Electric Traction

1. INTRODUCTION

The movement of trains and their energy consumption can be most conveniently studied by means of the speed–distance and the speed–time curves.

The motion of any vehicle may be at constant speed or it may consist of periodic acceleration and retardation. The speed–time curves have significant importance in traction. If the frictional resistance to the motion is known value, the energy required for motion of the vehicle can be determined from it.

Moreover, this curve gives the speed at various time instants after the start of run directly.

2. TYPES OF SERVICES

There are mainly three types of passenger services, by which the type of traction system has to be selected, namely:

1. Main line service.
2. Urban or city service.
3. Suburban service.

A. Main line services:

In the main line service, the distance between two stops is usually more than 10km. High balancing speeds should be required. Acceleration and retardation are not so important.

B. Urban service:

In the urban service, the distance between two stops is very less and it is less than 1km. It requires high average speed for frequent starting and stopping.

C. Suburban service:

In the suburban service, the distance between two stations is between 1 and 8 km.

This service requires rapid acceleration and retardation as frequent starting and stopping is required.

3. Speed–time And Speed–distance Curves For Different Services

The curve that shows the instantaneous speed of train in kmph along the ordinate and time in seconds along the abscissa is known as ***'speed–time' curve.***

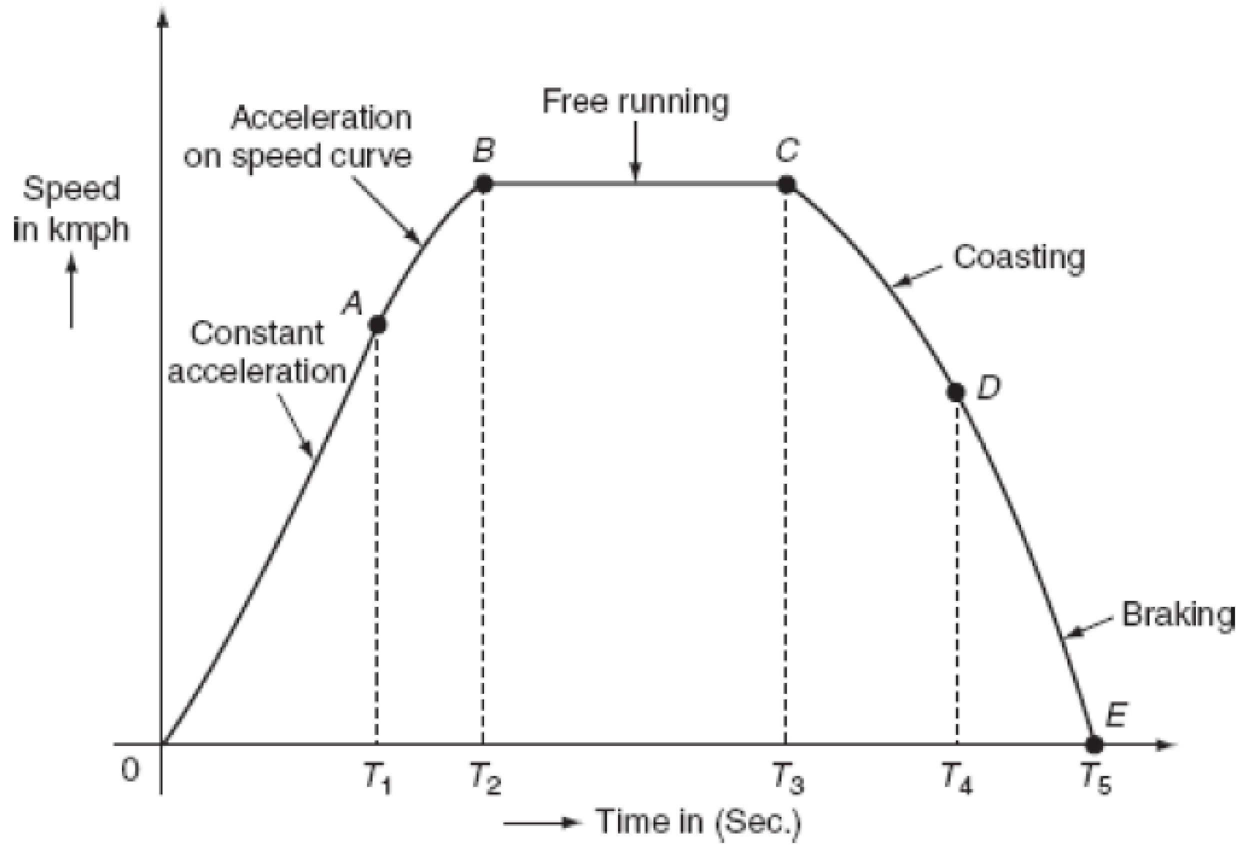
The curve that shows the distance between two stations in km along the ordinate and time in seconds along the abscissa is known as ***'speed–distance' curve.***

The area under the speed–time curve gives the distance travelled during, given time internal and slope at any point on the curve toward abscissa gives the acceleration and retardation at the instance, out of the two speed–time curve is more important.

3.1 Speed–time curve for main line service

Typical speed–time curve of a train running on main line service is shown in Figure, It mainly consists of the following time periods:

1. **Constant accelerating period.**
2. **Acceleration on speed curve.**
3. **Free-running period.**
4. **Coasting period.**
5. **Braking period.**



Speed–time curve for mainline service

A. Constant acceleration

During this period, the traction motor accelerate from rest. The curve 'OA' represents **the constant accelerating period**. During the instant 0 to *T1, **the current is*** **maintained approximately constant and the voltage across the motor is gradually increased by cutting out the starting resistance slowly moving from one notch to the other.** Thus, **current taken by the motor and the tractive efforts are practically constant and therefore acceleration remains constant during this period.**

Hence, this period is also called **as notch up accelerating period or rehostatic accelerating period.**

Typical value of acceleration lies between 0.5 and 1 kmph. **Acceleration is denoted with the symbol '*α*'.**

B. Acceleration on speed-curve

During the running period from *T1 to T2*, ***the voltage across the motor remains* constant and the current starts decreasing, this is because cut out at the instant '*T1*'.**

According to the characteristics of motor, **its speed increases with the decrease in the current and finally the current taken by the motor remains constant.** But, **at the same time**, **even though train accelerates, the acceleration decreases with the increase in speed.**

Finally, the acceleration reaches to zero for certain speed, at which the tractive effort excreted by the motor is exactly equals to the train resistance. This is also known as decreasing accelerating period. This period is shown by the curve '*AB*'.

C. Free-running or constant-speed period

The train runs freely during the period *T2 to T3 at the speed attained by the train at* the instant '*T2*'.

