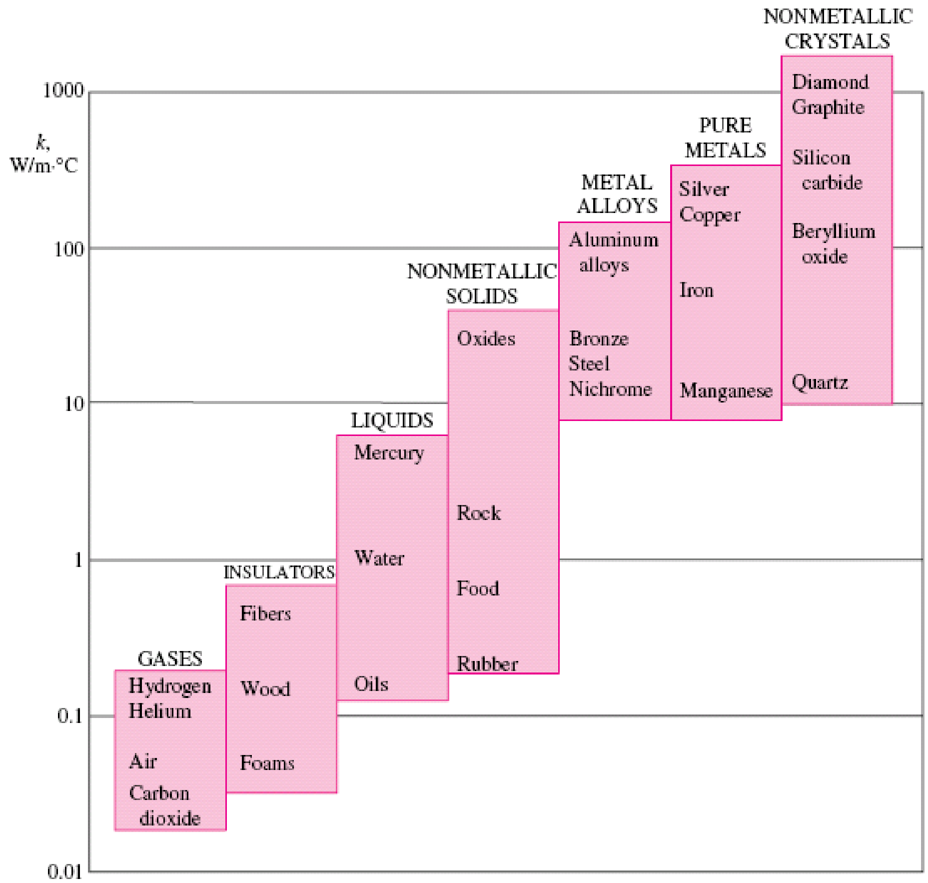
$$q''_{rad} = \frac{q}{A} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)$$

- **Blackbody:**

It refers to an object or system which absorbs all radiation incident upon it and re-radiates energy which is characteristic of this radiating system only, not dependent upon the type of radiation which is incident upon it.

2. Arrange the following materials from the highest to the lowest thermal conductivity:

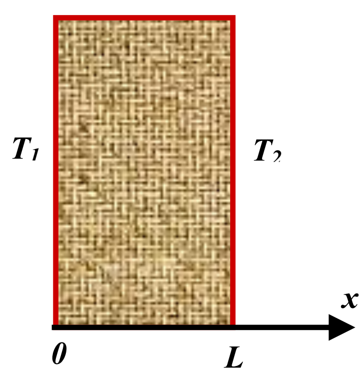
	Air	**Alloys**	**pure metal**	**nonmetallic**	**Water**
Rank	5	2	1	3	4



3. A certain insulation has a thermal conductivity of 10 W/m°C. What thickness is necessary to effect a temperature drop of 500 °C for a heat flux of 400 W/m^2?

Known:

$\Delta T = 500\,°C;\ q'' = 400\ W/m^2;\ k = 10\ W/m°C$



Find: the thickness, L

Solution:

Assumptions: 1D heat conduction, steady state, without heat generated

We apply Fourier's law: $q'' = -k\frac{dT}{dx}$

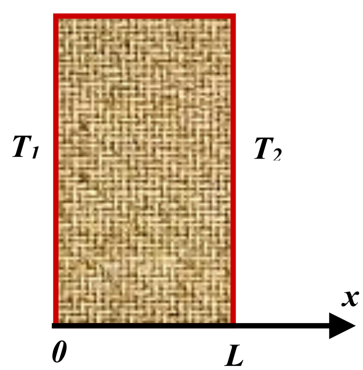
Integrating this equation: $q'' = k\frac{\Delta T}{L} \Rightarrow L = \frac{k\Delta T}{q''}$

$$\textbf{\textit{NA:}}\ L = \frac{10 \times 500}{400}\,\text{m} = 12.5\,\text{m}$$

4. The wall of an industrial furnace is constructed from 0.2 m thick fireclay brick having a thermal conductivity of 2.0 W/mK. Measurements made during steady state operation reveal temperatures of 1500 and 1250 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall which is 0.5 m by 4 m on a side?

