### 3. Single phase fully wave controlled full wave Rectifiers

### The first type: Center-Tap

#### 1. With a Resistive Load

Figure 5 shows the basic arrangement of a single-phase, center-tap controlled rectifier with a resistive load. Phase control of both the positive and the negative halves of the AC supply is now possible, thus increasing the DC voltage and reducing the ripple compared to those of half-wave rectifiers.



Figure 5 Full-wave center-tap controlled rectifier circuit

During the positive half-cycle of the input .voltage, SCR<sub>1</sub> is forward-biased. If we apply the gate signal at a SCR<sub>1</sub> turns on. The output Voltage  $(V_o)$  follows the input voltage. The load current  $(i_o = v_o/R|)$  has the same waveform as the load voltage. At  $\pi$ , when the current through SCR<sub>1</sub> becomes zero, it turns off naturally. During the negative half-cycle; SCR<sub>2</sub> is forward biased. SCR<sub>2</sub> is fired at  $(\pi + \alpha)$ . The output voltage again follows the input voltage."The current through SCR<sub>2</sub> becomes zero at 2  $\pi$ ; and it turns off. SCR<sub>2</sub> is fired again at  $(2\pi+\alpha)$  and SCR<sub>2</sub> at  $(3\pi+\alpha)$ , and the cycle repeats. Figure 5 (b) shows the resulting voltage and current waveforms. The average value of the load voltage is twice that given by single-phase half wave rectifier. The output voltage and current are given by

$$V_{o(avg)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$
$$V_{o(avg)} = \frac{V_m}{\pi} . (1 + \cos \alpha)$$
$$I_{o(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_m}{\pi R} . (1 + \cos \alpha)$$

The rms values for the output voltage and current are given by:

$$V_{o(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (\sqrt{2}V\sin(\omega t))^2 d\omega t} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (V_m \sin(\omega t))^2 d\omega t}$$
$$V_{o(rms)} = \frac{V_m}{\sqrt{2}} \cdot \sqrt{(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi})}$$
$$I_{o(rms)} = \frac{I_m}{\sqrt{2}} \cdot \sqrt{(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi})}$$

The average and the rms values for SCR are given by:

$$I_{thav} = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{\sqrt{2}V}{R} \sin(\omega t) d\omega t = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m}{R} \sin(\omega t) d\omega t = \frac{I_{o(avg)}}{2}$$
$$I_{th(rms)} = \sqrt{\frac{1}{2\pi}} 2 \int_{\alpha}^{\pi} (\frac{\sqrt{2}V}{R} \sin(\omega t))^2 d\omega t = \sqrt{\frac{1}{2\pi}} \int_{\alpha}^{\pi} (\frac{V_m}{R} \sin(\omega t))^2 d\omega t = \frac{I_{o(rms)}}{\sqrt{2}}$$

The output current is given by:

$$P_o = I_{o(rms)}^2 R = \frac{V_{o(rms)}^2}{R}$$

The input (supply) power factor is given by:

$$PF = \frac{I_{orms}^2 R}{V_s I_{orms}} = \frac{I_{orms} R}{V_s}$$

### 2. with an Inductive (RL) Load:

Figure 6 shows the waveforms for voltage, and current, assuming a hilly inductive load *so* that the load current is continuous (i.e., present at all times). SCR<sub>1</sub> conducts for 180° from  $\alpha$  to ( $\pi$ + $\alpha$ ), and the load voltage follows the input voltage. At ( $\pi$ + $\alpha$ ); SCR<sub>2</sub> is fired. SCR<sub>1</sub> now turns off, since the supply voltage immediately appears across it and applies a reverse bias. SCR<sub>2</sub> now conducts for 180° from ( $\pi$ + $\alpha$ ) to ( $2\pi$ + $\alpha$ ) and supplies power to the load.

