Chapter 2

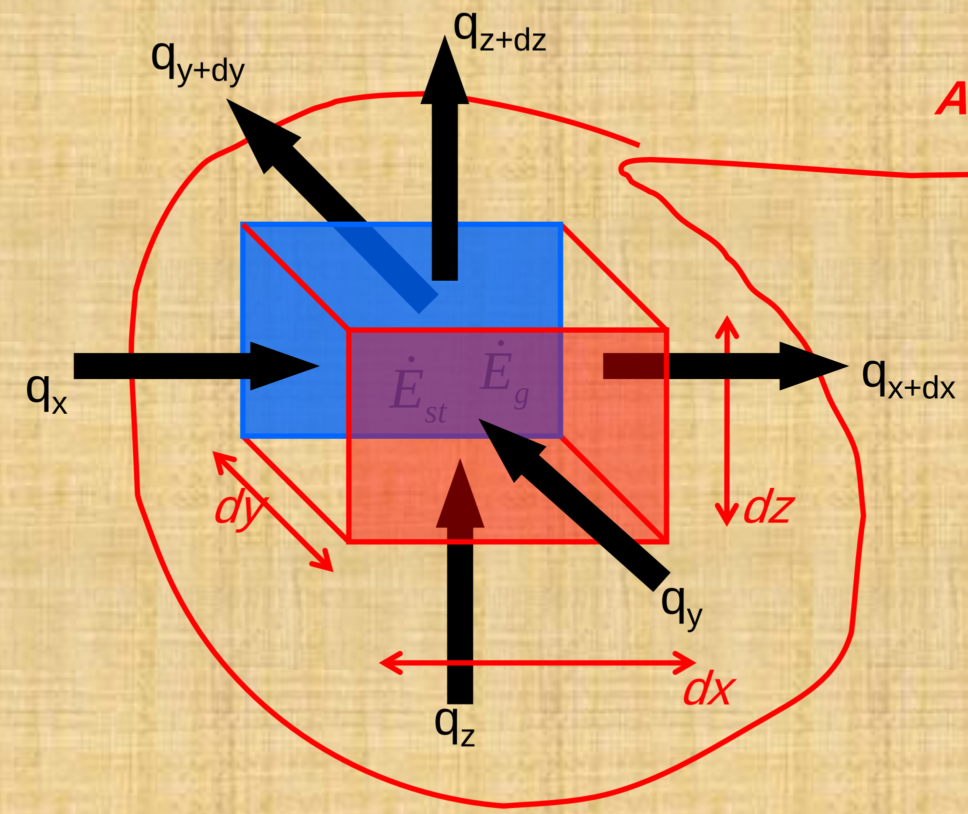
One Dimensional, Steady-State Heat Conduction

Chapter 2: One Dimensional, steady-state heat Conduction

Objectives

- To determine expressions for the temperature distribution and heat transfer rate in common (planar, cylindrical, and spherical) geometries.
- To introduce the concept of thermal resistance and the use of thermal circuits to model heat flow.

The heat diffusion equation



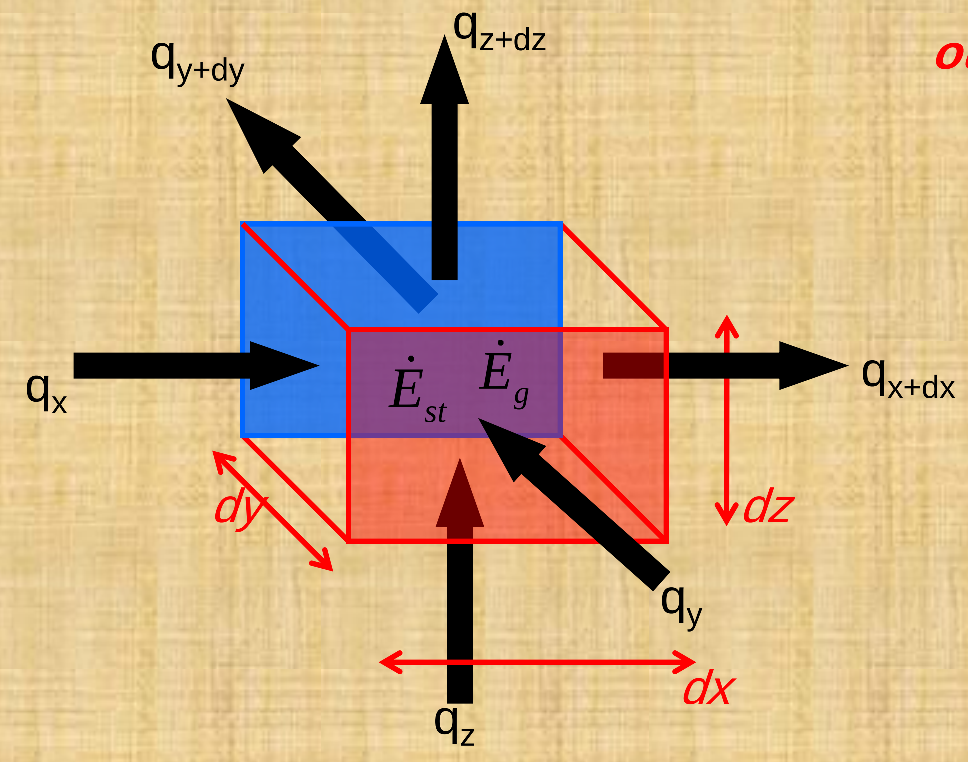
A homogenous medium in which

Bulk velocity = 0

(No advection)

T(x,y,z)

The heat diffusion equation



out

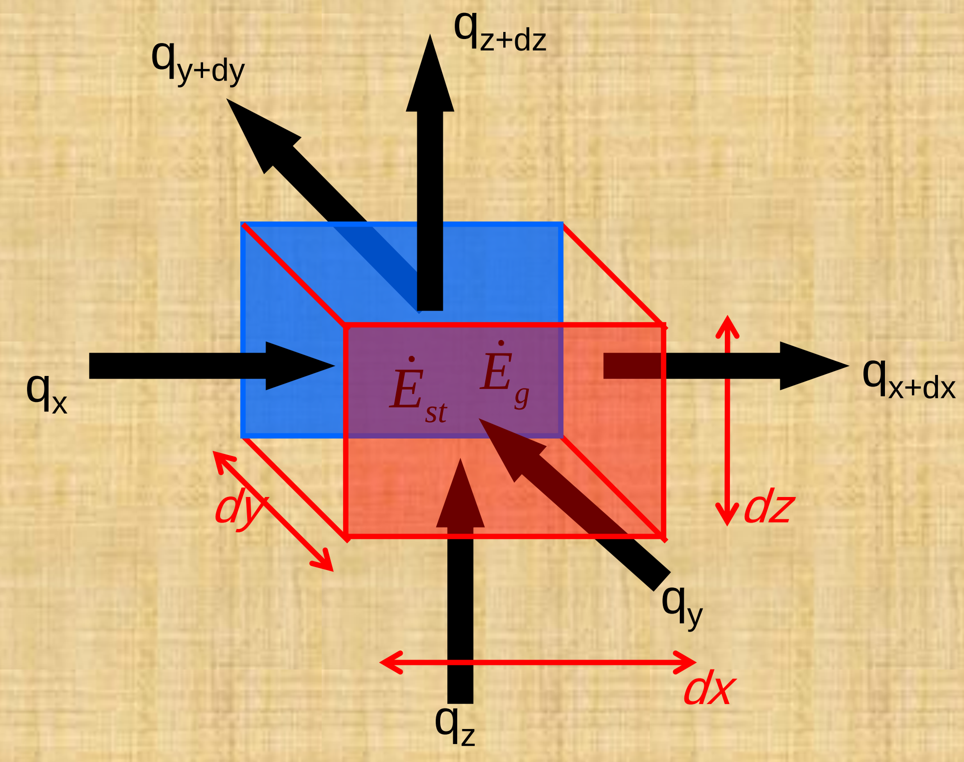
in

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

The heat diffusion equation



$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\dot{E}_g = \dot{q} dxdydz$$

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

The energy balance (law of conservation energy) may be made :

Energy conducted in the element + Heat generated with element

= Change in internal energy + Energy conducted out the element

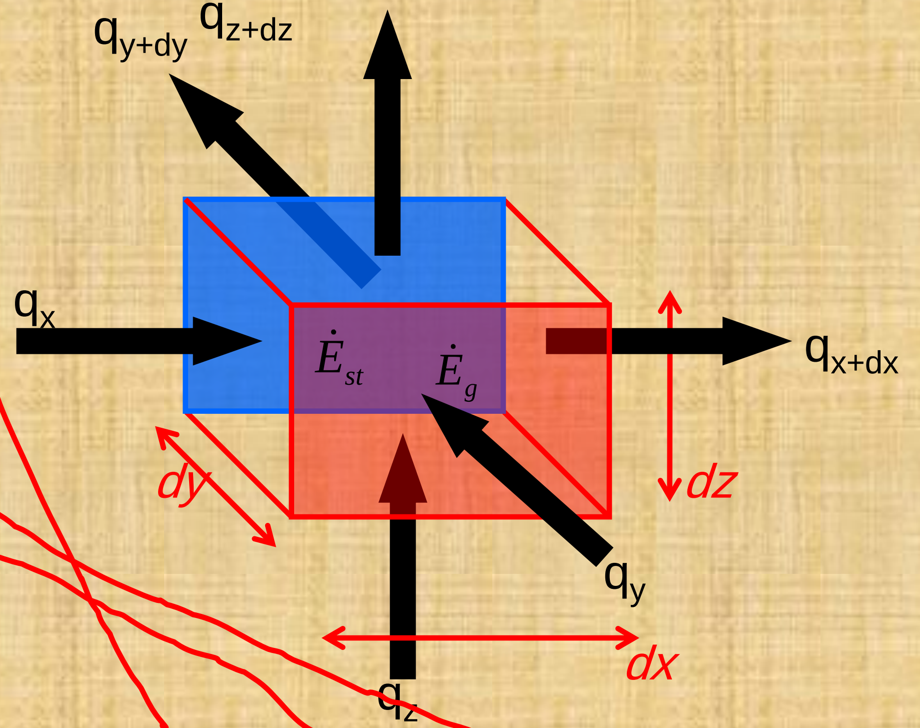
$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