*During this speed, **the motor draws constant power from the supply*** **lines.** This period is shown by the curve *BC.*

D. Coasting period

This period is from *T3 to T4, i.e., from C to D. At the instant 'T3'* ***power supply to*** **the traction, the motor will be cut off and the speed falls on account of friction, windage resistance, etc.**

During this period, the train runs due to the momentum attained at that particular instant. **The rate of the decrease of the speed during coasting period** is known as coasting retardation. Usually, it is denoted with the symbol 'β_c'.

E. Braking period

Braking period is from *T4 to T5, i.e., from D to E. At the end of the coasting period,* i.e., at ‘*T4*’ ***brakes are applied to bring the train to rest***.

During this period, ***the speed*** **of the train decreases rapidly and finally reduces to zero.**

In main line service, **the free-running period will be more, the starting and braking periods are very negligible**, since the distance between the stops for the main line service **is more than 10 km.**

3.2 Speed–time curve

A. for suburban service

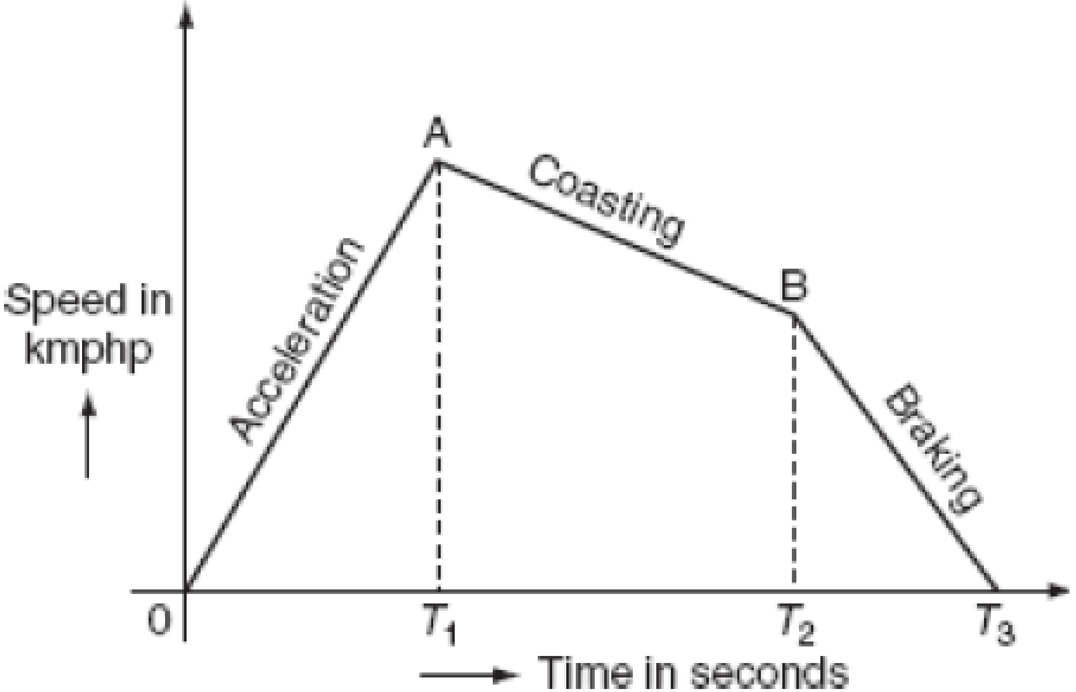
In suburban service, the distance between two adjacent stops for electric train is lying between 1 and 8 km. In this service, the distance between stops is more than the urban service and smaller than the main line service. The typical speed–time curve for suburban service is shown in Figure.

The speed–time curve for urban service consists of three distinct periods. They are:

1. Acceleration.

2. Coasting.

3. Retardation.



Typical speed–time curve for suburban service

For this service, there is no free-running period. The coasting period is comparatively longer since the distance between two stops is more. **Braking or retardation period is comparatively small**. **It requires relatively high values of acceleration and retardation**.

Typical acceleration and retardation values are lying between 1.5 and 4 kmphp and 3 and 4 kmphp, respectively.

B. Speed–time curve for urban or city service

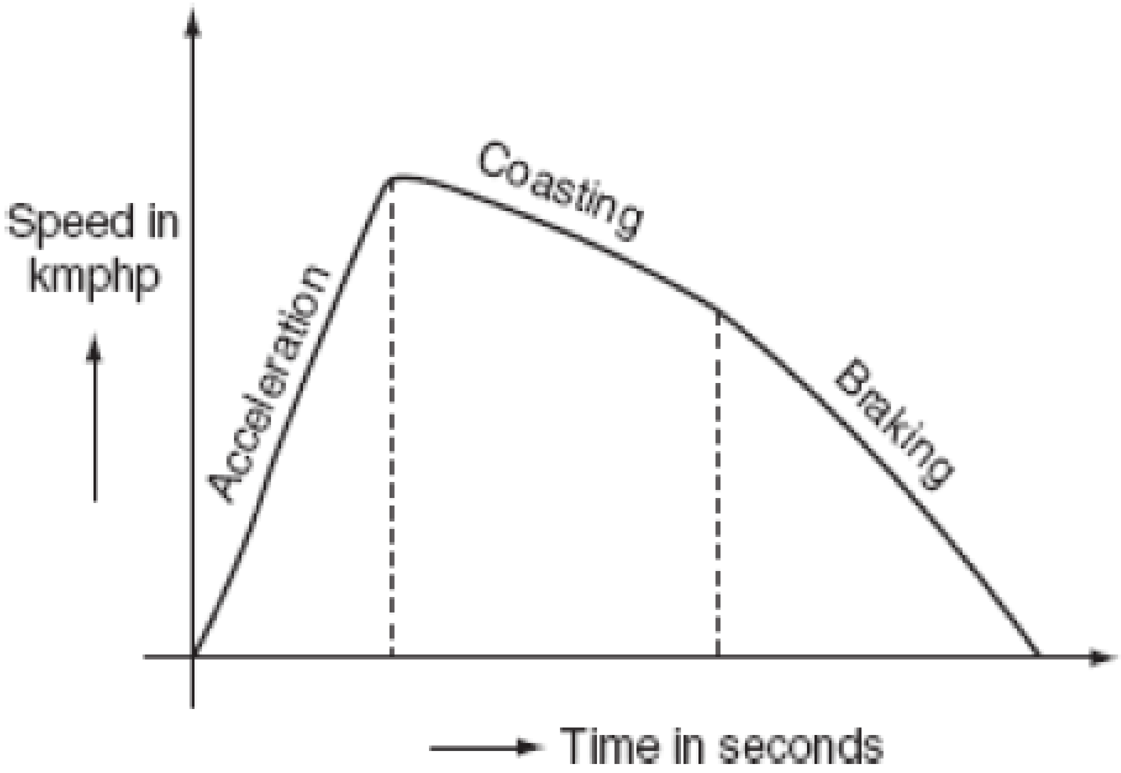
The speed–time curve urban or city service is almost similar to suburban service and is following shown in Figure.