The average value of the load voltage is given by:

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} V_m \sin(\omega t) d(\omega t) = \frac{2V_m}{\pi} \cos \alpha$$

The output voltage is at its maximum when  $\alpha = 0^{\circ}$ , zero when  $\alpha = 90^{\circ}$ , and its negative maximum when  $\alpha = 180^{\circ}$ . The normalized average output voltage is

$$V_{n} = \frac{V_{o(avg.)}}{V_{do}} = \cos \alpha$$

Figure 6 Full wave rectifier center tap with a RL load.

The control chara*cteristic* (the plot of  $V_n$  as a function of  $\alpha$ ) is shown in Figure 7. The RMS output voltage is given by:

 $V_{o(RMS)} = V_{s(RMS)}$ 



The average value of output current is given by:

$$I_{o(av)} = \frac{V_{o(avg)}}{R} = \frac{2V_m \cos(\alpha)}{\pi R}$$

If the load circuit has an EMF the output average current is given by:

$$I_{o(av)} = \frac{V_{o(avg)} - E_b}{R}$$

The average and rms values of the thyristor current are given by:

$$I_{th(av)} = \int_{\alpha}^{\pi+\alpha} I_{o(av)} d\omega t = \frac{I_{o(av)}}{2}$$

$$I_{th(rms)} = \sqrt{\int_{\alpha}^{\pi+\alpha} (I_{o(av)})^2 d\omega t} = \frac{I_{o(av)}}{\sqrt{2}}$$

The ram's value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi}} 2 \int_{\alpha}^{\pi+\alpha} I_{o(av)}^2 d\omega t = I_{o(av)}$$

The average value of the supply current equal zero because its positive part area equal to its negative part. But the average for half cycle is equal to the average of load current.

The output power is given by:

$$P_o = I_{o(av)} V_{o(av)} = I_{o(rms)}^2 R$$

If the load circuit has an EMF the output average Power is given by:

$$P_o = I_{o(av)} V_{o(av)} = I_{o(rms)}^2 R + E I_{0(av)}$$

The input power factor is given by:

$$PF = \frac{P_o}{S_{in}} = \frac{I_{o(av)}V_{o(av)}}{V_s I_{s(rms)}}$$

### 3. with an Inductive (RL) Load and with Freewheeling Diode:

A freewheeling diode connected across *the* inductive load (as shown in Figure 8) modifies the voltage and current waveforms of Figure 6. As the load voltage tends to go negative, the FWD becomes forward-biased and

starts conducting. Thus, the load voltage is clamped to zero volts. A nearly constant load current is maintained by the freewheeling current through the diode. The *aver*age load voltage and load voltage are given by:



Figure 8 Center-tap rectifier with RL load and FWD (a) circuit (b) voltage and current waveforms.

The freewheeling diode carries the load current during the delay period  $\alpha$  when the SCRs are off. Therefore, the average and rms values of current through the FWD (D) are given by:

$$I_{D(avg)} = \frac{1}{2\pi} 2\int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\alpha}{\pi} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \frac{\alpha}{\pi}$$
$$I_{D(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\alpha}{\pi}}$$

The average and rms values of the current through the thyristor are given by:

$$I_{th(avg)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{2\pi} = \frac{V_m (1 + \cos(\alpha)) (\pi - \alpha)}{\pi R}$$
$$I_{th(rms)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\pi - \alpha}{2\pi}} = \frac{V_m (1 + \cos(\alpha)) (\pi - \alpha)}{\pi R} \sqrt{\frac{\pi - \alpha}{2\pi}}$$

The rms value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\pi - \alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\pi - \alpha}{\pi}}$$

The average value of the supply current equal to zero for complete cycle, but for half cycle is given by:

$$I_{s(av)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{\pi} = \frac{V_m \cos(\alpha)}{\pi R} \frac{\pi - \alpha}{\pi}$$

### The second type: bridge circuit

#### 1. With a Resistive Load

Figure 9 shows a full-wave controlled bridge rectifier circuit with a resistive load. In this circuit, diagonally opposite pairs of SCRs turn on and off together. The circuit operation is similar that of the full-wave center-tap circuit discussed in Section 1. The average DC output voltage can be controlled from zero to its maximum value by varying the firing angle. The SCRs are controlled and fire in pairs with a delay angle of  $\alpha$ . The current and voltage waveform become full-wave, as shown in figure 10.