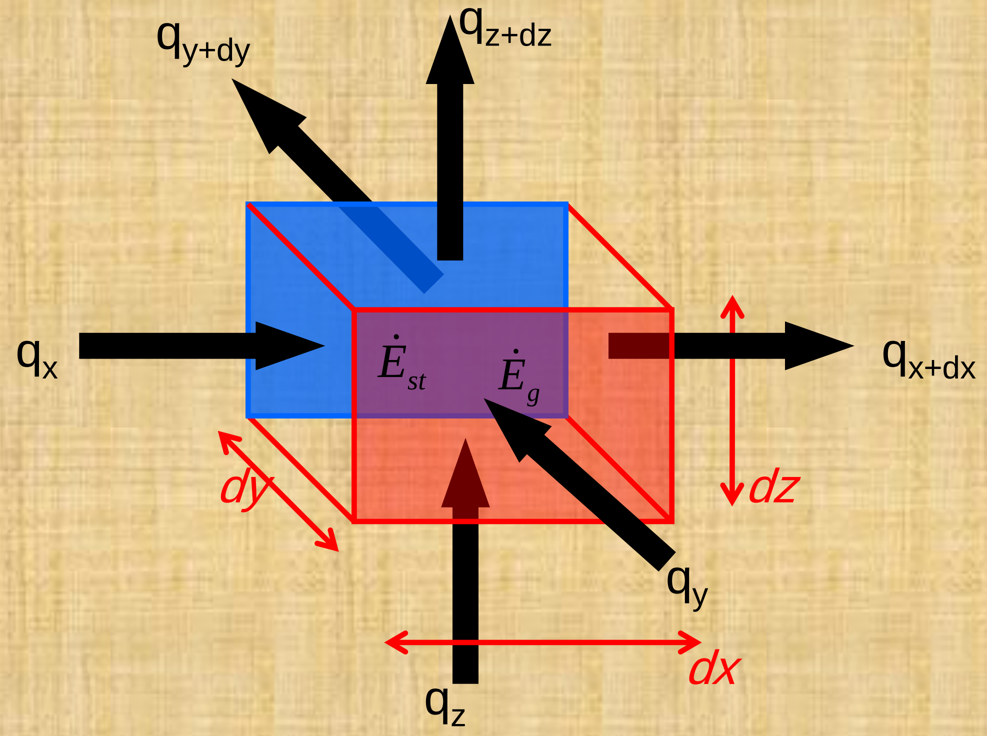
$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$\dot{E}_g = \dot{q}\, dxdydz$$

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$



$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{st} = \dot{E}_{out}$$



$$q_x + q_y + q_z + \dot{q}dxdydz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

$$q_x + q_y + q_z + \dot{q}dxdydz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} dxdydz$$

Recall that

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy$$

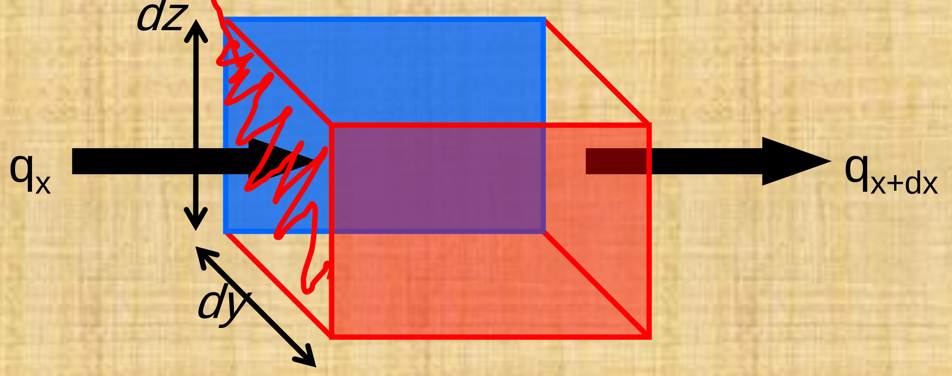
$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz$$

$$-\frac{\partial q_x}{\partial x}dx-\frac{\partial q_y}{\partial y}dy-\frac{\partial q_z}{\partial z}dz+\dot{q}dxdydz=\rho c_p\frac{\partial T}{\partial t}dxdydz$$

Recall Fourier's Law

$$q_x=-k\,dydz\frac{\partial T}{\partial x}$$

Area



$$-\frac{\partial q_x}{\partial x}dx-\frac{\partial q_y}{\partial y}dy-\frac{\partial q_z}{\partial z}dz+\dot{q}dxdydz=\rho c_p\frac{\partial T}{\partial t}dxdydz$$

$$q_x=-kdydz\frac{\partial T}{\partial x}$$

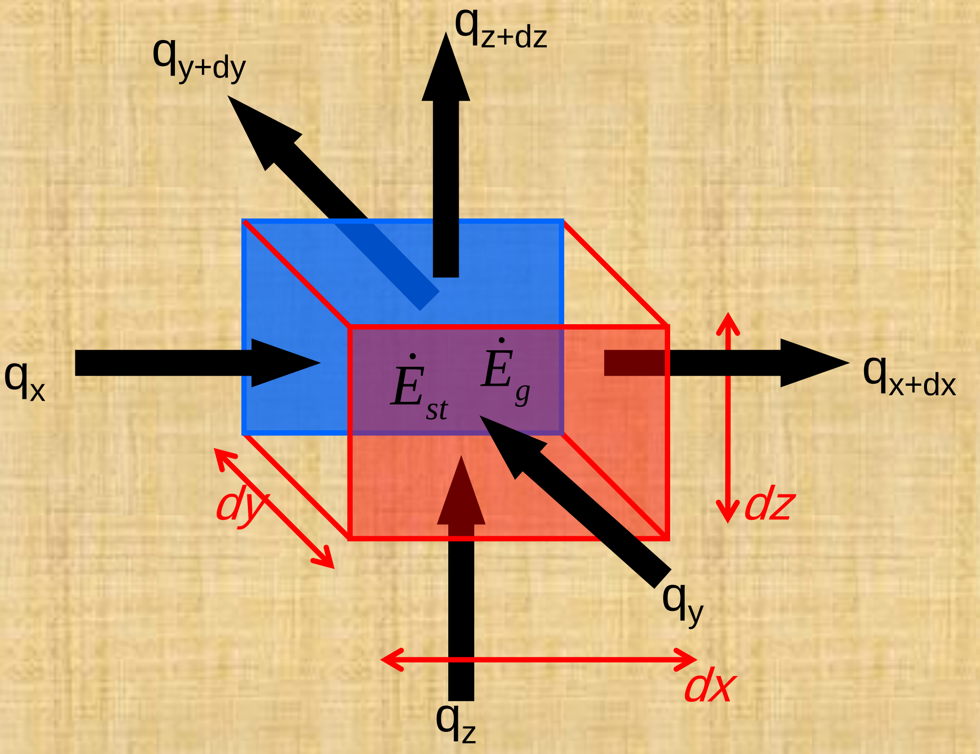
$$q_y=-kdxdz\frac{\partial T}{\partial y}$$

$$qz=-kdxdy\frac{\partial T}{\partial z}$$

Finally divide the whole equation by the volume ***dxdydz***

The Diffusion equation:

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$



If thermal conductivity is constant, you can divide the whole equation by *k* and this leads to the simplification

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

Where α is the thermal diffusivity given by

$$\alpha = \frac{k}{\rho c_p}$$

Thermal diffusivity has units of square meters per seconds (m^2/s).

Under steady state conditions *and with no heat generation* then the storage quantity reduces to zero and the heat equation reduces to

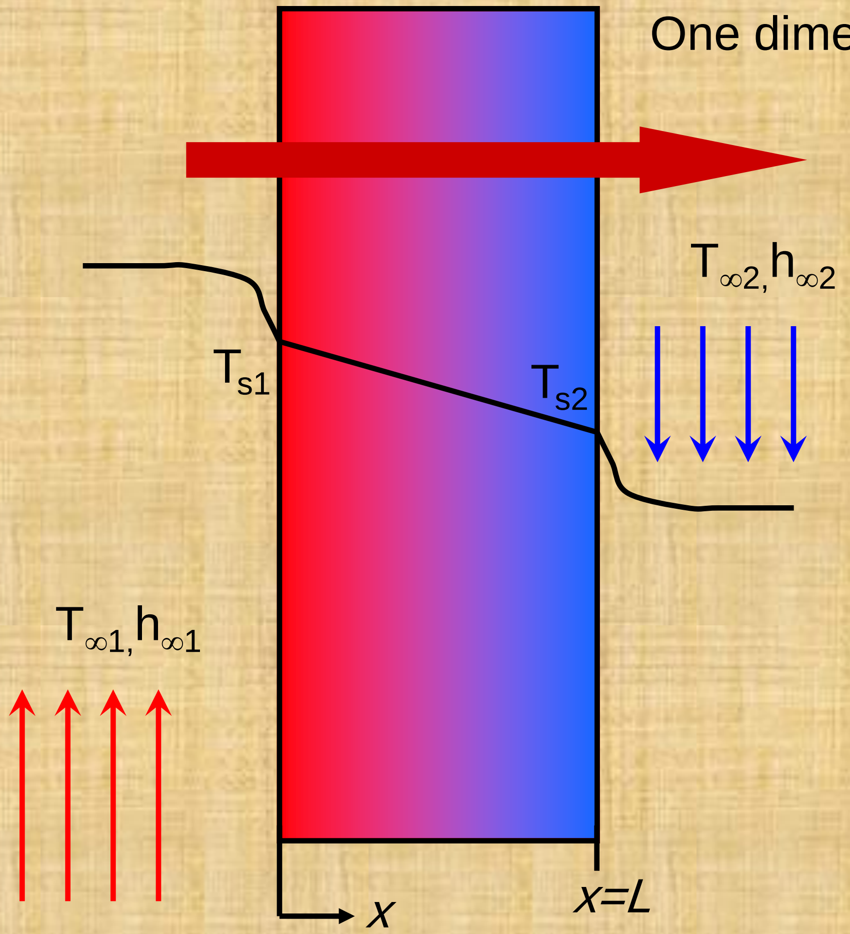
$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0$$

For one dimensional steady state heat transfer (w/o heat generated)

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0$$

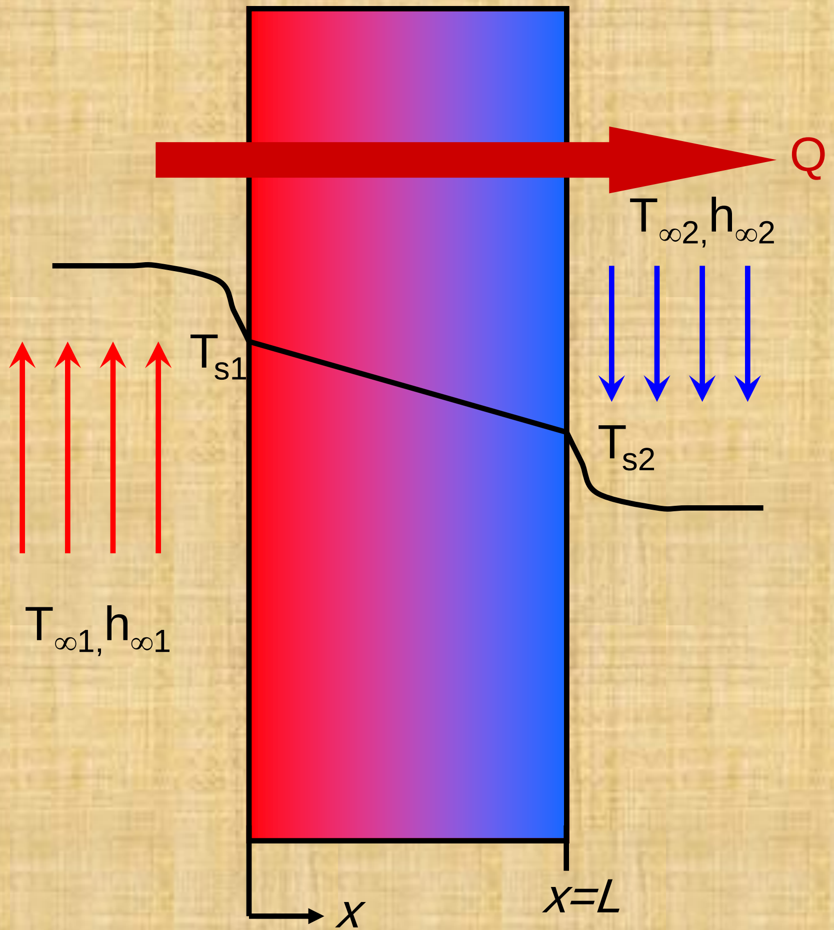
i.e the heat flux is constant in the direction of the heat transfer.