Known:

L = 0.2m; $k = 2$ W/mK; $T_1 = 1500$ K; $T_2 = 1250$ K; $A = (0.5 \times 4)\,\text{m}^2 = 2\,\text{m}^2$



Find: the rate of heat loss through this wall q

Solution:

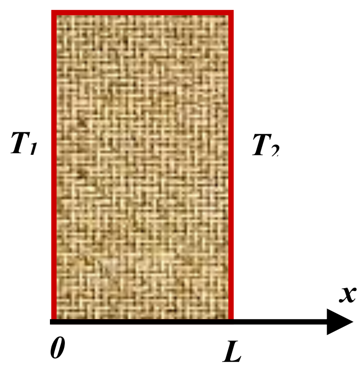
Assumptions: 1D heat conduction, steady state, without heat generated

We apply Fourier's law: $q = -kA\frac{dT}{dx}$

Integrating this equation: $q = kA\dfrac{T_1 - T_2}{L}$

$$\textbf{\textit{NA:}}\ q = 2\times 2\times \frac{1500-1250}{0.2}\ \mathrm{W} = 5000\ \mathrm{W} = 5\ \mathrm{kW}$$

5. Consider steady-state conditions from one-dimensional conduction in a plane wall having a thermal conductivity k = 10 W/mK and thickness L = 0.2 m, with no internal generation.



Determine the unknown quantities for each case, indicating the direction of the heat flux.

Known:

L = 0.2m; $k = 10$ W/mK;

Solution:

Assumptions: 1D heat conduction, steady state, without heat generated

To complete this table, we apply the following equations:

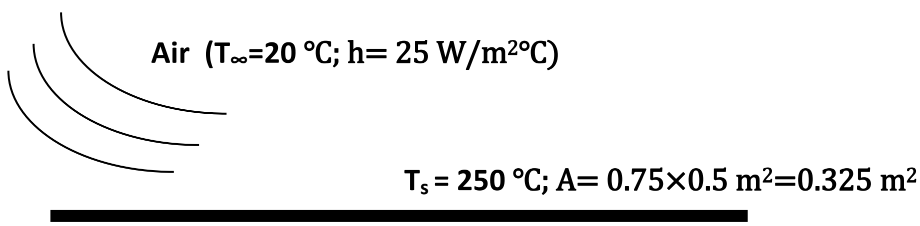
- ✓ $\dfrac{dT}{dx} = \dfrac{T_2 - T_1}{x_2 - x_1} = \dfrac{T_2 - T_1}{L}$ or $\dfrac{dT}{dx} = -\dfrac{q''}{k}$
- ✓ Fourier's law: $q'' = -k\dfrac{dT}{dx}$
- ✓ $\dfrac{dT}{dx} = \dfrac{T_2 - T_1}{L} \Rightarrow \begin{cases} T_1 = T_2 - L\dfrac{dT}{dx} \\ T_2 = T_1 + L\dfrac{dT}{dx} \end{cases}$

✓ Direction of heat transfer is, always, from hot to cold face

Case	T_1 (°C)	T_2 (°C)	$\frac{dT}{dx}$ (K/m)	Heat Flux q'' (W/m²)	Direction
A	**-40**	**-10**	**150**	**-1500**	**From T_2 to T_1**
B	**25**	**25**	**0**	**0**	**No heat transfer**
C	**70**	**50**	**-100**	**1000**	**From T_1 to T_2**
D	**50**	**10**	**-200**	**2000**	**From T_1 to T_2**

6. Air at 20°C blows over a hot plate 50 by 75 cm maintained at 250°C. The convection heat-transfer coefficient is 25 W/m²·°C. Calculate the heat transfer rate.