Figure 9 Full wave bridge rectifier circuit with resistive load.

The average value of the output DC voltage is given by:

$$V_{o(avg)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$
$$V_{o(avg)} = \frac{V_m}{\pi} . (1 + \cos \alpha)$$
$$I_{o(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_m}{\pi R} . (1 + \cos \alpha)$$

The rms values for the output voltage and current are given by:

$$V_{o(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (\sqrt{2}V\sin(\omega t))^2 d\omega t} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (V_m \sin(\omega t))^2 d\omega t}$$
$$V_{o(rms)} = \frac{V_m}{\sqrt{2}} \cdot \sqrt{(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi})}$$
$$I_{o(rms)} = \frac{I_m}{\sqrt{2}} \cdot \sqrt{(1 - \frac{\alpha}{\pi} + \frac{\sin 2\alpha}{2\pi})}$$

The average and the rms values for SCR are given by:

$$I_{thav} = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{\sqrt{2V}}{R} \sin(\omega t) d\omega t = \frac{1}{2\pi} \int_{\alpha}^{\pi} \frac{V_m}{R} \sin(\omega t) d\omega t = \frac{I_{o(avg)}}{2}$$
$$I_{th(rms)} = \sqrt{\frac{1}{2\pi} 2 \int_{\alpha}^{\pi} (\frac{\sqrt{2V}}{R} \sin(\omega t))^2 d\omega t} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} (\frac{V_m}{R} \sin(\omega t))^2 d\omega t} = \frac{I_{o(rms)}}{\sqrt{2}}$$

The output power is given by:

$$P_o = I_{o(rms)}^2 R = \frac{V_{o(rms)}^2}{R}$$

The input (supply) power factor is given by:

-

$$PF = \frac{I_{orms}^2 R}{V_s I_{orms}} = \frac{I_{orms} R}{V_s}$$



Figure 10 Waveforms of the bridge rectifier with a resistive load.

### 2. with an Inductive (RL) Load:

Figure 11 shows the bridge rectifier with the addition of an inductive load. The load current tends to keep flowing since the inductor induces a voltage that acts to oppose an increase or decrease in current. Therefore, SCR keeps conducing even though the voltage may have fallen to zero. The current maintains conduction in the SCR even after the voltage across the SCR has reversed.

When the inductance is small or the delay angle  $\alpha$  is kept large, the DC output current reaches zero every half cycle at  $(\pi + \beta)$ , as shown in Figure 12. During this period, neither pair of SCRs is on, nor therefore is the current said to be *discontinuous*.

The average values of the output voltage is



Figure 11 Bridge rectifier with a RL load.



Figure 12 Waveform of bridge rectifier with small inductive load.

If the load inductance is assumed to be large or  $\alpha$  becomes small, the load current cannot reach zero and it flows continuously, as shown in Figure 13. Therefore, one pair of SCRs is conducting at all times. The current is said to be *continuous*.

During the positive half –cycle, SCR<sub>1</sub> and SCR<sub>2</sub> conduct. Applying KVL around the loop containing, V<sub>s</sub>, SCR1 and SCR<sub>2</sub> at an instant when  $i_s > 0$ (between  $\alpha$  and  $(\pi$ - $\alpha$ )) gives

 $V_s = V_{SCR1} - V_{SCR2}$ 

Hence,  $V_{SCR1}=0$  since SCR<sub>1</sub> is conducting, therefore  $V_{SCR2} = -v_s$ , which means that SCR<sub>2</sub> is reverse-biased. Applying KVL around the i m p containing  $V_s$ , SCR1, the load, and SCR<sub>4</sub> gives

 $V_s = V_{SCR1} + v_o + V_{SCR4}$ 

Again SCR<sub>1</sub> and SCR<sub>4</sub> are conducting and have zero voltage across them. Therefore,  $v_0 = v_s$  (during the interval from  $\alpha$  to  $\pi$ ).