One dimensional steady state heat transfer (w/o heat generated)



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

0 0 0 0



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0$$

Integrate twice wrt x

$$T(x)=C_1 x + C_2$$

$$T(x) = C_1 x + C_2$$

In order to calculate C1 and C2 we need to apply the ***BOUNDARY CONDITIONS:***

@x=0 $T=T_{s,1}$

@x=l $T=T_{s,2}$

$C_2=T_{s,1}$

$$T(x) = C_1 x + T_{s,1}$$

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$

$$T(x) = \left(T_{s,2} - T_{s,1}\right)\frac{x}{L} + T_{s,1}$$

For one dimensional steady state conduction in a plane wall with no heat generation and constant thermal conductivity the temperature varies ***linearly*** with x ,

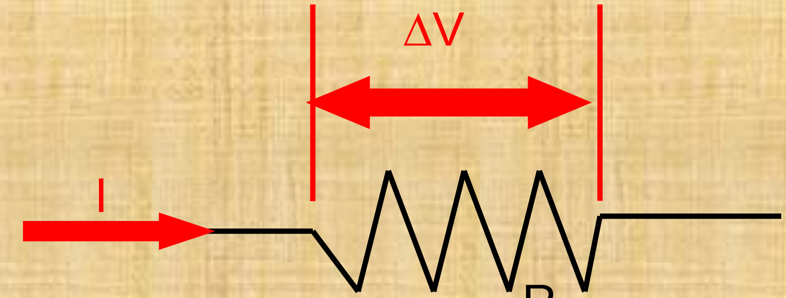
Fourier's law can now be stated as

$$q_x = -kA\frac{dT}{dx} = k\frac{A}{L}\left(T_{s,1} - T_{s,2}\right)$$

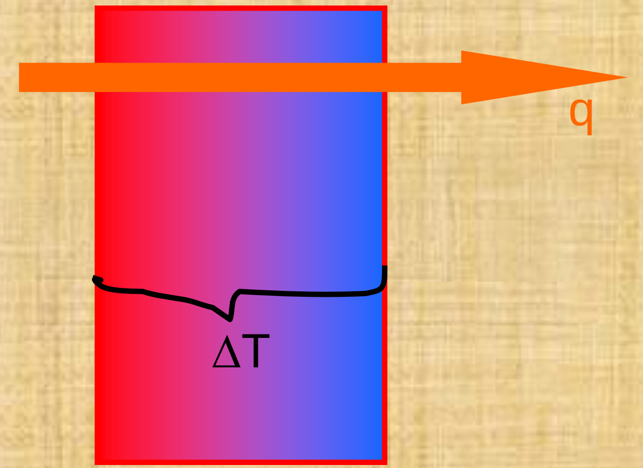
i.e the flux is

$$q''_x = \frac{k}{L}\left(T_{s,1} - T_{s,2}\right)$$

The electrical resistance analogy



$$R = \frac{\Delta V}{I}$$ Ohm's law

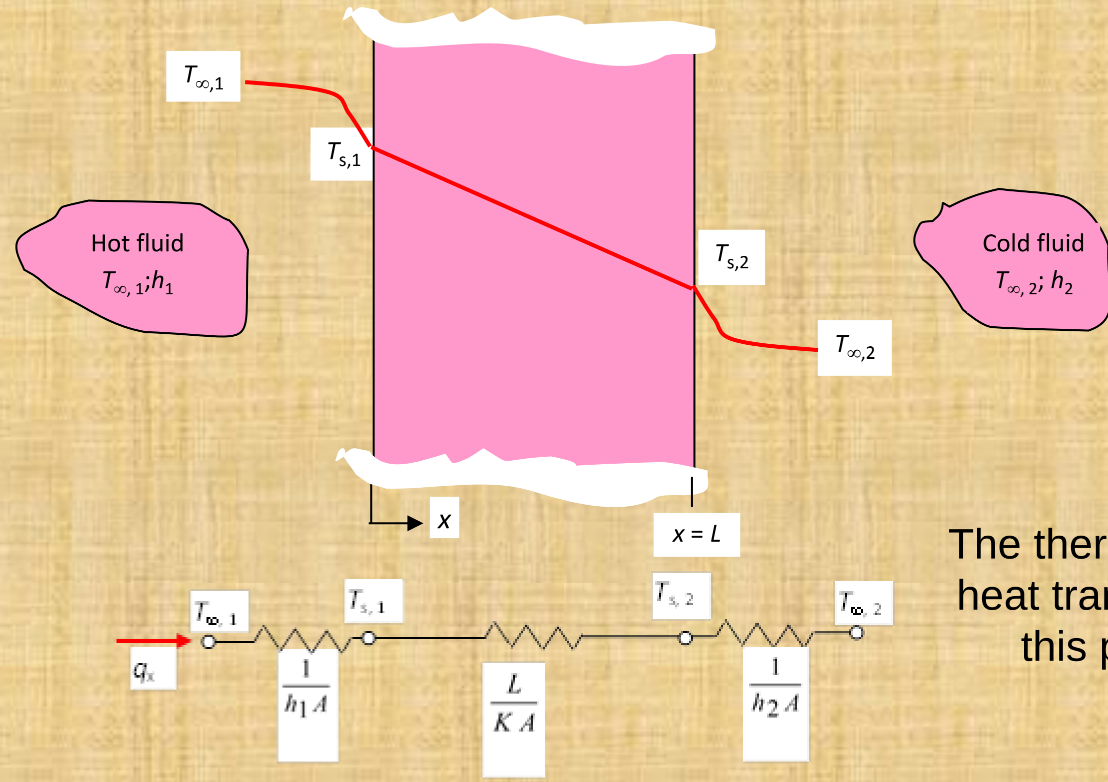


❖ conduction resistance : $R_{t,cond} = \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA}$ based on: $q = \frac{kA}{L}\left(T_{s,1} - T_{s,2}\right)$

❖ convection resistance : $R_{t,conv} = \frac{T_s - T_\infty}{q} = \frac{1}{hA}$ based on: $q = hA\left(T_s - T_\infty\right)$

❖ Radiation resistance : $R_{t,rad} = \frac{T_s - T_{sur}}{q_{rad}} = \frac{1}{h_r A}$ based on: $q_{rad} = h_r A\left(T_s - T_{sur}\right)$

where $h_r = \varepsilon \sigma \left(T_s + T_{sur}\right)\left(T_s^2 + T_{sur}^2\right)$



The thermal circuit for heat transfer through this plane wall

Since q_x is constant throughout the network, it follows that

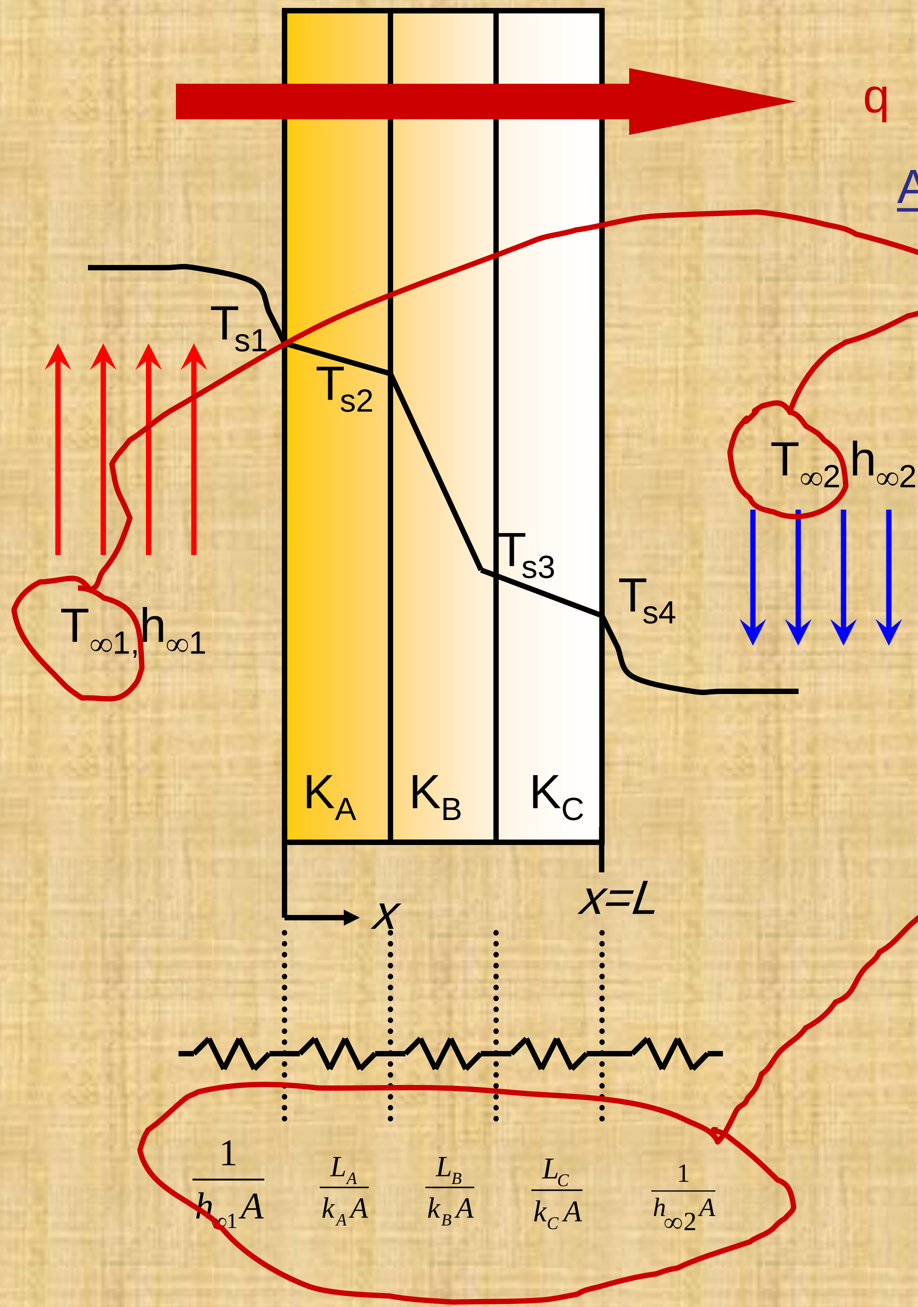
$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A}$$