In this service also, **there is no free-running period**. The distance between two stop is less about 1 km. Hence, **relatively short coasting and longer braking period is required.** **The relative values of acceleration and retardation are high to achieve moderately high average between the stops.**

Here, **the small coasting period is included to save the energy consumption.**

The acceleration for the urban service lies between 1.6 and 4 kmphp..

The coasting retardation is about 0.15 kmphp and the braking retardation is lying between 3 and 5 kmphp. Some typical values of various services are shown in Table 1.



Typical speed–time curve for urban service

Table 1 Types of services

	Mainline service	*Suburban service*	*Urban service*
Distance between stops in km	More than 10	1–8	1
Maximum speed in kmph	160	120	120
Acceleration in kmphp	0.5–0.9	1.5–4	1.5–4
Retardation in kmphp	1.5	3–4	3–4
Features	Long free-run period, coasting and acceleration braking periods are small	No free-running period, coasting period is long	No free-running period, coasting period is small

4. SOME DEFINITIONS

Crest speed

The maximum speed attained by the train during run is known as crest speed. It is denoted with '*V_m*'.

Average speed

It is the mean of the speeds attained by the train from start to stop, i.e., it is defined as the ratio of the distance covered by the train between two stops to the total time of rum. It is denoted with '*V_a*'.

$$\therefore \text{Average speed} = \frac{\text{distance between stops}}{\text{actual time of run}} \Longrightarrow V_a = \frac{D}{T}$$

where *V_a is the average speed of train in km/h, D is the distance between stops in* km, and *T is the actual time of run in hours.*

Schedule speed

The ratio of the distance covered between two stops to the total time of the run including the time for stop is known as schedule speed. It is denoted with the symbol '*Vs*'.

$$\therefore \text{Schedule speed} = \frac{\text{distance between stops}}{\text{total time of run} + \text{time for stop}} = \frac{\text{distance between stops}}{\text{shedule time}}$$

$$V_s = \frac{D}{T_s},$$ where *Ts is the schedule time in hours.*

Schedule time

It is defined as the sum of time required for actual run and the time required for stop.

$$\text{i.e., } T_s = T_{\text{run}} + T_{\text{stop}}.$$

5. FACTORS AFFECTING THE SCHEDULE SPEED OF A TRAIN

The factors that affect the schedule speed of a train are:

1. Crest speed.
2. The duration of stops.
3. The distance between the stops.
4. Acceleration.
5. Braking retardation.

A. Crest speed

It is the maximum speed of train, which affects the schedule speed as for fixed acceleration, retardation, and constant distance between the stops. If the crest speed increases, the actual running time of train decreases. For the low crest speed of train it running so, the high crest speed of train will increases its schedule speed.

B. Duration of stops

If the duration of stops is more, then the running time of train will be less; so that, this leads to the low schedule speed.
Thus, for high schedule speed, its duration of stops must be low.

C. Distance between the stops

If the distance between the stops is more, then the running time of the train is less; hence, the schedule speed of train will be more.

D. Acceleration

If the acceleration of train increases, then the running time of the train decreases provided the distance between stops and crest speed is maintained as constant.
Thus, the increase in acceleration will increase the schedule speed.

E. Breaking retardation

High breaking retardation leads to the reduction of running time of train. These will cause high schedule speed provided the distance between the stops is small.

6. SIMPLIFIED TRAPEZOIDAL AND QUADRILATERAL SPEED TIME CURVES

Simplified speed–time curves gives the relationship between acceleration, retardation average speed, and the distance between the stop, which are needed **to estimate the performance of a service at different schedule speeds**. So that, the actual speed–time curves for the main line, urban, and suburban services are **approximated to some from of the simplified curves. These curves may be of either trapezoidal or quadrilateral shape.**

6.1 Analysis of trapezoidal speed–time curve

Trapezoidal speed–time curve can be approximated from the actual speed–time curves of different services by assuming that:

- **The acceleration and retardation periods of the simplified curve is kept same as to that of the actual curve.**
- **The running and coasting periods of the actual speed–time curve are replaced by the constant periods.**

This known as trapezoidal approximation, a simplified trapezoidal speed–time curve is shown in following figure.

6.2 Calculations from the trapezoidal speed–time curve

Let

D be the distance between the stops in km,

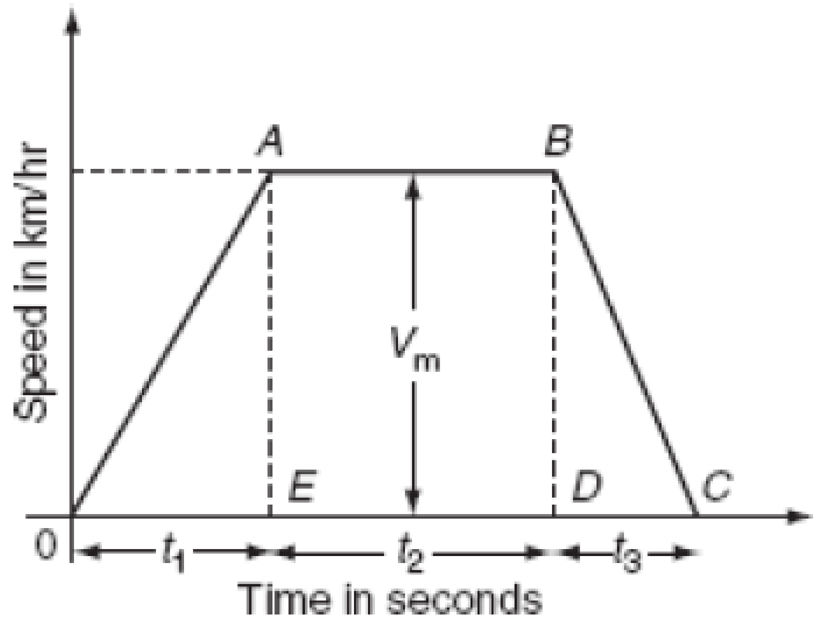
T be the actual running time of train in second,

α be the acceleration in km/h/sec,

β be the retardation in km/h/sec,

V_m *be* the maximum or the crest speed of train in km/h, and

V_a *be the average speed of* train in km/h.