During the negative half-cycle, the source voltage  $v_s \leq 0$ . The preceding equations do not change form, although some of the quantities change sign. Now, since  $v_s$  is negative, SCR<sub>2</sub> and SCR3 me forward-biased and will turn on when they receive a gate signal. Load current still flows in the same path through SCR<sub>1</sub> and SCR<sub>4</sub> until SCR<sub>2</sub> and SCR<sub>3</sub> are triggered. Therefore, from  $\pi$  to ( $\pi$ + $\alpha$ ), the load voltage is negative since  $v_s \leq 0$ . At ( $\pi$ + $\alpha$ ), SCR<sub>2</sub> and SCR<sub>3</sub> are triggered, which supplies a voltage  $v_o = -v_s$  to the load.

The average value of this output voltage varies with  $\alpha$ :

$$V_o = \frac{1}{\pi} \int_{\alpha}^{\pi + \alpha} V_m \sin(\omega t) d(\omega t) = \frac{2V_m}{\pi} \cos \alpha$$

The average value of output current is given by:

$$I_{o(av)} = \frac{V_{o(avg)}}{R} = \frac{2V_m \cos(\alpha)}{\pi R}$$



*Figure 13 Waveform of bridge rectifier with L* >>>>*R*.

If the load circuit has an EMF the output average current is given by:

$$I_{o(av)} = \frac{V_{o(avg)} - E_b}{R}$$

The average and rms values of the thyristor current are given by:

$$I_{th(av)} = \int_{\alpha}^{\pi+\alpha} I_{o(av)} d\omega t = \frac{I_{o(av)}}{2}$$
$$I_{th(rms)} = \sqrt{\int_{\alpha}^{\pi+\alpha} (I_{o(av)})^2 d\omega t} = \frac{I_{o(av)}}{\sqrt{2}}$$

The rms value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi}} 2 \int_{\alpha}^{\pi+\alpha} I_{o(av)}^2 d\omega t = I_{o(av)}$$

The average value of the supply current equal zero because its positive part area equal to its negative part. But the average for half cycle is equal to the average of load current.

The output power is given by:

$$P_o = I_{o(av)} V_{o(av)} = I_{o(rms)}^2 R$$

If the load circuit has an EMF the output average Power is given by:

$$P_{o} = I_{o(av)} V_{o(av)} = I_{o(rms)}^{2} R + E I_{0(av)}$$

The input power factor is given by:

$$PF = \frac{P_o}{S_{in}} = \frac{I_{o(av)}V_{o(av)}}{V_s I_{s(rms)}}$$

Note that when a becomes lager than  $90^{\circ}$ , the average value of output voltage becomes negative. This means that from 90 to  $180^{\circ}$ , power flows from the DC load side to the AC source side and the circuit operates as an inverter. When rectification and inversion are obtained from one converter, the process is called two-*quadrant operation* and the converter is called a *full converter*.

### 3. with an Inductive (RL) Load and with Freewheeling Diode:

If a diode is connected across the load (R and large L), the circuit can operate only as a rectifier because the diode prevents negative values of  $v_o$ from appearing across the load. Figure 14 shows the bridge rectifier circuit with the addition of a freewheeling diode (D). The diode provides an extra pa t h for the flow of load current.

Three paths are now possible:  $SCR_1$  and  $SCR_4$ ,  $SCR_2$  and,  $SCR_3$ , and the path through diode D.



Figure 14 Full-wave bridge rectifier with FWD

Negative values of  $v_o$  will forward-bias D and provide zero voltage across the load. Therefore, the negative portions of  $v_o$  in previous case are now replaced by  $v_o = 0$ , as shown in Figure 15 During this interval, the load current freewheels through D and the SCR currents and source current are zero. To illustrate this, let us apply KVL around the path that contains  $v_s$ , SCR<sub>1</sub>,  $v_o$ , and SCR<sub>4</sub>

$$v_s = V_{SCR1} + V_0 + V_{SCR4}$$



Figure 15 Voltage and current waveforms for figure 14.