In terms of the overall temperature difference, the heat transfer rate may also be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{tot}}$$ where $$R_{tot} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A}$$

resistances in series

The Composite Wall



A series composite wall separating two fluids

The 1-D heat transfer rate for the system may be expressed as

$$q = \frac{T_{\infty1} - T_{\infty2}}{\sum R}$$

$$q_x = \frac{T_{\infty 1} - T_{\infty 2}}{\dfrac{1}{h_{\infty 1} A} + \dfrac{L_A}{K_A A} + \dfrac{L_B}{K_B A} + \dfrac{L_C}{K_C A} + \dfrac{1}{h_{\infty 2} A}}$$

Alternatively, q_x can be related to the temperature difference and resistance associated with each element :

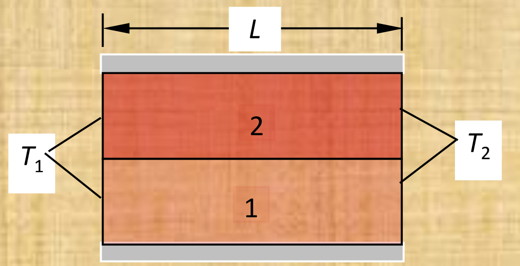
$$q_x = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1 A)} = \frac{T_{s,1} - T_2}{(L_A/k_A A)} = \frac{T_2 - T_3}{(L_B/k_B A)} = \ldots$$

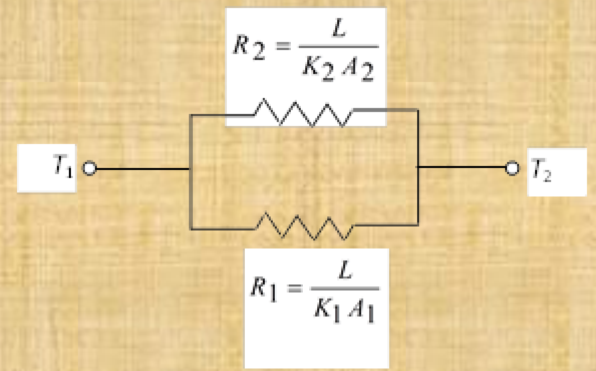
$$q_x = UA\Delta T$$

Where $$U = \frac{R_{tot}}{A} = \frac{1}{\dfrac{1}{h_{\infty 1}} + \dfrac{L_A}{K_A} + \dfrac{L_B}{K_B} + \dfrac{L_C}{K_C} + \dfrac{1}{h_{\infty 2}}}$$

In general, we may write $R_{tot} = \sum R_t = \dfrac{\Delta T}{q_x} = \dfrac{1}{U A}$

A parallel composite of two materials



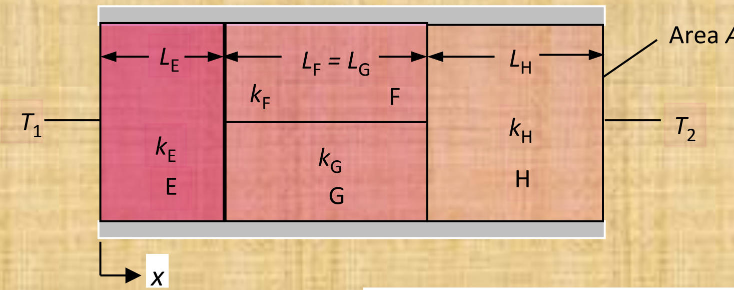


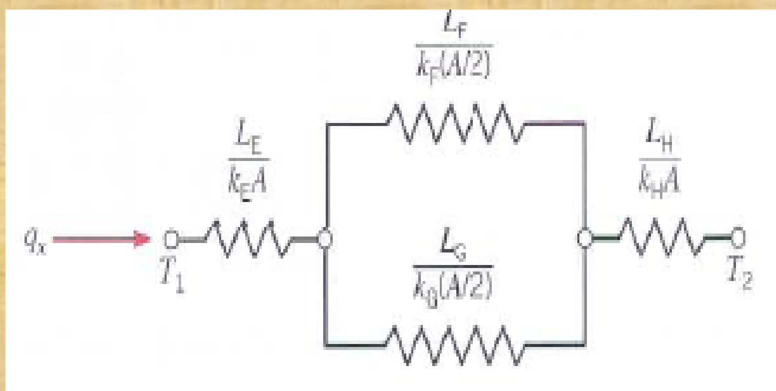
The heat transfer rate in the network is $q_x = \frac{T_1 - T_2}{R_{tot}}$ where $R_{tot} = \frac{1}{1/R_1 + 1/R_2}$

The heat transfer rate can be calculated as the sum of heat transfer rates in the individual materials:

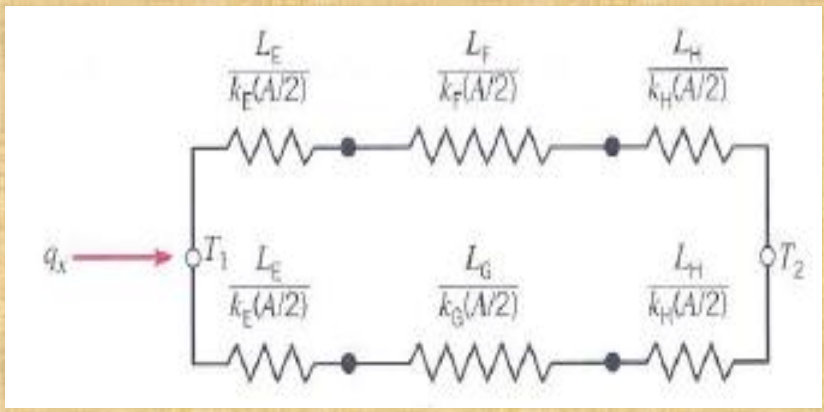
$$q_x = q_{1x} + q_{2x} = \frac{T_1 - T_2}{R_1} + \frac{T_1 - T_2}{R_2}$$

Series-parallel configurations





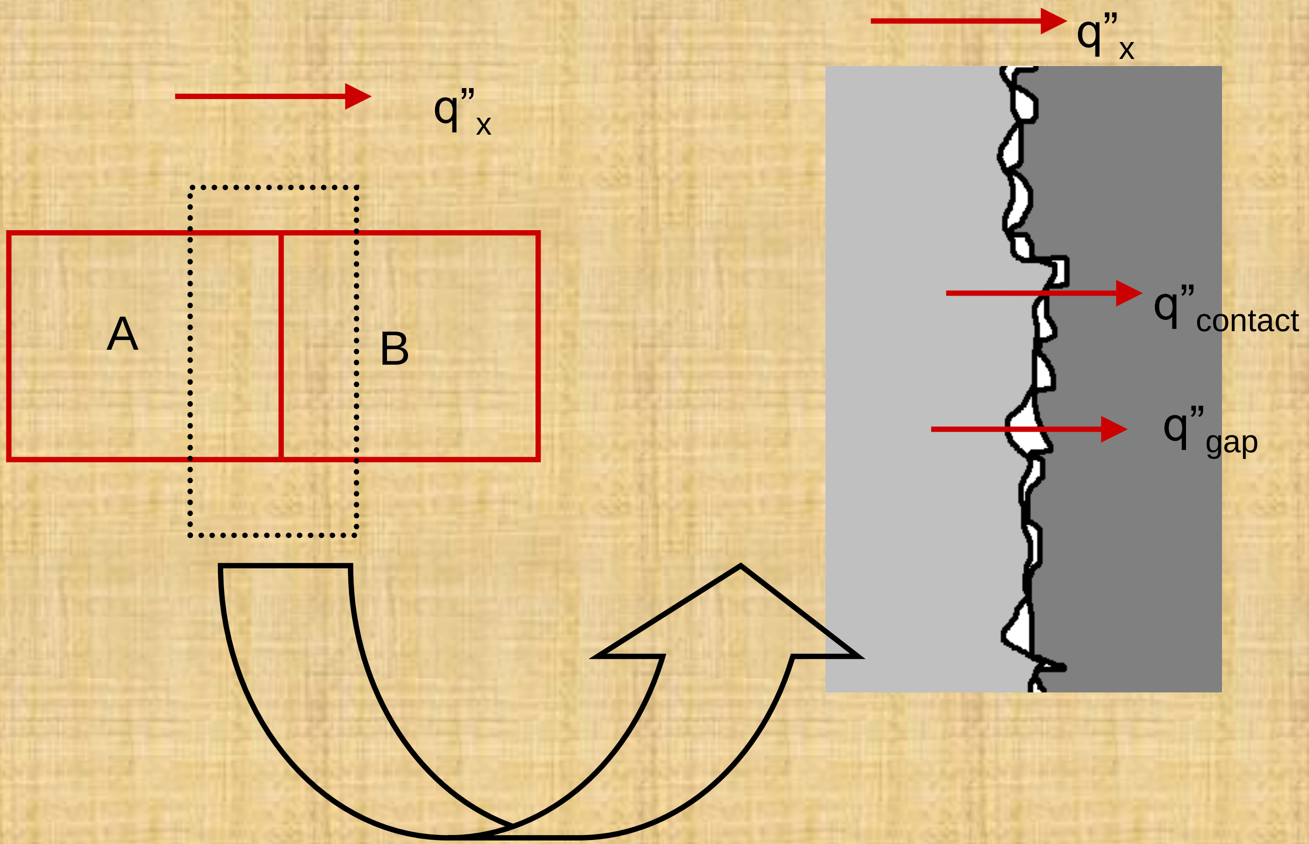
(a) Surfaces normal to the x direction are isothermal



(b) Surfaces parallel to the x direction are adiabatic

The actual value of q lies between the values obtained with circuits (a) and (b).