Trapezoidal speed–time curve

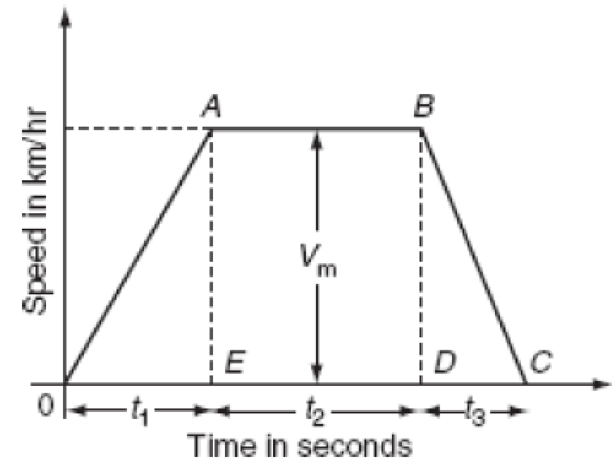
From the Figure

Actual running time of train, $T = t_1 + t_2 + t_3$.

Time for acceleration, $t_1 = \dfrac{V_m - 0}{\alpha} = \dfrac{V_m}{\alpha}$.

Time for retardation, $t_3 = \dfrac{V_m - 0}{\beta} = \dfrac{V_m}{\beta}$.

$$t_2 = T - \left[\frac{V_m}{\alpha} + \frac{V_m}{\beta}\right]$$



∴ **The distance between the stops (*D*)** *= area under triangle OAE + area of* rectangle *ABDE + area of triangle DBC*
= The distance travelled during acceleration + distance travelled during free running period + distance travelled during retardation.

Now:

The distance travelled during acceleration = average speed during accelerating period × time for acceleration

$$\textit{The distance travelled during acceleration} = \frac{1}{2} * V_m * t_1 \frac{km}{h} * s = \frac{V_m * t_1}{2 * 3600}\ km$$

The distance travelled during free-running period = average speed × time of free running

$$\text{The distance travelled during free} - \text{running period} = V_m * t_2 \frac{km}{h} * s = \frac{V_m * t_2}{3600}\ km$$

The distance travelled during retardation period = average speed × time for retardation

$$\text{The distance travelled during retardation period} = \frac{V_m}{2} * t_3 \frac{km}{h} * s = \frac{V_m * t_3}{2 * 3600}\ km$$

The distance between the two stops is:

$$D = \frac{V_m}{2} \times \frac{t_1}{3,600} + V_m \times \frac{t_2}{3,600} + \frac{V_m}{2} \times \frac{t_3}{3,600}$$

$$D=\frac{V_m t_1}{7,200}+\frac{V_m}{3,600}\left[T-V_m(t_1+t_2)\right]+\frac{V_m t_3}{7,200}$$

$$D=\frac{V_m{}^2}{7,200\alpha}+\frac{V_m}{3,600}\left[T-V_m\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\right]+\frac{V_m{}^2}{7,200\beta}$$

$$3,600\times D=\frac{V_m{}^2}{2\alpha}+\frac{V_m{}^2}{\beta}-V_m{}^2\left(\frac{1}{\alpha}+\frac{1}{\beta}\right)+V_m T$$

$$3,600\ D=V_m{}^2\left(\frac{1}{2\alpha}-\frac{1}{\alpha}\right)+V_m{}^2\left(\frac{1}{2\beta}-\frac{1}{\beta}\right)+V_m T$$

$$3,600\ D=\frac{-V_m{}^2}{2\alpha}-\frac{V_m{}^2}{2\beta}+V_m T$$

$$\therefore V_m{}^2\left[\frac{1}{2\alpha}+\frac{1}{2\beta}\right]-V_m T+3,600D=0.$$

Let $\dfrac{1}{2\alpha}+\dfrac{1}{2\beta}=X=\dfrac{\alpha+\beta}{2\alpha\beta}$

$$\therefore V_m{}^2 X-V_m T+3,600D=0.$$

$$V_m = \frac{T + \sqrt{T^2 - 4 \times X \times 3,600D}}{2 \times X}.$$

$$= \frac{T}{2X} \pm \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}}.$$

By considering positive sign, we will get high values of crest speed, which is practically not possible, **so negative sign should be considered:**

$$V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}}$$

Or, $$V_m = \frac{\alpha\beta}{\alpha+\beta}T - \sqrt{\left(\frac{\alpha\beta}{\alpha+\beta}\right)^2 T^2 - 7,200\left(\frac{\alpha\beta}{\alpha+\beta}\right)D}.$$

6.3 Analysis of quadrilateral speed–time curve

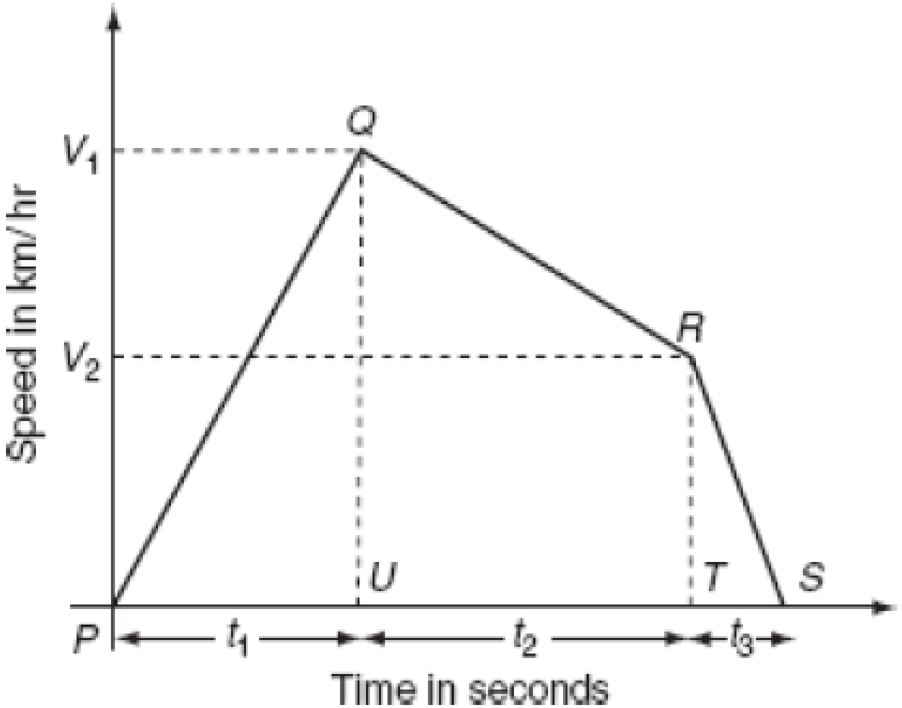
Quadrilateral speed–time curve for urban and suburban services for **which the distance between two stops is less**. The assumption for simplified quadrilateral speed–time curve is the **initial acceleration and coasting retardation periods are extended, and there is no free-running period**. Simplified quadrilateral speed–time curve is shown in Figure.