For the negative portion when  $v_s < 0$ , the FWD is on and  $v_o = 0$ . Therefore,

 $V_{SCR1}+v_{SCR4}<0$ 

This means that  $SCR_1$  and  $SCR_4$  in series and reverse-biased and turn off.  $SCR_2$  and  $SCR_3$  are already off since  $v_s$  is negative. The load current therefore transfers to the freewheeling diode.

The load voltage waveform is the same as that for the resistive load case and also as full-wave rectifier center-tap rectifier.

The average load voltage and load voltage are given by:

$$V_{o(avg)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$
$$V_{o(avg)} = \frac{V_m}{\pi} . (1 + \cos \alpha)$$
$$I_{o(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_m}{\pi R} . (1 + \cos \alpha)$$

The freewheeling diode carries the load current during the delay period  $\alpha$  when the SCRs are off. Therefore, the average and rms values of current through the FWD (D) are given by:

$$I_{D(avg)} = \frac{1}{2\pi} 2\int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\alpha}{\pi} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \frac{\alpha}{\pi}$$
$$I_{D(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\alpha}{\pi}}$$

The average and rms values of the current through the thyristor are given by:

$$I_{th(avg)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{2\pi} = \frac{V_m (1 + \cos(\alpha)) (\pi - \alpha)}{\pi R}$$
$$I_{th(rms)} = \sqrt{\frac{1}{2\pi} \int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\pi - \alpha}{2\pi}} = \frac{V_m (1 + \cos(\alpha)) (\pi - \alpha)}{\pi R} \sqrt{\frac{\pi - \alpha}{2\pi}}$$

The rms value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\pi - \alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\pi - \alpha}{\pi}}$$

The average value of the supply current equal to zero for complete cycle, but for half cycle is given by:

$$I_{s(av)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} (I_{o(av)}) d\omega t = I_{o(av)} \frac{\pi - \alpha}{\pi} = \frac{V_m \cos(\alpha)}{\pi R} \frac{\pi - \alpha}{\pi}$$

### <u>The Third type: Single-Phase Full-Wave Half-Controlled Rectifiers</u> <u>with RL Load</u>

Full or two quadrant converters can operate with both positive and negative average DC load voltages. In the rectifying mode, they supply power from the AC source to the DC load. In the inversion mode, they remove power from DC load and return it to the AC source.

There are various applications that require power flow only from the AC

source to the DC load and therefore are operated in only the rectifying mode.

This is accomplished in bridge rectifiers by replacing half of the SCRs with diodes. These circuits are called one-quadrant or semi controlled bridge rectifiers. An alternative method of obtaining one-quadrant operation in bridge rectifiers is to connect a freewheeling diode across the output terminals of the rectifier.

A basic semi-controlled bridge circuit is shown in Figure 16. Its operation is the same as that of a fully controlled bridge rectifier with a resistive load. When the source voltage is positive, SCR<sub>1</sub> and D<sub>4</sub> are forward-biased. If we trigger SCR<sub>1</sub> at  $\alpha$ , current will flow through D<sub>4</sub>, the load, and SCR<sub>1</sub>. SCR<sub>1</sub> turns off at  $\pi$  when the source reverses. The load voltage is the same as the input voltage during this period  $<\alpha$  to  $\pi$ ). At ( $\pi + \alpha$ ), SCR<sub>2</sub> is triggered, causing current to flow through D<sub>3</sub> and the load. At time  $2\pi$ , SCR<sub>2</sub> turns off and the cycle repeats.



Figure 16 single-phase full-wave Half controlled bridge rectifier circuit.

With an inductive load, commutation (current transfer) occurs every half-cycle to bypass the load current through the diode, as shown in Figure 17. The current through SCK<sub>1</sub> and D<sub>3</sub> during the interval  $\pi$  to ( $\pi$ + $\alpha$ ) and through SCR<sub>2</sub> and D<sub>4</sub> during the interval  $2\pi$  to ( $2\pi$ + $\alpha$ ). As a result, the negative portion of the output voltage is cut off, and the waveform of the output voltage becomes the same as with a pure resistive load. The average output voltage can therefore be varied from its maximum positive value to zero as the firing angle is varied from 0 to  $180^{\circ}$ . The average value of the output voltage is given by:

$$V_{o(avg)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$
$$V_{o(avg)} = \frac{V_m}{\pi} \cdot (1 + \cos \alpha)$$

That is, the voltage is the same as that of a full converter with FWD



Figure 17 waveform for half controlled bridge rectifier with a R-L load.