Known:



Find: the heat transfer rate, q

Solution:

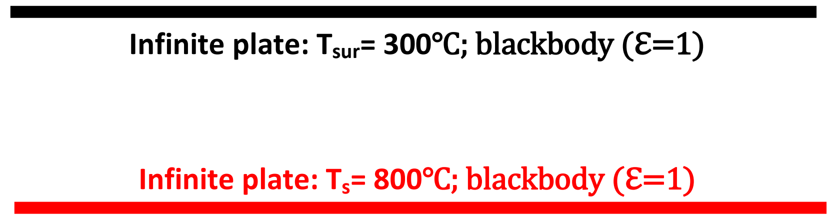
From Newton's law of cooling $q = hA(T_s - T_\infty)$

NA: $q = (25)(0.50)(0.75)(250-20)$

$= 2.156$ kW

7. Two infinite black plates at 800 and 300°C exchange heat by radiation. Calculate the heat transfer per unit area.

Known:



$$T_s = 800 \ \ °\text{C} = (800 + 273.15) \ \text{K} = 1073.15 \ \ \text{K}$$
$$T_{sur} = 300 \ \ °\text{C} = (300 + 273.15) \ \text{K} = 573.15 \ \ \text{K}$$

Find: the heat transfer per unit area (heat flux), q''

Solution:

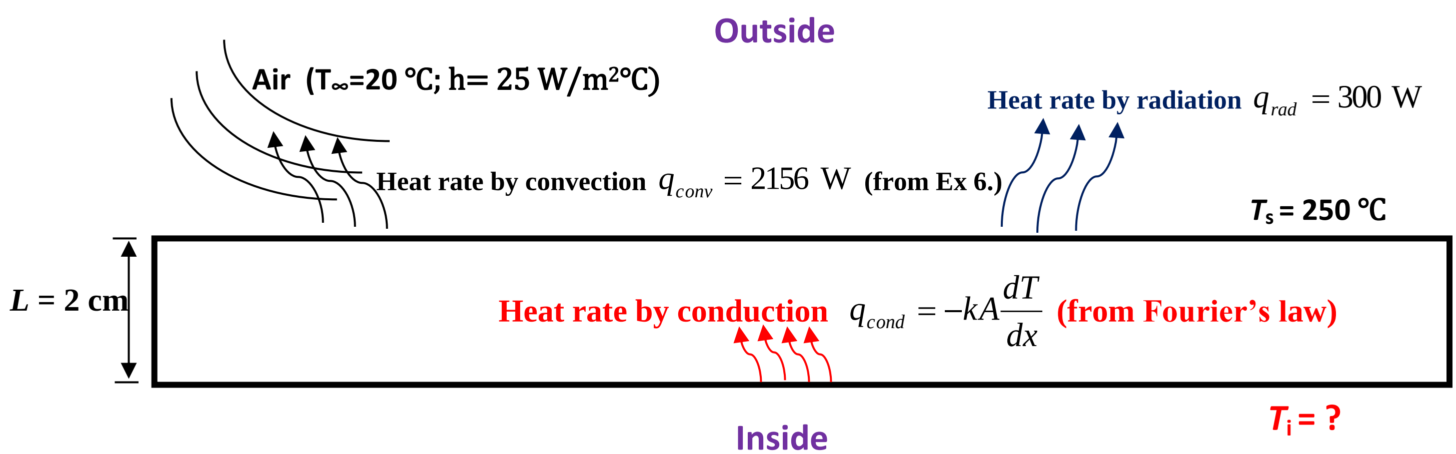
From Stefan-Boltzmann law $q^{"} = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right)$

$$\textbf{\textit{NA:}} \ \ q^{"} = 1 \times 5.67 \cdot 10^{-8}\left(1073.15^4 - 573.15^4\right) \ \ \ \text{W/m}^2$$

$$\Rightarrow q^{"} = 69082 \ \ \ \text{W/m}^2 = 69.082 \ \ \ \text{kW/m}^2$$

8. Assuming that the plate in Ex. 6 is made of carbon steel (1%) 2 cm thick and that 300 W is lost from the plate surface by radiation, calculate the inside plate temperature.

Known:



$\boldsymbol{A}$ **= 0.75×0.5 m²=0.325 m²;** ***k*** **is taken from data base :** ***k*** **= 43 W/mK**

Find: the inside plate temperature, *Ti*

Solution:

Assumptions: 1D heat conduction, steady state, without heat generated

The heat conducted through the plate must be equal to the sum of convection and radiation heat losses (**the energy balance equation**):

$$E_{in} + E_g = E_{out} + E_{st} \quad \text{where } E_g = 0 \;;\; E_{st} = 0 \,(\text{steady-state})$$