Let

V_1 be the speed at the end of accelerating period in km/h,

V_2 be the speed at the end of coasting retardation period in km/h, and

β_c be the coasting retardation in km/h/sec.



Quadrilateral speed–time curve

Time for acceleration, $t_1 = \frac{V_1 - 0}{\alpha} = \frac{V_1}{\alpha}$.

Time for coasting period, $t_2 = \frac{V_2 - V_1}{\beta}$.

Time period for braking retardation period, $t_3 = \frac{V_2 - 0}{\beta} = \frac{V_2}{\beta}$.

Total distance travelled during the running period *D* = the area of triangle *PQU + the area of rectangle UQRS + the area of* triangle *TRS.*

= the distance travelled during acceleration + the distance travelled during coasting retardation + the distance travelled during breaking retardation.

But, the distance travelled during acceleration = average speed × time for acceleration

The distance travelled during acceleration $= \frac{0+V_1}{2} \times t_1 \text{ km/h} \times \text{sec} = \frac{V_1}{2} \times \frac{t_1}{3{,}600} \text{km.}$

The distance travelled during coasting retardation $= \frac{V_2+V_1}{2} \times t_2 \text{ km/h} \times \text{sec}$

$$= \frac{V_2+V_1}{2} \times \frac{t_2}{3{,}600} \text{km.}$$

The distance travelled during breaking retardation $= \frac{0+V_2}{2} \times t_3 \text{km/h} \times \text{sec} = \frac{V_2}{2} \times \frac{t_3}{3{,}600} \text{km.}$

∴ Total distance travelled:

$$D = \frac{V_1}{2} \times \frac{t_1}{3{,}600} + \frac{(V_1+V_2)}{2} \frac{(t_2)}{3{,}600} + \frac{V_2}{2} \times \frac{t_3}{3{,}600}$$

$$= \frac{V_1 t_1}{7{,}200} + \frac{(V_1+V_2)t_2}{7{,}200} + \frac{V_2 t_3}{7{,}200}$$

$$= \frac{V_1}{7{,}200}(t_1+t_2) + \frac{V_2}{7{,}200}(t_2+t_3)$$

$$
\begin{aligned}
D &= \frac{V_1}{7,200}(T - t_3) + \frac{V_2}{7,200}(T - t_1) \\
&= \frac{(V_1 + V_2)T}{7,200} - \frac{V_1 t_3}{7,200} - \frac{V_2 t_1}{7,200} \\
&= \frac{(V_1 + V_2)T}{7,200} - \frac{V_1 V_2}{7,200\beta} - \frac{V_1 V_2}{7,200\alpha} \\
&= \frac{\mathrm{T}}{7,200}(V_1 + V_2) - \frac{V_1 V_2}{7,200}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)
\end{aligned}
$$

$$
7,200D = (V_1 + V_2)\mathrm{T} - V_1 V_2\left(\frac{1}{\alpha} + \frac{1}{\beta}\right).
$$

Example 1

The distance between two stops is 1.2 km. A schedule speed of 40 kmph is required to cover that distance. The stop is of 18-s duration. The values of the acceleration and retardation are 2 kmphp and 3 kmphp, respectively. Then, determine the maximum speed over the run. Assume a simplified trapezoidal speed–time curve.

Solution:

Acceleration α = *2.0 kmphp.*
Schedule speed *Vs = 40 kmph.*

Retardation β = 3 kmphp.
Distance of run, D = 1.2 km.

$$\text{Schedule time, } T_s = \frac{D \times 3,600}{V_s} = \frac{1.2 \times 3,600}{40} = 108 \text{ s.}$$

$$\text{Actual run time, } T = T_s - \text{stop duration} = 108 - 18 = 90 \text{ s.}$$

$$\text{Maximum speed } V_m = \frac{T}{2X} - \sqrt{\frac{T^2}{4X^2} - \frac{3,600D}{X}},$$

$$X = \frac{1}{2\alpha} + \frac{1}{2\beta} = \frac{1}{2 \times 2} + \frac{1}{2 \times 3} = 0.416.$$

$$\therefore V_m = \frac{90}{2\times0.416} - \sqrt{\frac{(90)^2}{4\times(0.416)^2} - \frac{3,600\times1.2}{0.416}}$$

$$= 108.173 - \sqrt{(1,1701.414) - (1,0384.61)}$$

$$= 71.88 \text{ kmph.}$$

Example 2

The speed–time curve of train carries of the following parameters:

1. Free running for 12 min.

2. Uniform acceleration of 6.5 kmphp for 20 s.

3. Uniform deceleration of 6.5 kmphp to stop the train.

4. A stop of 7 min. Then, determine

- **a. The distance between two stations,**
- **b. The average speed, and**
- **c. The schedule speed.**

Solution:

a. Acceleration (α) = *6.5 kmphps.* Acceleration period $t_1 = 20$ *s.*

Maximum speed $V_m = \alpha t_1$ = 6.5 × 20 = 130 kmph

Free-running time (t_2) = 12×60 = 720 s.

$$\text{Time for retardation, } (t_3) = \frac{V_m}{\beta} = \frac{130}{6.5} = 20\text{ s}.$$

The distance travelled during the acceleration period:

$$D_1 = \frac{1}{2}\frac{V_m t_1}{3,600} = \frac{1}{2} \times \frac{130 \times 20}{3,600} = 0.36\text{ km}.$$

The distance travelled during the free-running period:

$$D_2 = \frac{V_m t_2}{3,600} = \frac{130 \times 720}{3,600} = 26\text{ km}.$$

The distance travelled during the braking period

$$D_3 = \frac{V_m t_3}{7,200} = \frac{130 \times 20}{7,200} = 0.362\text{ km}.$$

The distance between the two stations:

$$D = D_1 + D_2 + D_3 = 0.36 + 26 + 0.362$$

$$= 26.724\text{ km}.$$

b. Average distance $(V_{avg}) = \frac{D \times 3600}{T}$

$$= \frac{26.724 \times 3600}{20 + 720 + 20} = 126.58 \text{ kmph.}$$

c. Schedule speed $(V_s) = \frac{D \times 3600}{T + \text{stoptime}}$

$$= \frac{26.724 \times 3,600}{20 + 720 + 20 + 70 \times 60} = 81.53 \text{ kmph.}$$

Example 3

An electric train is to have the acceleration and braking retardation of 0.6 km/hr/sec and 3 km/hr/sec, respectively. If the ratio of the maximum speed to the average speed is 1.3 and time for stop is 25 s.
Determine the schedule speed for a run of 1.6 km. Assume the simplified trapezoidal speed–time curve.

