# Course Specifications

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|---|---|
| **Course Title:** | **Measure Theory and Integration** |
| **Course Code:** | **23044315-3** |
| **Program:** | **Bachelor of Mathematics** |
| **Department:** | **Mathematics Department** |
| **College:** | **Jamoum University College** |
| **Institution:** | **Umm Al-Qura University** |

## Table of Contents

## A. Course Identification

| |
|---|
| **1. Credit hours: 4 credit hours** |
| **2. Course type**<br>**a.** University ☐ College ☐ Department ☑ Others ☐<br>**b.** Required ☑ Elective ☐ |
| **3. Level/year at which this course is offered: 8th level** |
| **4. Pre-requisites for this course** (if any)**:** Real Analysis (2) |
| **5. Co-requisites for this course** (if any)**:** |

**6. Mode of Instruction** (mark all that apply)

| No | Mode of Instruction | Contact Hours | Percentage |
|---|---|---|---|
| 1 | **Traditional classroom** | 45 | 100 |
| 2 | **Blended** | | |
| 3 | **E-learning** | | |
| 4 | **Correspondence** | | |
| 5 | **Other** | | |

**7. Actual Learning Hours** (based on academic semester)

| No | Activity | Learning Hours |
|---|---|---|
| **Contact Hours** | | |
| 1 | **Lecture** | 45 |
| 2 | **Laboratory/Studio** | |
| 3 | **Tutorial** | |
| 4 | **Others** (specify) | |
| | **Total** | 45 |
| **Other Learning Hours*** | | |
| 1 | **Study** | 60 |
| 2 | **Assignments** | 10 |
| 3 | **Library** | 0 |
| 4 | **Projects/Research Essays/Theses** | 0 |
| 5 | **Others** (specify) | 0 |
| | **Total** | 70 |

* The length of time that a learner takes to complete learning activities that lead to achievement of course learning outcomes, such as study time, homework assignments, projects, preparing presentations, library times

## B. Course Objectives and Learning Outcomes

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|---|
| **1. Course Description** |
| **2. Course Main Objective**<br>The course introduces the Lebesgue integral and develops the elements of measure theory. We give the notions of special set systems, rings and algebras of sets, generated set systems. Borel sets, additive and sigma-additive set functions, outer measure and measure, extension and |

completion of measure, construction of Lebesgue and Lebesgue-Stieltjes measure, measurable functions and their properties, simple functions, construction of Lebesgue integral and its properties, absolute convergence of integral, integrable functions, Lebesgue theorem on dominated convergence, Lebesgue-Stieltjes integral, Convergence theorems.

### 3. Course Learning Outcomes

| | CLOs | Aligned PLOs |
|---|---|---|
| 1 | **Knowledge:** | |
| 1.1 | After successful completion of the course, the student should be able to 1) Describe the Measure. 2) Determine the measurable functions. 3) Find out which functions can be integrated, and prove the main properties of the Lebesgue integral. 4) Apply and manipulate convergence theorem for the integrals. | |
| 1.2 | | |
| **2** | **Skills :** | |
| 2.1 | We motivate and introduce the notion of a measure, that is, a way to assign a size to as many subsets as possible in an abstract space and sigma algebra, sigma ring. 2) we can introduce and discuss so called measurable functions which, roughly speaking, form the class of functions which we will be able to integrate. 3) We then introduce and study integration of these measurable functions with respect to a measure. We discuss (among other things) the monotone and dominated convergence theorems concerning the interchangeability of limit and integral, the substitution rule, absolute continuity and the relation of this new integral to the Riemann integral. | |
| 2.2 | | |
| **3** | **Competence:** | |
| 3.1 | Punctual attendance of classes is required. 2) Students should demonstrate their sense of responsibility for learning by completing both reading and writing assignments in due time. 3) Students learn to manage their time. 4) Accustom students to take responsibility of self – learning 5) Students should act responsibly | |
| 3.2 | | |

## C. Course Content

| No | List of Topics | Contact Hours |
|---|---|---|
| 1 | Preliminaries on Set Theory and Topology: Basic concepts from set theory and topology. Rings, sigma-rings, algebras and sigma-algebras of sets. Generated set systems, connection between some types of generated systems, examples of rings and sigma-rings over intervals. Concept of set function and some types of set functions. Properties of additive functions defined on rings of sets. | 10 |
| 2 | Measure: Non-negative sigma-additive functions and their properties, measure, connection of measure with non-negative additive functions, examples of measures over intervals. Outer measure, measurability in | 12 |

| | | |
|---|---|---|
| | sense of Carathéodory. Extension and completion of measure, m*-measurability and completeness, system of m*-measurable sets with respect to induced measure. Existence and uniqueness of extension of a measure. Lebesgue measure | |
| 3 | Measurable functions: Measurable space, simple measurable functions, measurable functions and criteria of measurability of functions. Further properties of measurable functions, sequences of measurable functions, Borel and Lebesgue measurability. | 12 |
| 4 | Integral: Measure space, integral of simple functions. Integral of a non-negative measurable function, definition of integral of a measurable function and its properties. Integral and limit of a sequence of functions, integral as a set function, Lebesgue and Lebesgue-Stieltjes integral, convergence theorems. | 11 |
| **Total** | | 45 |