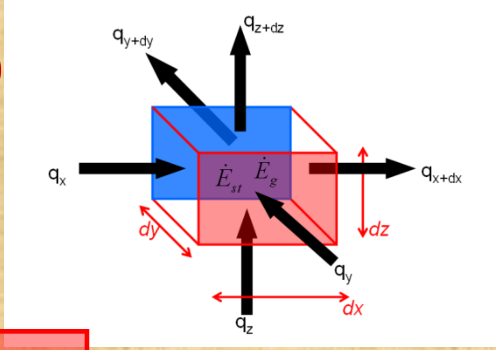
Contact resistance



Thermal contact resistance : $R''_{t,c} = \frac{T_A - T_B}{q''_x}$

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

0 0 0 0

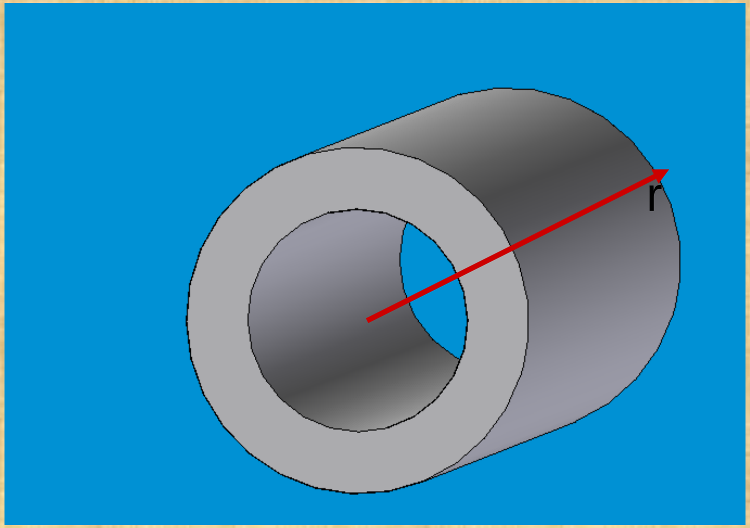


For a steady state one dimensional heat transfer and no energy generation

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)=0$$

Cylindrical coordinates

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2}+\frac{\partial^2 T}{\partial z^2}+\frac{\dot{q}}{k}=\frac{1}{\alpha}\frac{\partial T}{\partial t}$$



The heat equation for a steady state one dimensional heat transfer and no energy generation for a hollow cylinder

$$\frac{1}{r}\frac{d}{dr}\left(k\,r\frac{dT}{dr}\right)=0$$

Fourier's Law

We stated the phenomenologically found Fourier's law of conduction in one direction

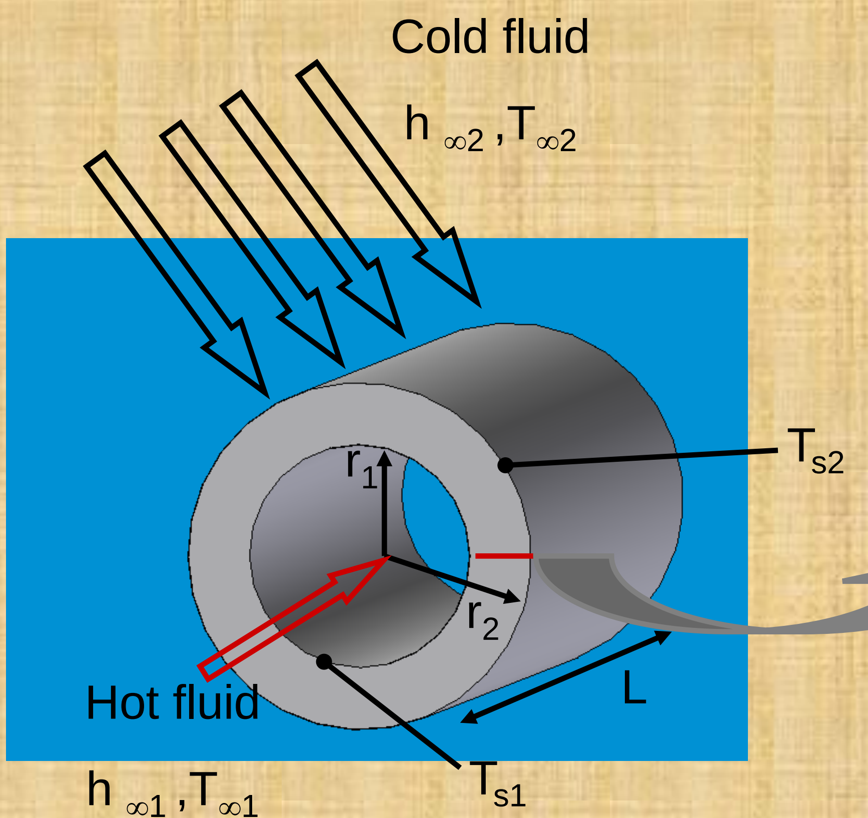
$$q_x = -kA\frac{dT}{dx}$$

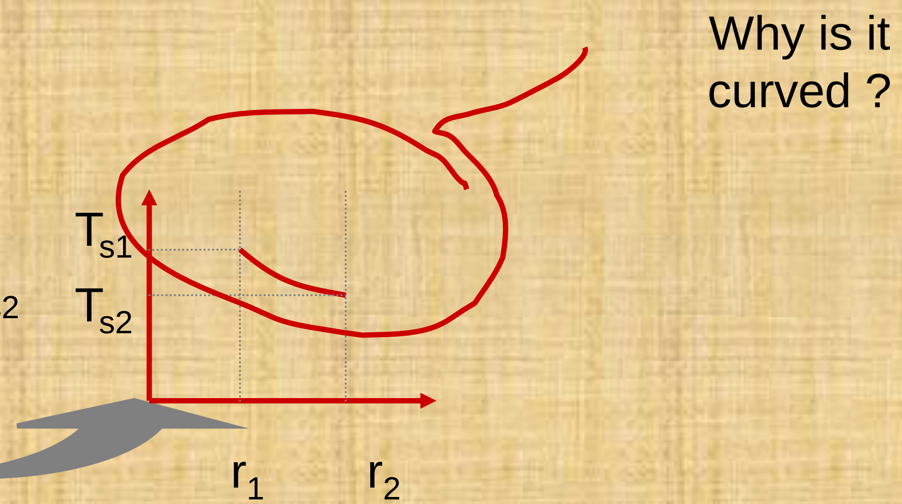
Fourier's law of conduction in one direction namely the radial direction

$$q_x = -kA\frac{dT}{dr} = -k\left(2\pi rL\right)\frac{dT}{dr}$$

Cylindrical heat transfer

Steady state conditions with no heat generation $\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)$





$$\frac{1}{h_1 2\pi r_1 L} \qquad \frac{\ln\left(r_2/r_1\right)}{2\pi Lk} \qquad \frac{1}{h_2 2\pi r_1 L}$$

We would like to solve for the radial temperature field

Assume the conduction coefficient is constant and integrate the heat equation twice

$$\int\int \frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0$$

$$T(r) = C_1 \ln r + C_2$$

Apply the boundary conditions

$T(r_1) = Ts_1$

$T(r_1) = Ts_2$

Which gives

$$Ts_1 = C_1 \ln r_1 + C_2$$
$$Ts_2 = C_1 \ln r_2 + C_2$$

Solving the two equations simultaneously gives C1 and C2 and substituting into the general solution gives

$$T(r) = \frac{Ts_1 - Ts_2}{\ln r_1 / r_2} \ln \frac{r}{r_2} + Ts_2$$

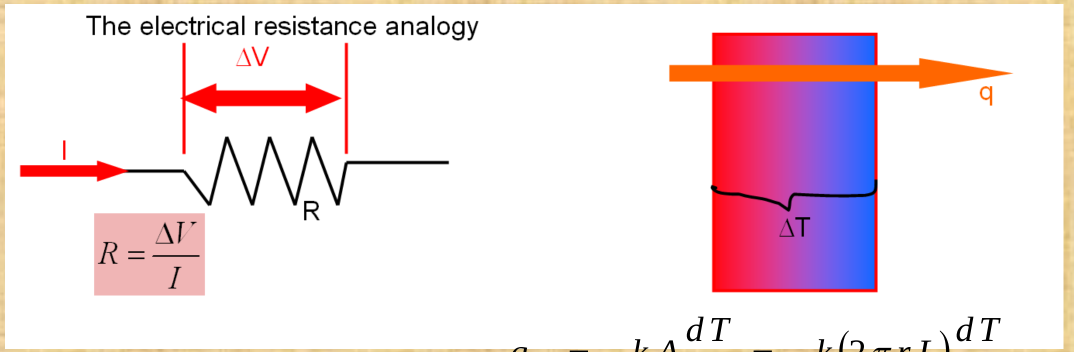
The wall temperature in the cylinder is logarithmic and not linear like the case for the plane wall under the same conditions

Take the derivative of T(r) wrt r and substitute dT/dr in Fourier's Law in cylindrical form

dT/dr

$$q_r = -k\left(2\pi r L\right)\frac{dT}{dr}$$

Which gives $$q_r = \frac{2\pi L k\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2 / r_1\right)}$$



$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr}$$

$$= \frac{2\pi Lk\left(T_{s,1} - T_{s,2}\right)}{\ln\left(r_2/r_1\right)}$$

Note that the heat rate is NOT a linear function of radius but a logarithmic function of the radius

Recall the electrical resistance analogy

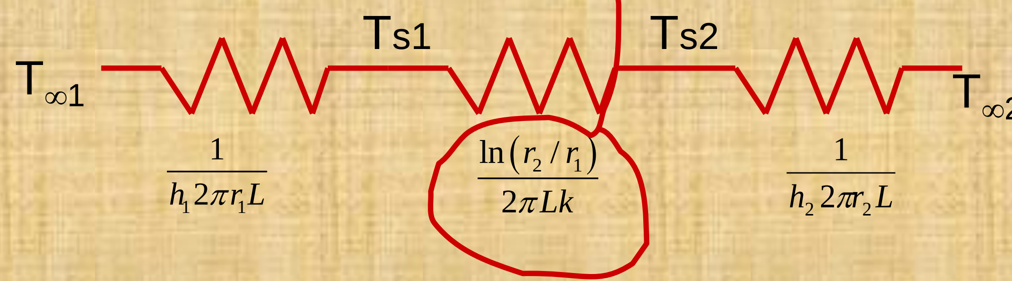
$$R_{t,cond} = \frac{T_{s1} - T_{s2}}{q_r}$$