Solution:

Acceleration $\alpha = 0.6$ *km/hr/s.*

Retardation $\beta = 3$ *km/hr/s.*

Distance of run $D = 1.6$ *km.*

Let the cultural time of run be 'T '*s.*

Average speed $V_a = \frac{3,600D}{T} = \frac{3,600 \times 1.6}{T}$

$$= \frac{5,760}{T} \text{kmph.}$$

Maximum speed $= 1.3V_a = 1.3 \times \frac{5,760}{T} = \frac{7,488}{T}$ km/hr

$$V_m^2\left[\frac{1}{2\alpha}+\frac{1}{2\beta}\right] - V_m T + 3,600 = D$$

$$V_m^2 = \frac{V_m T - 3,600D}{\left(\frac{1}{2\alpha}+\frac{1}{2\beta}\right)}$$

$$= \frac{\frac{7,488}{T} \times T - 3,600 \times 1.6}{\left(\frac{1}{2 \times 0.6}+\frac{1}{2 \times 3}\right)}$$

$$\therefore V_m = 41.59 \text{ km/hr.}$$

$$= \frac{7,488 - 5,760}{0.833 + 0.166}$$

$$= 1,729.729$$

$$\text{Average speed, } (V_a) = \frac{V_m}{1.3} = \frac{41.59}{1.3}$$

$$(V_a) = 31.9923 \text{ kmph.}$$

$$\text{Actual time of run } T = \frac{3{,}600D}{V_a}$$

$$= \frac{3{,}600 \times 1.6}{31.9923}$$

$$T = 180.0433 \text{ s.}$$

$$\text{Schedule time } T_s = \text{Actual time of run} + \text{time of stop}$$

$$= 180.0433 + 25$$

$$= 205.0433 \text{ s.}$$

$$\text{Schedule speed } V_s = \frac{D \times 3{,}600}{T_s}$$

$$= \frac{1.6 \times 3{,}600}{205.0433}$$

$$= 28.0916 \text{ kmph.}$$

7. TRACTIVE EEFFORT (FT)

It is the effective force acting on the wheel of locomotive, necessary to propel the train is known as '*tractive effort*'. *It is denoted with the symbol F_t.* ***The tractive* effort is a vector quantity always acting tangential to the wheel of a locomotive. It is measured in newton.**

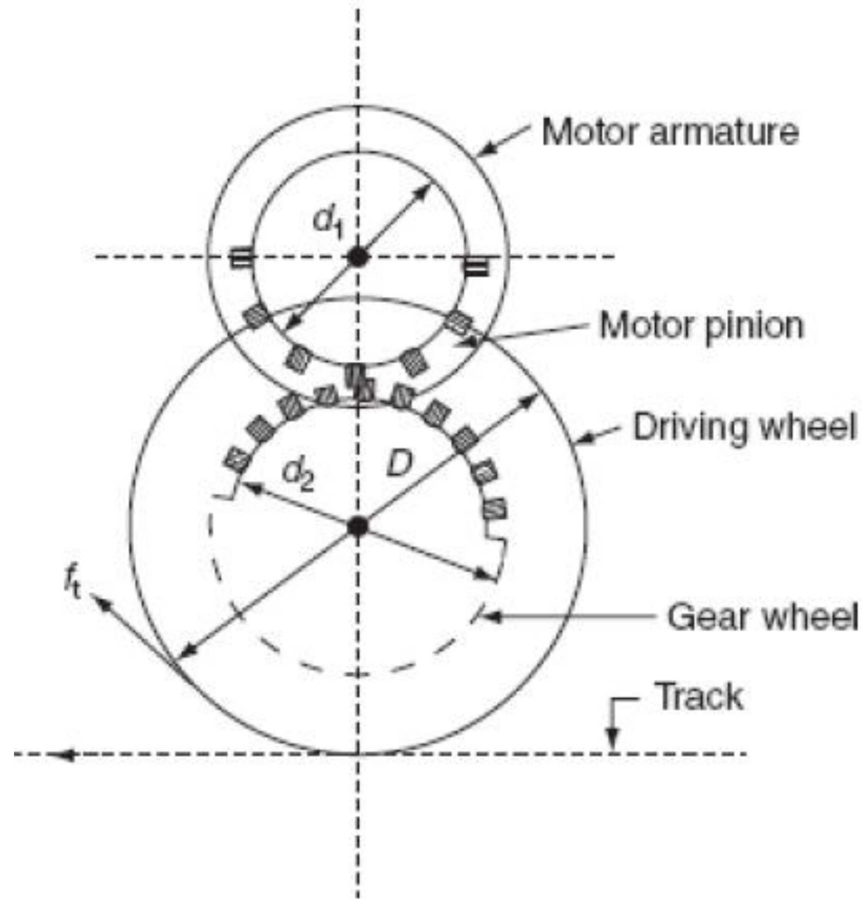
The net effective force or the total tractive effort (*Ft) on the wheel of a* locomotive or a train to run on the track is equals to the sum of tractive effort:

1. Required for linear and angular acceleration (F_a).
2. To overcome the effect of gravity (F_g).
3. To overcome the frictional resistance to the motion of the train (F_r).

$$\therefore F_t = F_a + F_g + F_r$$

A. Mechanics of train movement

The essential driving mechanism of an electric locomotive is shown in following Figure. The electric locomotive consists of pinion and gear wheel meshed with the traction motor and the wheel of the locomotive. Here, the gear wheel transfers the tractive effort at the edge of the pinion to the driving wheel.



Driving mechanism of electric locomotives

Let

T is the torque exerted by the motor in N-m,

Fp is tractive effort at the edge of the pinion in Newton,

Ft is the tractive effort at the wheel, *D is the diameter of* the driving wheel,

d_1 and d_2 are the diameter of pinion and gear wheel, respectively,

η is the efficiency of the power transmission for the motor to the driving axle.

Now, the torque developed by the motor $T = F_p \times \frac{d_1}{2}$ N-m.

$$\therefore F_p = \frac{2T}{d_1} \text{ N.}$$

The tractive effort at the edge of the pinion transferred to the wheel of locomotive is:

$$F_t = F_p \times \frac{d_2}{D} \text{ N.}$$

From above equations

$$F_t = \eta \times \frac{2T}{d_1} \times \frac{d_2}{D} = \eta \cdot T \cdot \frac{2}{D}\left(\frac{d_2}{d_1}\right) = \eta T \cdot \frac{2}{D} \cdot r,$$

where 'r' $= \left(\frac{d_2}{d_1}\right)$ is known as gear ratio.

$$\therefore F_t = 2\eta r \frac{T}{D} \text{ N.}$$

B. Force required for linear and angular acceleration (Fa)

According to the fundamental law of acceleration, the force required to accelerate the motion of the body is given by:

Force = Mass × acceleration ⟹ ***F = ma.***

Let the weight of train be '*W*' *tons being accelerated at 'α' kmphps:*

$\therefore$ The mass of train $m = 1{,}000$ W kg.