## D. Teaching and Assessment

### 1. Alignment of Course Learning Outcomes with Teaching Strategies and Assessment Methods

| Code | Course Learning Outcomes | Teaching Strategies | Assessment Methods |
|---|---|---|---|
| **1.0** | **Knowledge** | | |
| 1.1 | After successful completion of the course, the student should be able to 1) Describe the Measure. 2) Determine the measurable functions. 3) Find out which functions can be integrated, and prove the main properties of the Lebesgue integral. 4) Apply and manipulate convergence theorem for the integrals. | Lectures, Tutorials, exams | Written Exams |
| 1.2 | | | |
| 1.3 | | | |
| **2.0** | **Skills** | | |
| 2.1 | We motivate and introduce the notion of a measure, that is, a way to assign a size to as many subsets as possible in an abstract space and sigma algebra, sigma ring. 2) we can introduce and discuss so called measurable functions which, roughly speaking, form the class of functions which we will be able to integrate. 3) We then introduce and study integration of these measurable functions with respect to a measure. We discuss (among other things) the monotone and dominated convergence theorems concerning the interchangeability | Lectures and Tutorials | Written Exams |

| Code | Course Learning Outcomes | Teaching Strategies | Assessment Methods |
|---|---|---|---|
| | of limit and integral, the substitution rule, absolute continuity and the relation of this new integral to the Riemann integral. | | |
| 2.2 | | | |
| 2.3 | | | |
| **3.0** | **Competence** | | |
| 3.1 | Punctual attendance of classes is required. 2) Students should demonstrate their sense of responsibility for learning by completing both reading and writing assignments in due time. 3) Students learn to manage their time. 4) Accustom students to take responsibility of self –learning 5) Students should act responsibly | Working together Brainstorming: A Method of solving problems in which all members of a group suggest ideas and then discuss them. | Group study to do homework |

**2. Assessment Tasks for Students**

| # | Assessment task* | Week Due | Percentage of Total Assessment Score |
|---|---|---|---|
| 1 | Midterm Test (1) | 6th week | 20% |
| 2 | Midterm Test (2) | 12th week | 20% |
| 3 | Homework + Reports + Quizzes | During the semester | 10% |
| 4 | Final Examination | End of semester | 50% |

***Assessment task** (i.e., written test, oral test, oral presentation, group project, essay, etc.)

## E. Student Academic Counseling and Support

**Arrangements for availability of faculty and teaching staff for individual student consultations and academic advice :**

-Each group of students is assigned to a particular faculty where he or she will provide academic advising during specific academic hours. Each staff will provide at least one session/week.

-There will be an academic advisor how will be a responsible for helping the student by doing the general supervision.

- The people in the library will support the students during the time of the course.

## F. Learning Resources and Facilities

## 1.Learning Resources

| | |
|---|---|
| **Required Textbooks** | 1. An introduction to classical real analysis, (Karl, R. Stromberg)<br>2. Rudin, W.: Real and Complex Analysis, Third Edition, McGraw-Hill Book Company<br>(1987). |
| **Essential References Materials** | 1. A. Mukherjea and K. Pothoven, Real and Functional Analysis, Plenum Press, New York,<br>1978.<br>2. Stein, E. M. and Shakarchi, R. Real Analysis - measure theory, integration and Hilbert<br>spaces. (Princeton Lectures in Analysis III) Princeton University Press (2005). |
| **Electronic Materials** | http://ebookee.org/ |
| **Other Learning Materials** | Microsoft Excel |

## 2. Facilities Required

| Item | Resources |
|---|---|
| **Accommodation**<br>(Classrooms, laboratories, demonstration rooms/labs, etc.) | -The size of the room should be proportional to the number of students<br>- Provide enough seats for students.<br>- The number of student not exceed on 30 in the classroom<br>- Library |
| **Technology Resources**<br>(AV, data show, Smart Board, software, etc.) | -Hall is equipped with a computer.<br>- Provide overhead projectors and related items<br>-Smart board |
| **Other Resources**<br>(Specify, e.g. if specific laboratory equipment is required, list requirements or attach a list) | none |

## G. Course Quality Evaluation

| Evaluation Areas/Issues | Evaluators | Evaluation Methods |
|---|---|---|
| Student feedback through electronic facilities organized by the deanship of egistration and acceptance.<br>• Following completion of the prescribed course study in Pediatrics module, an evaluation should be conducted through the following: A student questionnaire feedback should be carried out on the | Faculty | Direct |
| Evaluation of the teachers by internal & external faculty members.<br>• Visiting to the classrooms.<br>• Mutual visits between colleagues and giving advices to each other after each lecture<br>• A staff questionnaire feedback about course | Faculty | Direct |
| | | |
| | | |

**Evaluation areas** (e.g., Effectiveness of teaching and assessment, Extent of achievement of course learning outcomes, Quality of learning resources, etc.)
**Evaluators** (Students, Faculty, Program Leaders, Peer Reviewer, Others (specify)
**Assessment Methods** (Direct, Indirect)

## H. Specification Approval Data

| | |
|---|---|
| **Council / Committee** | |
| **Reference No.** | |
| **Date** | |