$$\text{and } E_{in} = q''_{cond} \;;\; E_{out} = q''_{conv} + q''_{rad}$$

Thus,

$$q_{cond} = q_{conv} + q_{rad}$$

$$-kA\frac{\Delta T}{\Delta x} = 2156 + 300 = 2456 \text{ W}$$

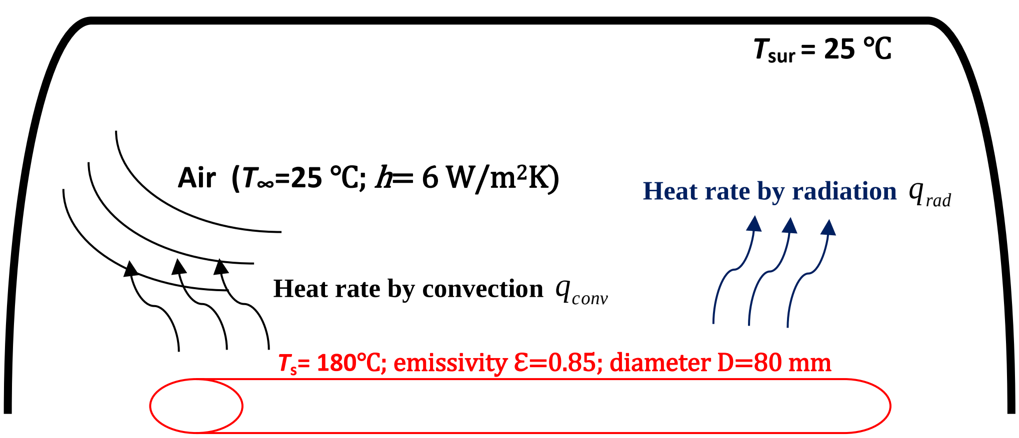
$$\Delta T = \frac{(-2456)(0.02)}{(0.5)(0.75)(43)} = -3.05°\text{C where } \Delta T = \text{Ts} - \text{T}_\text{i}$$

where the value of k is taken from data base (k = 43 W/mK). The inside plate temperature is therefore

$$T_\text{i} = 250 - (-3.05) = 253.05 \text{ °C}$$

9. An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of pipe is 80 mm, and its surface temperature and emissivity are 180°C and 0.85, respectively. If the free convection coefficient from the surface to the air is 6 W/m^{2}K, what is the rate of heat loss from the surface per unit length of pipe?

Known:



Find: the rate of heat loss from the surface per unit length of pipe, q'

Solution:

Assumptions: 1D heat conduction, steady state, without heat generated

The heat loss from the surface per unit length of pipe must be equal to the sum of convection and radiation heat losses:

$$q' = q'_{conv} + q'_{rad} \qquad ;\ where$$

$$q'_{conv} = hA'(T_s - T_\infty) \ ; \quad A' = \pi D \quad \text{is the surface per unit length}$$

$$q'_{rad} = \varepsilon\sigma A'\left(T_s^4 - T_{sur}^4\right)$$

$$\Rightarrow q' = hA'(T_s - T_\infty) + \varepsilon\sigma A'\left(T_s^4 - T_{sur}^4\right)$$

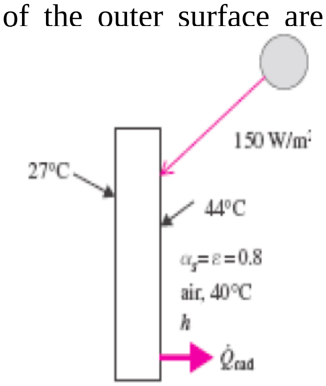
$$\Rightarrow q' = h\pi D(T_s - T_\infty) + \varepsilon\sigma\pi D\left(T_s^4 - T_{sur}^4\right)$$

$$\textbf{\textit{NA:}} \quad q'_{conv} = 6 \times \pi \times 0.08(180 - 25) \ \text{W/m} = 233.734 \ \text{W/m}$$