And, the acceleration $= \alpha$ kmphps

$$= \alpha \times \frac{1{,}000}{3{,}600} \text{ m/s}^2 = 0.2788\alpha \text{ m/s}^2.$$

The tractive effort required for linear acceleration:

$$F_a = 1{,}000\ \mathrm{W\ kg} \times 0.2778\alpha\ \mathrm{m/s^2}$$

$$= 27.88\ \mathrm{W}\alpha \Rightarrow \mathrm{kg - m/s^2\ (or)\ N.}$$

The above holds good only if the accelerating body has no rotating parts. Owing to the fact that **the train has rotating parts such as motor armature, wheels, axels, and gear system**.

The weight of the body being accelerated including the rotating parts is known as *effective weight or accelerating weight. It is denoted* with '*We*'. *The accelerating weight '(We)' is much higher (about 8–15%) than the* dead weight (*W) of the train.*

***Hence, these parts need to be given angular* acceleration at the same time as the whole train is accelerated in linear direction.**

∴ The tractive effort required-for linear and angular acceleration is:

$$F_a = 27.88\ W_e\alpha\ \mathrm{N.}$$

C. Tractive effort required to overcome the train resistance (Fr)

When the train is running at uniform speed on a level track, it has to overcome the opposing force due to the surface friction, i.e., the friction at various parts of the rolling stock, the fraction at the track, and also due to the wind resistance.

The magnitude of the frictional resistance depends upon the shape, size, and condition of the track and the velocity of the train, etc.

Let

'r' is the specific train resistance in N/ton of the dead weight 'W' and

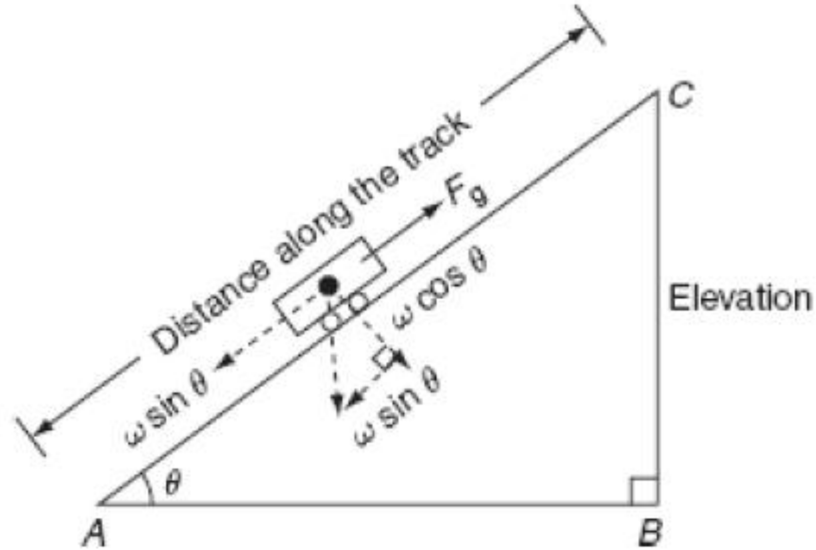
'W' is the dead weight in ton.

$\therefore$ The tractive effort required to overcome the train resistance $F_r = Wr$ N.

D. Tractive effort required to overcome the effect of gravity (Fg)

When the train is moving on up gradient as shown in Figure, the gravity component of the dead weight opposes the motion of the train in upward direction.

In order to prevent this opposition, the tractive effort should be acting in upward direction.



Train moving on up gradient

∴ The tractive effort required to overcome the effect of gravity:

$$F_g = \pm\, \text{mg} \sin\theta \text{ N}$$

$$= \pm 1{,}000 \text{ Wg} \sin\theta \quad [\because m = 1{,}000 \text{ Wkg}].$$

$$\text{Gradient} = \sin\theta = \frac{BC}{AC} = \frac{\text{Elevation}}{\text{distance along the track}}$$

$$\% \text{ Gradient } G = \sin\theta \times 100.$$

$$\therefore F_g = \pm 1,000\, W\, g \times \frac{G}{100}$$

$$= \pm\, 10 \times 9.81\ WG$$

$$= \pm\, 98.1\ WG\,N \qquad [\text{since } g = 9.81\ m/s^2].$$

+ve sign for the train is moving on up gradient.

–ve sign for the train is moving on down gradient.

This is due to when the train is moving on up a gradient, the tractive effort showing in the above Equation will be required to oppose the force due to gravitational force, but while going down the gradient, the same force will be added to the total tractive effort.

∴ The total tractive effort required for the propulsion of train *Ft = Fa + Fr ± Fg:*

$$F_t = 277.8\ W_e \alpha + Wr \pm 98.1\ WG\,N.$$

8. Power output from the driving axle:

Let F_t *is the tractive effort in N and v is the speed of train in kmph.*

∴ The power output (*P*) = *rate of work done*

$$= \text{Tractive effort} \times \frac{\text{distance}}{\text{time}}$$

$$= \text{Tractive effort} \times \text{speed}$$

$$= \frac{F_t \times \nu \times 1{,}000}{3{,}600}\,\text{W}$$

$$= \frac{F_t \times \nu}{3{,}600}\,\text{kW}.$$

If *'v' is in m/s, then* $P = F_t \times v$ *W.*

If *'η' is the efficiency of the gear transmission, then the power output of* motors,

$$P = \frac{F_t \nu}{\eta}\,\text{W} \qquad P = \frac{F_t \nu}{3{,}600\eta}\,\text{kW}.$$

9. SPECIFIC ENERGY CONSUMPTION

The energy input to the motors is called the *energy consumption*. *This is the energy* consumed by various parts of the train for its propulsion. **The energy drawn from the distribution system should be equals to the energy consumed by the various parts of the train and the quantity of the energy required for lighting, heating, control, and braking**.

This quantity of energy consumed by the various parts of train per ton per kilometer is known as specific energy consumption. It is expressed in **watt hours per ton per km.**

$$\therefore \left.\begin{matrix}\text{Specific energy}\\ \text{consumption}\end{matrix}\right\} = \frac{\text{total energy consumption in W} - \text{h}}{\text{the weight of the train in tons} \times \text{the distance covered by train in km}}$$

Or $$\text{The specific energy output} = \frac{\text{energy output in Whr}}{\text{weight of train in tons} \times \text{distance of running}}$$

Factors affecting the specific energy consumption

A. Distance between stations

The specific energy consumption is inversely proportional to the distance between stations. Greater the distance between stops is, the lesser will be the specific energy consumption. The typical values of the specific energy consumption is less for the main line service of 20–30 W*hr/ton*km and high for the urban and suburban services of 50–60 W*hr/ton*km.