The average load current when L is very large is given by

$$I_{o(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_m}{\pi R} \cdot (1 + \cos \alpha)$$

The average and rms values of the current through the thyristor are given by:

$$I_{th(avg)} = \frac{1}{2\pi} \int_{\alpha}^{\pi+\alpha} I_{o(av)} d\omega t = \frac{I_{o(av)}}{2}$$
$$I_{th(rms)} = \sqrt{\frac{1}{2\pi}} \int_{\alpha}^{\pi+\alpha} (I_{o(av)})^2 d\omega t = \frac{I_{o(av)}}{\sqrt{2}}$$

The average and the rms values of the current through diode are given by:

$$I_{D(avg)} = \frac{1}{2\pi} \int_{0}^{\pi} I_{o(av)} d\omega t = \frac{I_{o(av)}}{2}$$
$$I_{D(rms)} = \sqrt{\frac{1}{2\pi}} \int_{0}^{\pi} (I_{o(av)})^{2} d\omega t = \frac{I_{o(av)}}{\sqrt{2}}$$

The rms value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(\alpha v)})^2 d\omega t} = I_{o(\alpha v)} \sqrt{\frac{\pi - \alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\pi - \alpha}{\pi}}$$

The average value of the supply current equal to zero for complete cycle, but for half cycle is given by:

$$I_{s(av)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{\pi} = \frac{V_m \cos(\alpha)}{\pi R} \frac{\pi - \alpha}{\pi}$$

If the circuit in Figure 16 contains a highly inductive load the load current will flow throughout the entire negative half-cycle (even if the gate signal is removed) and therefore the circuit will lose control. If a freewheeling diode is used (Figure 18), the FWD becomes forward-biased and begins to conduct as the had voltage tends to reverse. The load current freewheels through the FWD. Therefore, during the internal  $\pi$  to ( $\pi$ +  $\alpha$ ), the

output voltage becomes zero. Similarly, during the interval  $2\pi$  to  $(2\pi + \alpha)$ , the FWD will clamp the negative voltage excursions to zero.



*Figure 18 Half-controlled bridge rectifier with FWD*. (a) Circuit (b) waveform.

The average voltage is given by:

$$V_{o(avg)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} \sqrt{2} V \sin(\omega t) d\omega t = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} V_m \sin(\omega t) d\omega t$$
$$V_{o(avg)} = \frac{V_m}{\pi} . (1 + \cos \alpha)$$

The average load current when L is very large is given by

$$I_{o(avg)} = \frac{V_{o(avg)}}{R} = \frac{V_m}{\pi R} \cdot (1 + \cos \alpha)$$

The average and rms values of the current through the thyristor and diode are given by:

$$I_{th(avg)} = I_{D(av)} = \frac{1}{2\pi} \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{2\pi}$$
$$I_{th(rms)} = I_{D(av)} = \sqrt{\frac{1}{2\pi}} \int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t = I_{o(av)} \sqrt{\frac{\pi - \alpha}{2\pi}}$$

The average and the rms values of the current through FWD are given by:

$$I_{FWD(avg)} = \frac{1}{2\pi} 2 \int_{0}^{\alpha} I_{o(av)} d\omega t = I_{o(av)} \frac{\alpha}{\pi}$$
$$I_{FWD(rms)} = \sqrt{\frac{1}{2\pi}} 2 \int_{0}^{\alpha} (I_{o(av)})^{2} d\omega t = I_{o(av)} \sqrt{\frac{\alpha}{\pi}}$$