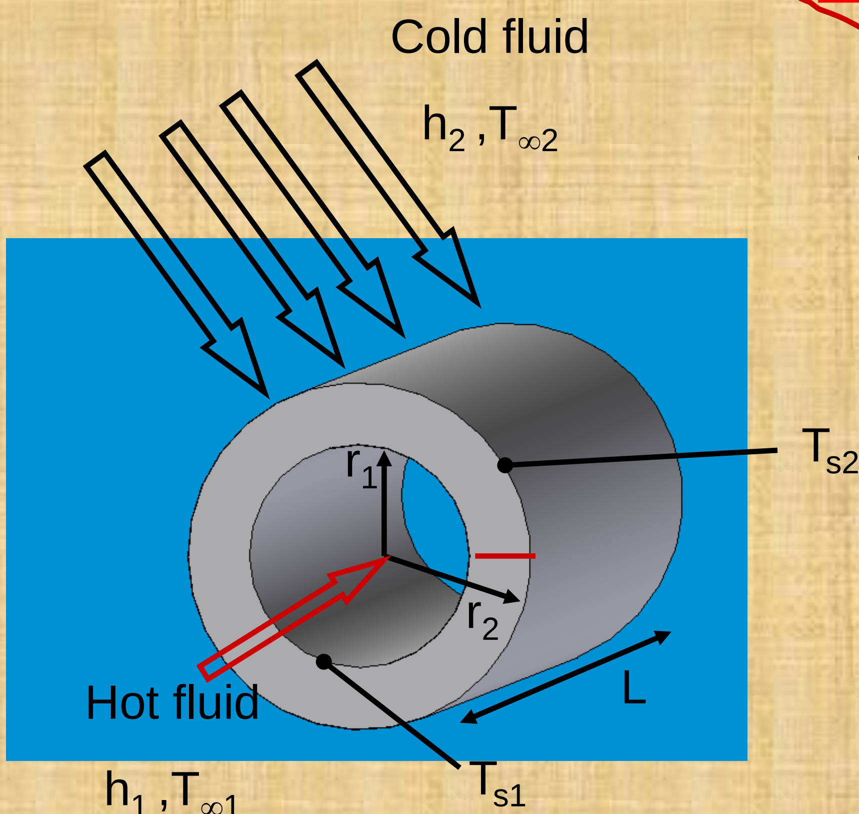
Which gives the conductivity resistance

$$R_{t,cond} = \frac{\ln\left(r_2/r_1\right)}{2\pi Lk}$$

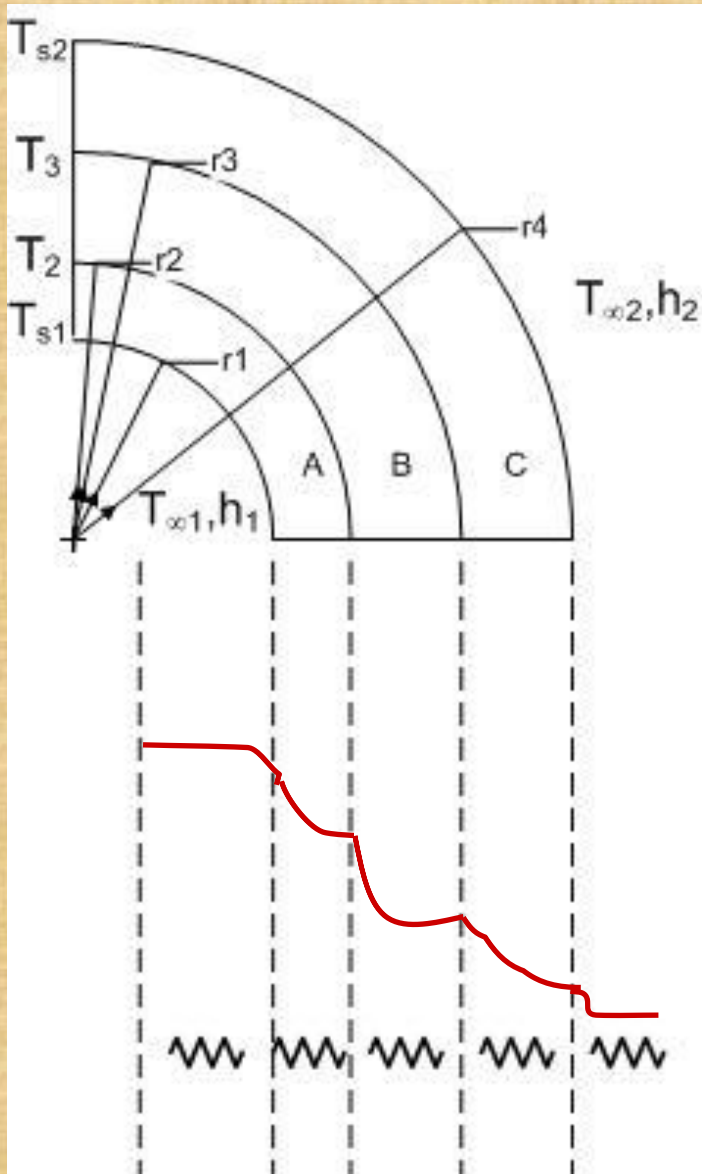
Cylindrical heat transfer

$$R_{t,cond} = \frac{\ln\left(r_2 / r_1\right)}{2\pi Lk}$$



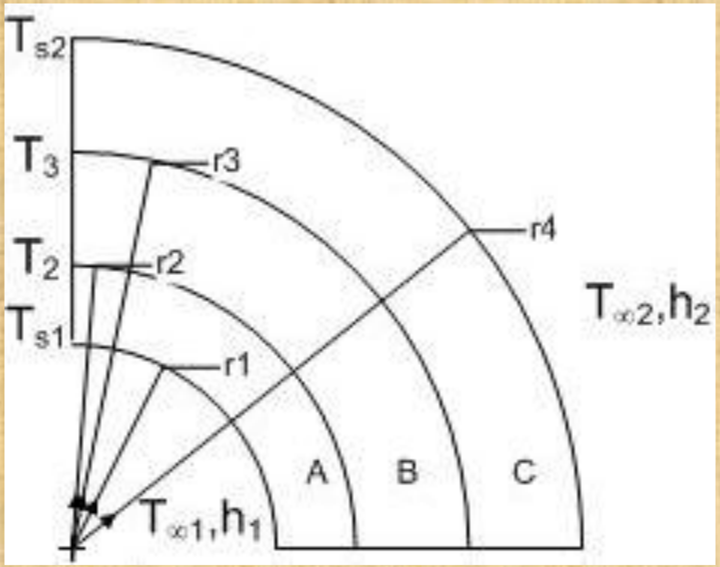


Composite cylindrical wall



$$\frac{1}{h_1 2\pi r_1 L} \quad \frac{\ln(r_2/r_1)}{2\pi k_A L} \quad \frac{\ln(r_3/r_2)}{2\pi k_B L} \quad \frac{\ln(r_4/r_3)}{2\pi k_C L} \quad \frac{1}{h_2 2\pi r_4 L}$$

The heat transfer rate



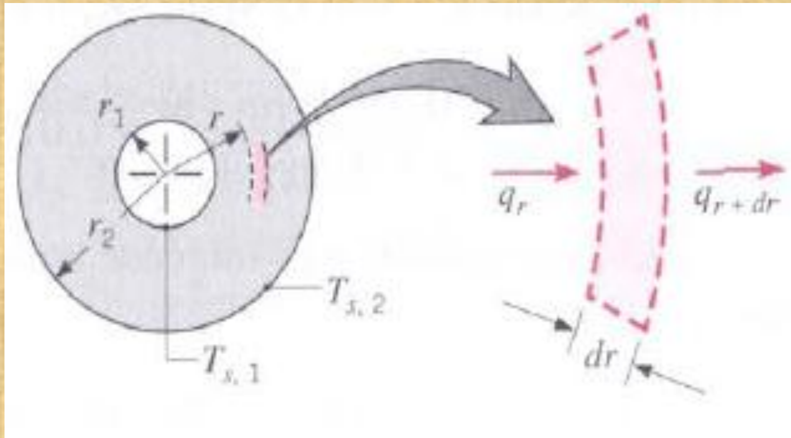
$$q_r = \frac{T_{\infty 1} - T_{\infty 2}}{\frac{1}{h_1 2\pi r_1 L} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{\ln(r_4/r_3)}{2\pi k_C L} + \frac{1}{h_2 2\pi r_4 L}}$$

Spherical heat transfer

The appropriate form of Fourier's law is

$$q_r = -kA\frac{dT}{dr} = -k\left(4\pi r^2\right)\frac{dT}{dr}$$

- The heat transfer rate is then (assuming constant *k*)



$$q_r = \frac{4\pi k\left(T_{s,1} - T_{s,2}\right)}{\left(1/r_1\right) - \left(1/r_2\right)}$$

The thermal resistance is

$$R_{t,cond} = \frac{\Delta T}{q_r} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Note: Spherical composites may be treated the same way as composite walls and cylinders.

Critical radius of insulation

- **For a plane wall exposed to a fluid, an increase in the thickness of the wall results in an increase in the conduction resistance R_{cond} = L/(kA) but does not change the convection resistance R_{conv}. Hence, the heat transfer rate will reduce as the wall thickness increases.**

- **For geometries with non-constant cross-sectional area (e.g. a cylinder, a sphere), increase in the wall thickness does not always bring about a decrease in the heat transfer rate.**

- **The critical radius of insulation for a cylinder exposed to convection is** $r_{cr} = \frac{k}{h}$

where *k* is the of thermal conductivity of the insulation material and *h* is the convection heat transfer coefficient on the insulation.

- **The critical radius of insulation for a sphere exposed to convection is** $r_{cr} = \frac{2k}{h}$

Wall with Heat generation

- We looked at a wall with no heat generation. Many cases require the consideration of a wall with heat generation.
- One such case is heat generation due to resistance.

The rate at which energy is generated by passing a current *I* through the resistance *R*