B. Acceleration and retardation

For a given schedule speed, the specific energy consumption will accordingly be less for more acceleration and retardation.

C. Maximum speed

For a given distance between the stops, the specific energy consumption increases with the increase in the speed of train.

D. Gradient and train resistance

From the specific energy consumption, it is clear that both gradient and train resistance are proportional to the specific energy consumption.

Normally, the coefficient of adhesion will be affected by the running of train, parentage gradient, condition of track, etc. for the wet and greasy track conditions. The value of the coefficient of adhesion is much higher compared to dry and sandy conditions.

10. IMPORTANT DEFINITIONS

1 Dead weight

It is the total weight of train to be propelled by the locomotive. It is denoted by ‘*W*’.

2 Accelerating weight

It is the effective weight of train that has angular acceleration due to the rotational inertia including the dead weight of the train. **It is denoted by ‘*We*’. This effective train is also known as accelerating weight. The effective weight of the train will be more than the dead weight.** Normally, it is taken as 5–10% of more than the dead weight.

3 Adhesive weight

The total weight to be carried out on the drive in wheels of a locomotive is **known as adhesive weight**

4 Coefficient of adhesion

It is defined as the ratio of the tractive effort required to propel the wheel of a locomotive to its adhesive weight.

$$F_t \propto W$$

$$= \mu W,$$

where *F_t is the tractive effort and W is the adhesive weight.*

$$\therefore \mu = \frac{F_t}{W}.$$

Example 4

A 250-ton motor coach having four motors each developing 6,000 N-m torque during acceleration, starts from rest. If the gradient is 40 in 1,000, gear ration is 4, gear transmission efficiency is 87%, wheel radius is 40 cm, train resistance is 50 N/ton, the addition of rotational inertia is 12%. Calculate the time taken to attain a speed of 50 kmph. If the line voltage is 3,000-V DC and the efficiency of motors is 85%. Find the current during notching period.

Solution:

The weight of train $W = 250$ *ton.*

Gear ratio $r = 4$.

Wheel diameter $D = 2 \times 40 = 80$ *cm*$=0.8$*m*

Train resistance $r = 50$ *N/ton.*

Rotational inertia = 12%.

Accelerating weight of the train $We = 1.10 \times 250 = 275$ *ton.*

Total torque developed $T = 4 \times 6{,}000 = 24{,}000$ *Nm.*

Parentage gradient $G = \dfrac{40}{1{,}000} \times 100 = 4\%.$

Tractive effort $F_t = \dfrac{\eta T 2r}{D} = \dfrac{0.87 \times 24{,}000 \times 2 \times 4}{0.8} = 208{,}800\,\text{N}.$

But

$$F_t = 277.8\ W_e\alpha + 98.1\ WG + Wr$$

$$208{,}800 = 277.8 \times 275\ \alpha + 98.1 \times 250 \times 4 + 250 \times 50$$

$$\therefore \alpha = 1.285 \text{ kmphps.}$$

The time taken for the train to attain the speed of 50 kmph:

$$t = \frac{V_m}{\alpha} = \frac{50}{1.285} = 38.89 \text{s.}$$

Power output from the driving axles $= \frac{F_t \times V_m}{3{,}600} = \frac{208{,}800 \times 50}{3{,}600} = 2{,}900 \text{ kW.}$

$$\text{Power input} = \frac{\text{power output}}{\eta_m}$$

$$= \frac{2{,}900}{0.85} = 3{,}411.76 \text{kW.}$$

$$\text{Total current drawn} = \frac{\text{power input}}{V}$$

$$= \frac{3,411.76 \times 10^3}{3,000} = 1,137.25\ \text{A}.$$

$$\text{Current drawn by the each motor} = \frac{1,137.25}{4} = 284.31\ \text{A}.$$

Example 5

An electric train of weight 250 ton has eight motors geared to driving wheels, each is 85 cm diameter. The tractive resistance is of 50/ton. The effect of rotational inertia is 8% of the train weight, the gear ratio is 4–1, and the gearing efficiency is 85% determine The torque developed by each motor to accelerate the train to a speed of 50 kmph in 30 s up a gradient of 1 in 200.

Solution:

The weight of train *W = 250 ton.*
Tractive resistance, *r = 50N/ton.*
Gearing efficiency *η = 0.85.*
Time of acceleration =30 s

The diameter of driving wheel *D =0.85 m.*
Gear ratio *r = 4.*
Maximum speed *Vm = 50 kmph.*

Accelerating weight of the train:

$$W_e = 1.10 \times W = 1.10 \times 250 = 275 \text{ ton.}$$

$$\text{Acceleration } \alpha = \frac{V_m}{t_1} = \frac{50}{30} = 1.66 \text{ kmpmph.}$$

$$\text{Tractive effort } F_t = 277.8\, W_e \alpha + 98.1\, WG + Wr$$

$$= 126{,}815.7 + 12{,}262.5 + 12{,}500$$

$$= 151{,}578.2 \text{ N.}$$

$$\text{Total torque developed } T = \frac{F_t \times D}{\eta \times 2\gamma} = \frac{151{,}578.2 \times 0.85}{0.85 \times 2 \times 4} = 18.947.25 \text{ N-m.}$$

$$\text{Torque developed by each motor} = \frac{18{,}947.25}{8}$$

$$= 2{,}368.409 \text{ N-m.}$$

Example 6

A tram car is equipped with two motors that are operating in parallel, the resistance in parallel. The resistance of each motor is 0.5 Ω. Calculate the current drawn from the supply mains at 450 V when the car is running at a steady-state speed of 45 kmph and each motor is developing a tractive effort of 1,600 N. The friction, windage, and other losses may be assumed as 3,000 W per motor.

Solution:

The resistance of each motor = 0.5 Ω. Voltage across each motor *V = 450 V.*

Tractive effort *Ft = 1,600 N.* Maximum speed *Vm = 45 kmph.*

Losses per motor = 3,000 W.

The power output of each motor $= \dfrac{F_t \times V_m}{3,600} = \dfrac{1,600 \times 45 \times 10^3}{3,600} = 20,000$ W.

$$\text{Copper losses} = I^2 R_m = I^2 \times 0.5$$

$$\text{Motor input} = \text{motor output} + \text{constant loss} + \text{copper losses}$$

$$450 \times I = 20,000 + 3,000 + 0.5 I^2$$

$$0.5\, I^2 - 450 I + 23,000 = 0.$$

$$0.5\,I^2 - 450I + 23{,}000 = 0.$$

After solving, we get $I = 54.39$ A.

Total current drawn from supply mains $= 2 \times 54.39$

$= 108.78$ A.