The rms value of the supply current is given by:

$$I_{s(rms)} = \sqrt{\frac{1}{2\pi} 2\int_{\alpha}^{\pi} (I_{o(av)})^2 d\omega t} = I_{o(av)} \sqrt{\frac{\pi - \alpha}{\pi}} = \frac{V_m (1 + \cos(\alpha))}{\pi R} \sqrt{\frac{\pi - \alpha}{\pi}}$$

The average value of the supply current equal to zero for complete cycle, but for half cycle is given by:

$$I_{s(av)} = \frac{1}{2\pi} 2 \int_{\alpha}^{\pi} I_{o(av)} d\omega t = I_{o(av)} \frac{\pi - \alpha}{\pi} = \frac{V_m \cos(\alpha)}{\pi R} \frac{\pi - \alpha}{\pi}$$

- **Example 1** Draw the output voltage waveform for a full-wave semi controlled rectifier like the one shown in Figure 16 for the following delay angles: a)  $\alpha = 0^{\circ}$ 
  - b)  $\alpha = 45^{\circ}$ . c)  $\alpha = 90^{\circ}$

Solution

a)  $\alpha = 0^{\circ}$ . During the-positive half-cycle, SCR<sub>1</sub> and D<sub>4</sub> are conducting, and during the negative half cycle, SCR<sub>2</sub> and D<sub>3</sub> are-conducting. The output voltage is the same as that of a diode bridge rectifier (Figure Ex1 (a)). b)  $\alpha = 45^{\circ}$ . During the positive half-cycle, the voltage across the load is zero until SCR<sub>1</sub> is turned on at 45°. The load current flows through SCR<sub>1</sub> and D<sub>4</sub>, and the source voltage is applied to the load. During the negative half-cycle. SCR<sub>1</sub> becomes reverse-biased at  $\pi$ . If we assume an inductive load, SCR<sub>1</sub> maintain conduction until SCR<sub>2</sub> is turned on. The load current will is freewheeling through SCR<sub>1</sub> and D<sub>3</sub>.



Figure Ex1. The output voltage waveform for (a).  $\alpha 0^{\circ}$ . (b)  $\alpha = 45^{\circ}$ . (c)  $\alpha = 90^{\circ}$ .

When SCR<sub>2</sub> is turned on at  $(\pi + 45^{\circ})$ , SCR<sub>1</sub> turns off and the load current flows through SCR<sub>2</sub> and D<sub>3</sub> until SCR<sub>2</sub> and D<sub>3</sub> are reverse-biased. At this point, SCR<sub>2</sub> remains in conduction the load current freewheels through SCR<sub>2</sub> and D<sub>4</sub>. During the period the load current is freewheeling, no current is supplied from the AC source. Both SCRs remain in conduction for 180°. Figure Ex1 (b) shows the output voltage waveform

(c)  $\alpha = 90^{\circ}$ . The operation is exactly the same as in the case when up  $45^{\circ}$  (see Figure Ex1(c)).

#### 2. An alternative circuit configuration is shown, in Figure 19.

The freewheeling current is limited to the path that includes the two diodes,  $D_2$  and  $D_4$ , *in* series. Therefore, the period of conduction for the diode increase and that of the SCRS decreases.



Figure 19 Semi controlled bridge converter, an alternative circuit configuration.

#### Quizzes

- 1. Try to draw the complete waveforms and the formulas of the average and rms values of output voltage, output current, supply current, the thristor current and diode current.
- 2. Find the output power, PIV, and the input power factor.

# 3. <u>Another possible arrangement of the semi controlled bridge</u> <u>converter is shown in Figure 20.</u>

Here the AC input voltage is rectified by the diode bridge to give fullwave voltage output. The output is then controlled by the SCR. The FWD allows the flow of current through the load during the time the SCR off. The average output voltage is the same as in the case of the full-bridge fully control with FWD.



Figure 20 Semi controlled bridge converter, alternative arrangement.

## Quizzes

- 1. Try to draw the complete waveforms and the formulas of the average and rms values of output voltage, output current, supply current, the thristor current and diode current and FWD.
- 2. Find the output power, PIV, and the input power factor.
- 3. Discuss the case if the load is inductive load in series with emf.