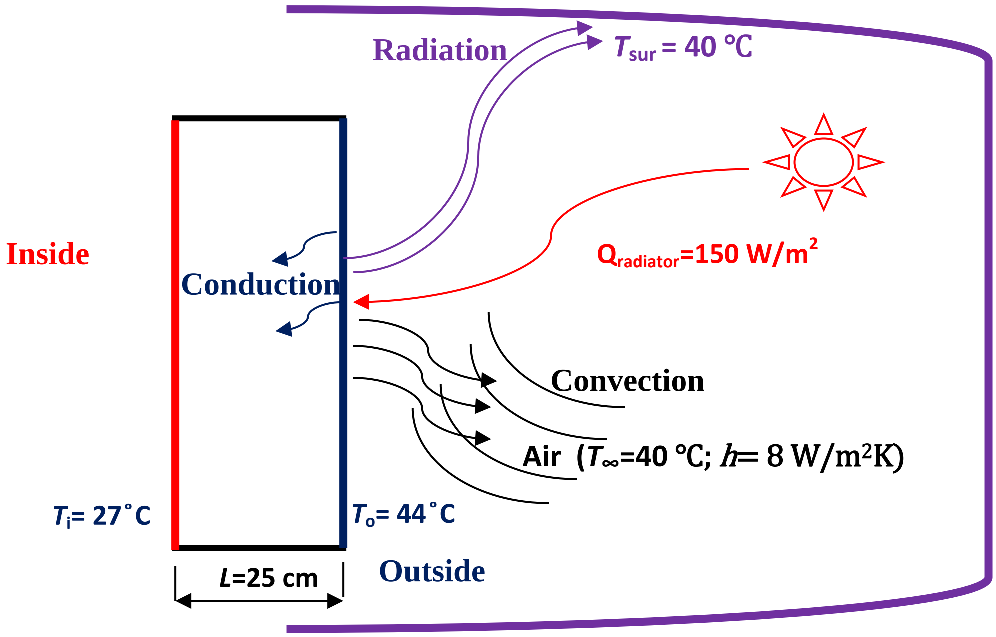
$$q'_{rad} = 0.85 \times 5.67 \cdot 10^{-8} \times \pi \cdot 0.08 \times \left[(180 + 273.15)^4 - (25 + 273.15)^4\right] \ \text{W/m} = 415.036$$

$$\Rightarrow q' = q'_{con} + q'_{rad} = (233.734 + 415.0.36)\ \text{W/m} = 648.77 \ \text{W/m}$$

10. The inner and outer surfaces of a 25-cm-thick wall in summer are at 27°C and 44°C, respectively. The outer surface of the wall exchanges heat by radiation with surrounding surfaces at 40°C, and convection with ambient air also at 40°C with a convection heat transfer coefficient of 8 W/m^2·°C. Solar radiation is incident on the surface at a rate of 150 W/ m^2. If both the emissivity and the solar absorptivity of the outer surface are 0.8, determine the effective thermal conductivity of the wall.



Known:



Find: the effective thermal conductivity of the wall, *k*

Solution:

Assumptions: 1D heat conduction, steady state, without heat generated

The heat flux conducted through the plate must be equal to the sum of convection and radiation heat fluxes (**the energy balance equation**):

$$E_{in} + E_g = E_{out} + E_{st} \quad \text{where } E_g = 0 \ ; \ E_{st} = 0 \, (\text{steady-state})$$

$$\text{and } E_{in} = \dot{Q}_{rad} \ ; \ E_{out} = q''_{cond} + q''_{conv} + q''_{rad}$$

Thus,

$$q''_{cond} + q''_{conv} + q''_{rad} = \dot{Q}_{rad}$$

$$\therefore \quad q''_{cond} = \dot{Q}_{rad} - q''_{conv} - q''_{rad} \qquad (1)$$

where

- Conduction heat flux: $q''_{cond} = -k\dfrac{dT}{dx} = -\dfrac{k(T_i - T_o)}{L} = \dfrac{k(T_o - T_i)}{L}$

- Convection heat flux: $q''_{conv} = h(T_o - T_\infty) = 8 \times (44 - 40)\ \text{W/m}^2 = 32\ \text{W/m}^2$

- Radiation heat flux to cold walls:

$$\begin{aligned} q''_{rad} &= \varepsilon\sigma\left(T_o^4 - T_{sur}^4\right) = \varepsilon\sigma\left(T_o^4 - T_{sur}^4\right) \\ &= 5.67 \cdot 10^{-8}\left[(44 + 273.15)^4 - (40 + 273.15)^4\right] \\ &= 22.72\ \text{W/m}^2 \end{aligned}$$

- Radiative heat flux absorbed by the solar radiator:

$$\begin{aligned} q''_{rad} &= \dot{Q}_{rad} = \alpha \times Q_{radiator} \\ &= (0.8 \times 150)\ \text{W/m}^2 \\ &= 120\ \text{W/m}^2 \end{aligned}$$

Thus, equation (1) becomes

$$\Rightarrow q''_{cond} = \frac{k(T_o - T_i)}{L} = (120 - 22.72 - 32)\ \text{W/m}^2 = 65.28\ \text{W/m}^2$$

$$\Rightarrow k = \frac{L \times q''_{cond}}{(T_o - T_i)} \quad \text{where } T_o = 44\ {}^\circ\text{C};\ T_i = 27\ {}^\circ\text{C};\ L = 25\ \text{cm} = 0.25\ \text{m and } q''_{\text{cond}} = 65.28\ \text{W/m}^2$$

$$\Rightarrow k = \frac{0.25 \times 65.28}{(44 - 27)}\ \text{W/m}\cdot\text{K} = 0.96\ \text{W/m}\cdot\text{K}$$