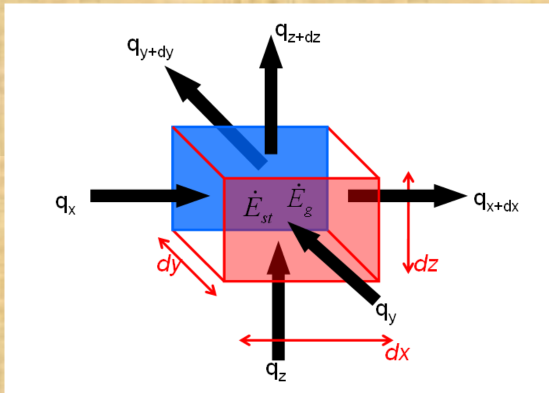
$$\dot{E}_g = I^2 R$$

If you assume the power generated is uniform In this case

$$\dot{q} = \frac{\dot{E}_g}{Volume} = \frac{I^2 R}{Volume}$$

Let us solve for the temperature field starting with the heat diffudion equation

Plane wall with uniform heat generation



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t}$$

0 0 0 ss

Assume conductivity is constant

Plane wall

Uniform means this term is constant

The heat rate equation simplifies to

$$\frac{d^2T}{dx}+\frac{\dot{q}}{k}=0$$

Integrate twice gives

$$T=\frac{-\dot{q}}{2k}x^2+C_1x+C_2$$

Plane wall with uniform heat generation

Solving for C1 and C2 depends on the boundary conditions

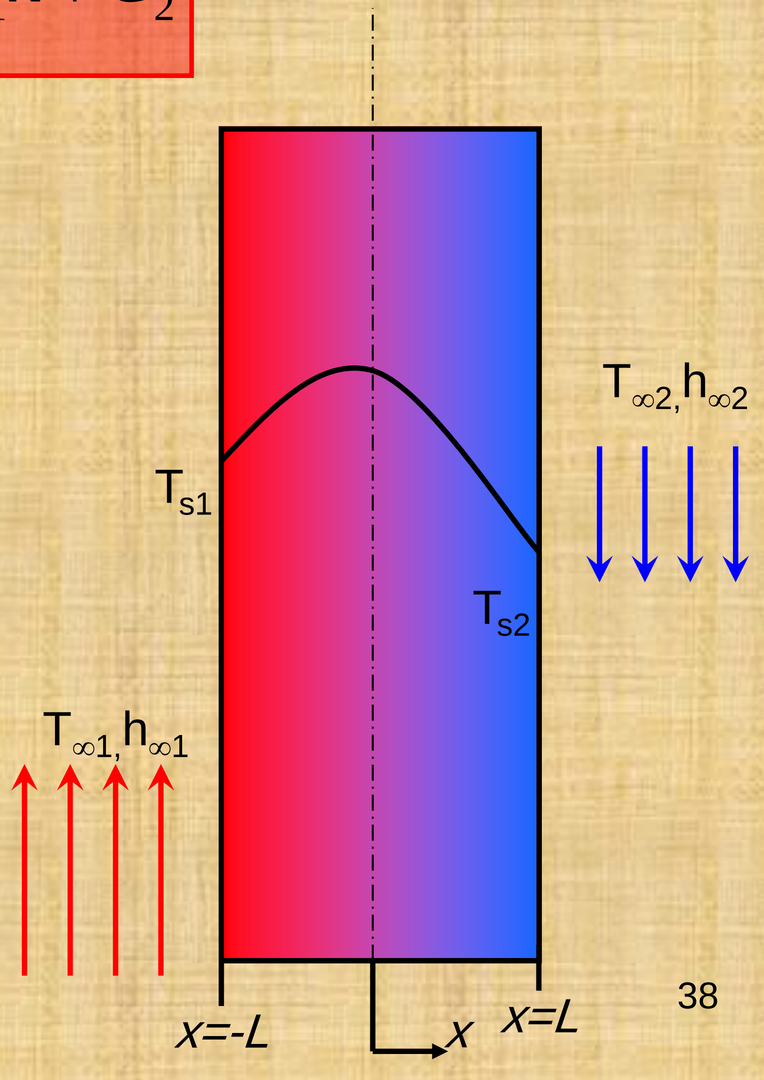
$$T = \frac{-\dot{q}}{2k}x^2 + C_1 x + C_2$$

Case 1

The boundary conditions are

$T(-L)=T_{s,1}$ and $T(L)=T_{s,2}$

This gives



Plane wall with uniform heat generation

$$T = \frac{-\dot{q}}{2k}x^2 + C_1 x + C_2$$

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L}$$

$$C_2 = \frac{\dot{q}L^2}{2k} + \frac{T_{s,1} + T_{s,2}}{2}$$

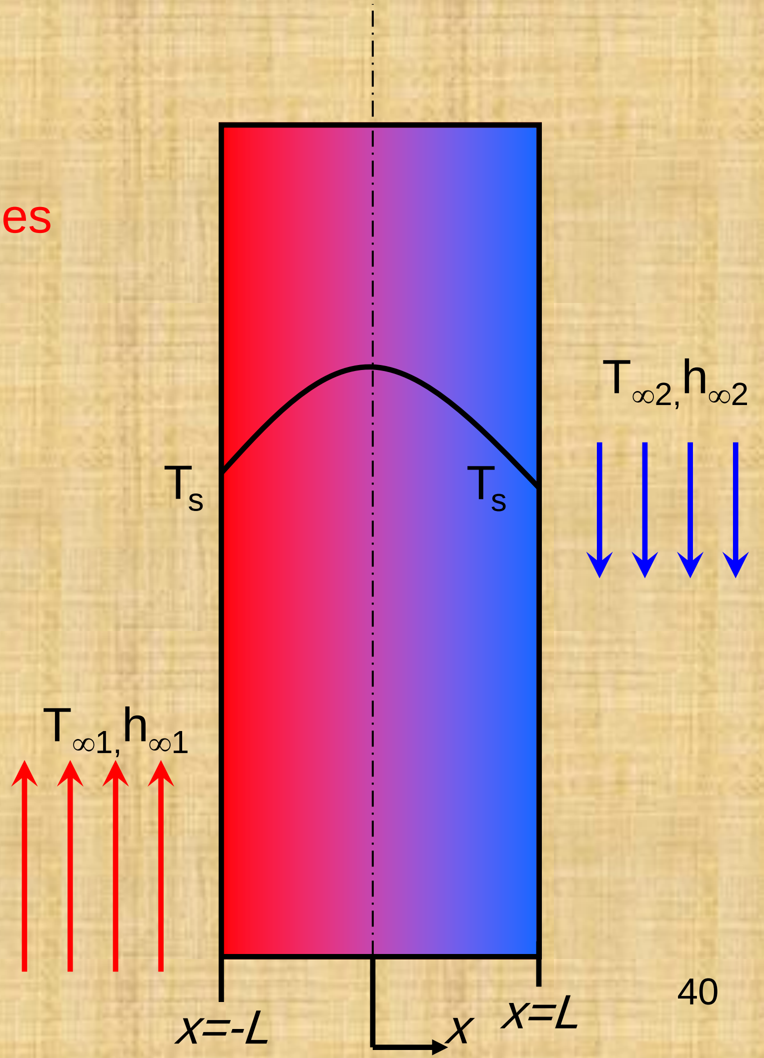
$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_{s,2} - T_{s,1}}{2}\right)\frac{x}{L} + \frac{T_{s,2} + T_{s,1}}{2}$$

Solving for C1 and C2 depends on the boundary conditions

$$T = \frac{-\dot{q}}{2k}x^2 + C_1 x + C_2$$

Case 2

Plane wall with uniform heat generation, both sides maintained at the same temperature



The boundary conditions are

$T(-L)=T_s$ and $T(L)=T_s$

This gives a symmetrical temperature distribution

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$$

- The maximum temperature for this case is at the center and is given by

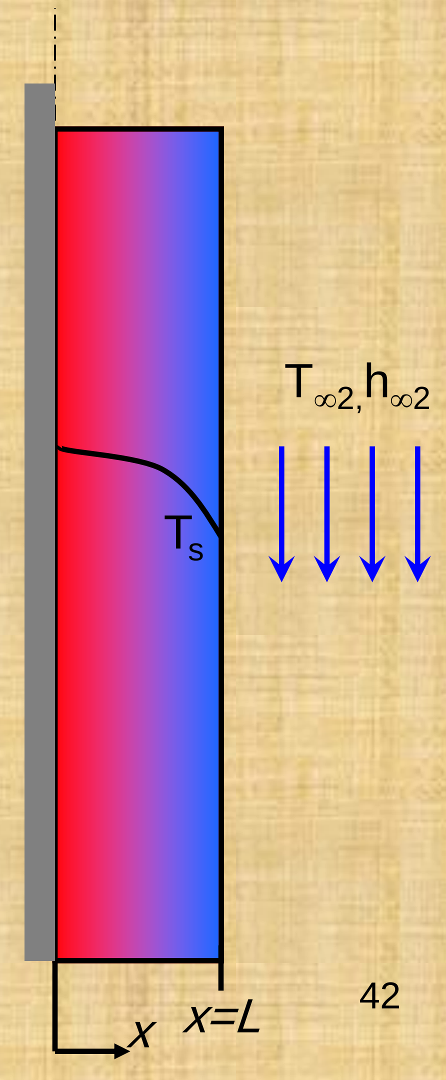
$$T(0) = \frac{\dot{q}L^2}{2k} + T_s$$

- The temperature gradient at this location is

$$\frac{d}{dx}\big(T(x)\big) = 0$$

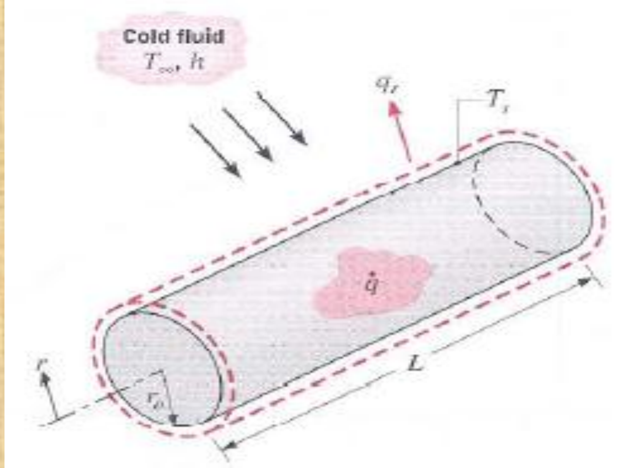
- Which means that no heat crosses the mid-plane

The problem may be represented with an adiabatic mid-plane



Radial Systems with uniform heat generation

❖Cylindrical system



Heat diffusion equation: $$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0$$

Boundary Conditions: $\left.\frac{dT}{dr}\right|_{r=0}=0$ and $T(r_o)=T_s$

Temperature distribution: $$T(r)=\frac{\dot{q}r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right)+T_s$$

centerline temperature : $$T(0)=T_o=\frac{\dot{q}r_o^2}{4k}+T_s$$

$$\frac{T(r)-T_s}{T_o-T_s}=1-\left(\frac{r}{r_o}\right)^2$$

Note: To relate T_s to T_∞, apply an overall energy balance on the cylinder to obtain:

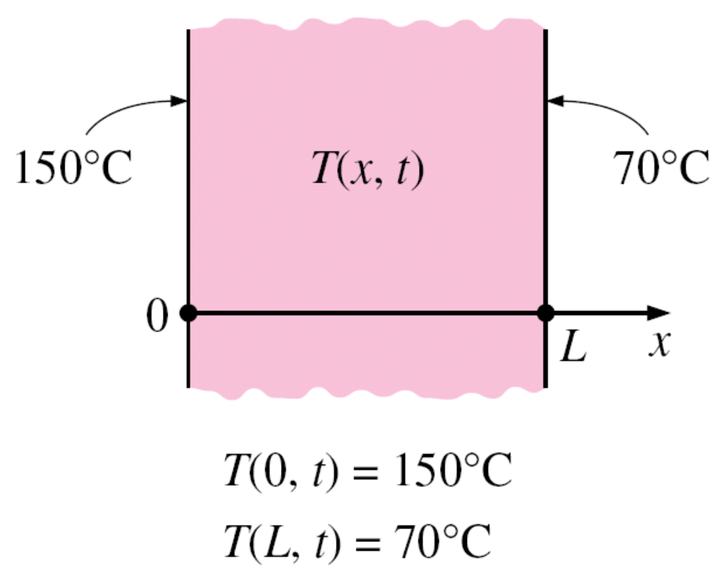
$$\dot{q}\left(\pi r_o^2 L\right)=h\left(2\pi r_o L\right)\left(T_s-T_\infty\right)$$

Boundary and Initial Conditions

- Specified Temperature Boundary Condition
- Specified Heat Flux Boundary Condition
- Convection Boundary Condition
- Radiation Boundary Condition
- Interface Boundary Conditions
- Generalized Boundary Conditions

Specified Temperature Boundary Condition

For one-dimensional heat transfer through a plane wall of thickness *L*, for example, the specified temperature boundary conditions can be expressed as

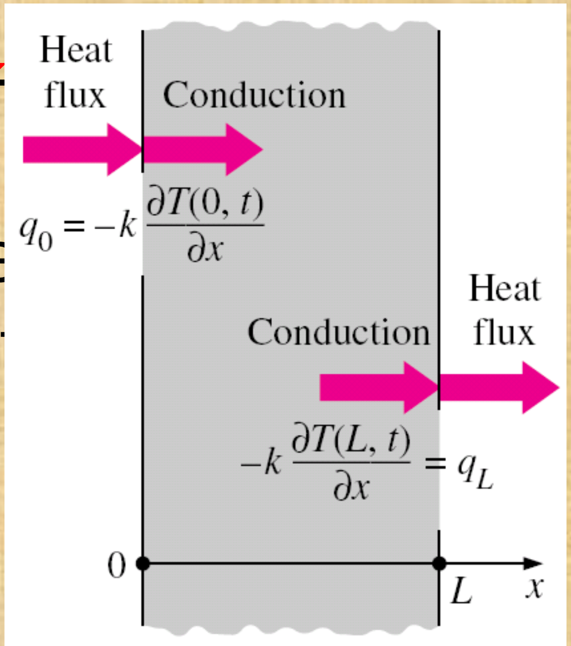


$$T(0, t) = T_1$$
$$T(L, t) = T_2$$

The specified temperatures can be constant, which is the case for steady heat conduction, or may vary with time.

Specified Heat Flux Boundary Condition

The heat flux in the positive *x*-direction anywhere in the medium, including the boundaries, can be expressed by *Fourier's law* of hea[t] conduction as

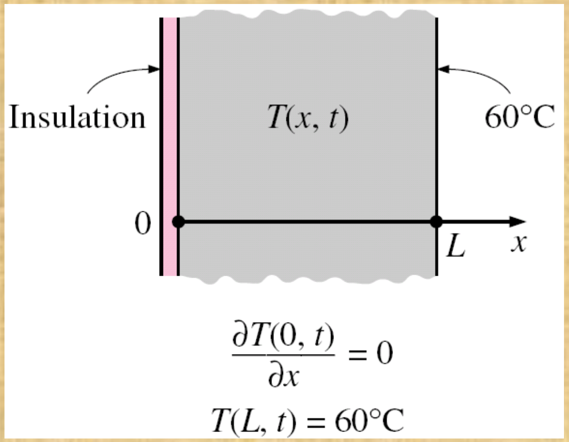


$$\dot{q} = -k\frac{dT}{dx} = \left(\text{Heat flux in the positive x-direction}\right)$$

The sign of the specified heat flux is determined by inspection: *positive* if the heat flux is in the positive direction of the coordinate axis, and *negative* if it is in the opposite direction.

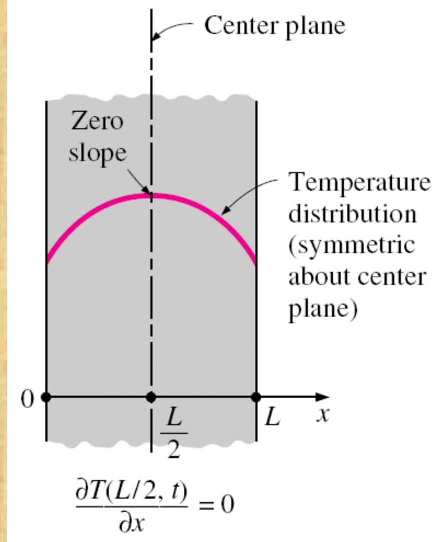
Two Special Cases

Insulated boundary



$$k\frac{\partial T(0,t)}{\partial x}=0 \quad \text{or} \quad \frac{\partial T(0,t)}{\partial x}=0$$

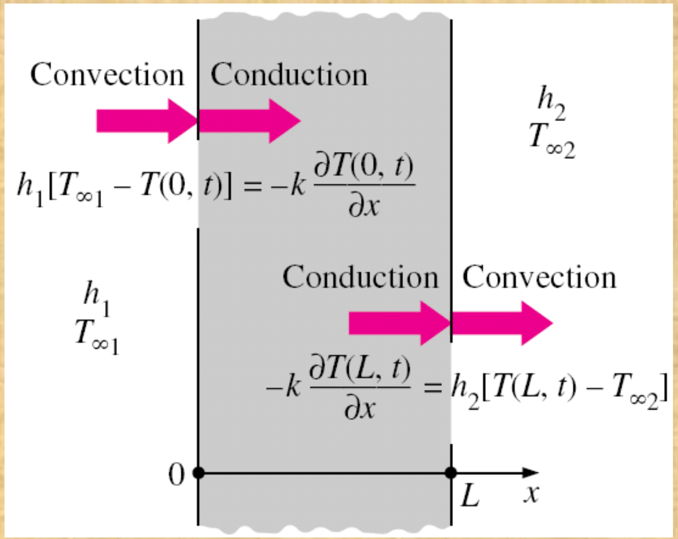
Thermal symmetry



$$\frac{\partial T\left(L/2,t\right)}{\partial x}=0$$

Convection Boundary Condition

(Heat conduction at the surface in a selected direction) **=** (Heat convection at the surface in the same direction)



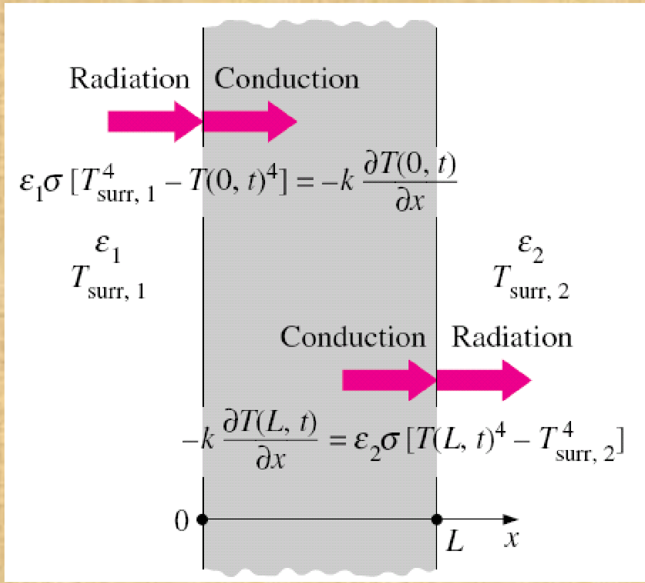
$$-k\frac{\partial T(0,t)}{\partial x} = h_1\left[T_{\infty 1} - T(0,t)\right]$$

and

$$-k\frac{\partial T(L,t)}{\partial x} = h_2\left[T(L,t) - T_{\infty 2}\right]$$

Radiation Boundary Condition

(Heat conduction at the surface in a selected direction) = (Radiation exchange at the surface in the same direction)



$$-k\frac{\partial T(0,t)}{\partial x} = \varepsilon_1\sigma\left[T^4_{surr,1} - T(0,t)^4\right]$$

and

$$-k\frac{\partial T(L,t)}{\partial x} = \varepsilon_2\sigma\left[T(L,t)^4 - T^4_{surr,2}\right]$$

Interface Boundary Conditions

At the interface the requirements are:

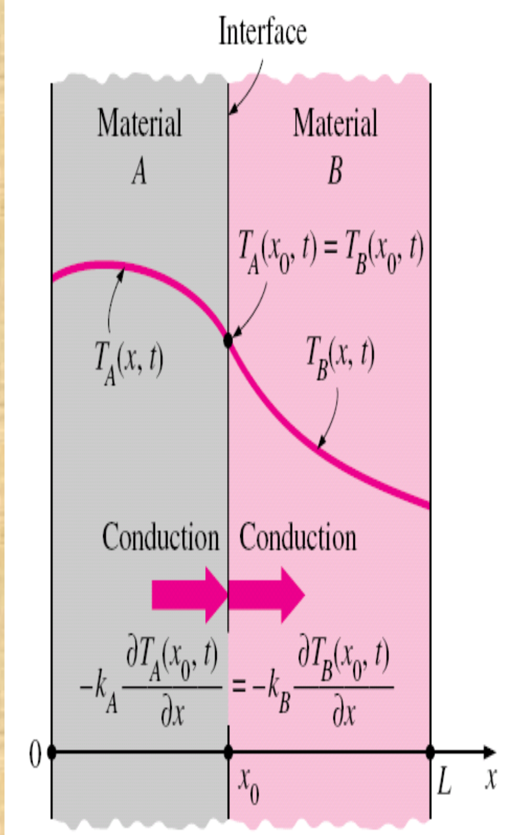
(1) two bodies in contact must have the *same temperature* at the area of contact,

(2) an interface (which is a surface) cannot store any energy, and thus the *heat flux* on the two sides of an interface *must be the same*.

$$T_A(x_0, t) = T_B(x_0, t)$$

and

$$-k_A \frac{\partial T_A(x_0,t)}{\partial x} = -k_B \frac{\partial T_B(x_0,t)}{\partial x}$$



Generalized Boundary Conditions

In general, a surface may involve convection, radiation, *and* specified heat flux simultaneously. The boundary condition in such cases is again obtained from a surface energy balance, expressed as

$$\left(\begin{array}{c} \textbf{Heat transfer} \\ \textbf{to the surface} \\ \textbf{in all modes} \end{array} \right) = \left(\begin{array}{c} \textbf{Heat transfer} \\ \textbf{from the surface} \\ \textbf{In all modes} \end{array} \right)$$

Heat Generation in Solids

The quantities of major interest in a medium with heat generation are the surface temperature T_s and the maximum temperature $T_{\max}$ that occurs in the medium in *steady* operation.

Heat Generation in Solids -The Surface Temperature

(Rate of heat transfer from the solid) = (Rate of energy generation within the solid)

For *uniform* heat generation within the medium

$$\dot{Q} = \dot{e}_{gen} V \qquad (\text{W})$$

The heat transfer rate by convection can also be expressed from Newton's law of cooling as

$$\dot{Q} = hA_s \left(T_s - T_\infty\right) \qquad (\text{W})$$

$$T_s = T_\infty + \frac{\dot{e}_{gen} V}{hA_s}$$

Heat Generation in Solids -The Surface Temperature

For a large *plane wall* of thickness 2L (A_s=2A_{wall} and V=2LA_{wall})

$$T_{s,\,plane\ wall} = T_\infty + \frac{\dot{e}_{gen} L}{h}$$

For a long solid *cylinder* of radius r_0 (A_s=2$\pi r_0 L$ and V=$\pi r_0^2 L$)

$$T_{s,cylinder} = T_\infty + \frac{\dot{e}_{gen} r_0}{2h}$$

For a solid *sphere* of radius r_0 (A_s=4πr_0^2 and V=$^4/_3\pi r_0^3$)

$$T_{s,sphere} = T_\infty + \frac{\dot{e}_{gen} r_0}{3h}$$