Q1.4 ① $r_{cm} = \dfrac{\sum m_i r_i}{m} = \dfrac{mr + mr + mr}{m}$

$= \dfrac{(i+j+k)+(i+k)+(k)}{3} = \frac{1}{3}(2i + j + 3k)$

② $V_{cm} = \dfrac{\sum m_i v_i}{m} = \frac{1}{3}\left[(-i) + (2j) + (i+j+k)\right] = \frac{1}{3}(3j+k)$

③ $P = \sum m_i v_i = mV_{cm}$

$= 3\left(\frac{1}{3}\right)(3\hat{j} + \hat{k}) = (3j + k)$

④ $\sum r_i \times p = \sum r_i \times m_i \vec{v}_i$

$= \left[(i+j+k) \times (-i) + (i+k) \times (2j) + (k) \times (i+j+k)\right]$

$= (k - j) + (2k - 2i) + (j - i)$

$= -3i + 3k$

⑤ $T = \sum \frac{1}{2} m_i v_i^2$

$= \left[(1) + (4) + (3)\right] = 4$

Q8/ $T = \sum \frac{1}{2} m_i v_i^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

$T = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} M v^2$

$v = v_1 - v_2$

$m = m_1 + m_2$

$v_{cm} = \frac{1}{m} \sum m_i v_i = \frac{1}{m}(m_1 v_1 + m_2 v_2)$

$\therefore \frac{1}{2} m v_{cm}^2 + \frac{1}{2} M v^2 = \frac{1}{2} \cancel{m} \left(\dfrac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{\cancel{m}} \right) + \frac{1}{2} \dfrac{m_1 m_2}{2m_1 + m_2} (\vec{v}_1 + \vec{v}_2)^2$

$= \frac{1}{2m}\left(m_1^2 v_1^2 + m_2^2 v_2^2 + m_1 m_2 v_1^2 + m_1 m_2 v_2^2\right)$

$= \frac{1}{2m}\left[m_1 v_1^2 (m_1 + m_2) + m_2 v_2^2 (m_1 + m_2)\right] = \frac{1}{2}\left[m_1 v_1^2 + m_2 v_2^2\right]$

$T = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} M v^2$

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Q9/ $T = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \mu v^2$

$Q = T - T'$ $\qquad v_{cm} = v'_{cm}$

$Q = T - T' = \frac{1}{2} m v_{cm}^2 + \frac{1}{2} \mu v^2 - \frac{1}{2} m v'^2 - \frac{1}{2} \mu v'^2 = \frac{1}{2} \mu v^2 - \frac{1}{2} \mu v'^2$

$\varepsilon = \frac{v'}{v} \implies v' = \varepsilon v$

$Q = \frac{1}{2} \mu v^2 - \frac{1}{2} \mu v'^2 = \frac{1}{2} \mu v^2 - \frac{1}{2} \mu \varepsilon^2 v^2 = \frac{1}{2} \mu v^2 (1 - \varepsilon^2)$

Q10/ $m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$

$v_2 = 0$

$m_1 v_1 = m_1 v'_1 + m_2 v'_2$

$m_1 v_1 = m_2 v'_2 \pm m_1 v'_1$

$v'_1 = v_1 - \frac{m_2}{m_1} v'_2 \implies v'^2_1 = v_1^2 - 2 \frac{m_2}{m_1} v_1 v'_2 + \frac{m_2^2}{m_1^2} v'^2_2$ --- ①

$\frac{1}{2} m v_1^2 = \frac{1}{2} m v'^2_1 + \frac{1}{2} m_2 v'^2_2$ --- ②

$\frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_1 \left(v_1^2 - 2 \frac{m_2}{m_1} v_1 v'_2 + \frac{m_2^2}{m_1^2} v'^2_2 \right) + \frac{1}{2} m_2 v'^2_2$

$\frac{1}{2} m v_1^2 = \frac{1}{2} m_1 v_1^2 - m_2 v_1 v'_2 + \frac{1}{2} \frac{m_2^2}{m_1} v'^2_2 + \frac{1}{2} m_2 v'^2_2$

$\frac{1}{2} \frac{m_2^2}{m_1} v'^2_2 + \frac{1}{2} m_2 v'^2_2 - m_2 v_1 v'_2 = 0 \implies \frac{m_2}{2} \left(\frac{m_2}{m_1} + 1 \right) v'^2_2 - m_2 v_1 v'_2 = 0$

$\left(\frac{m_1 + m_2}{m_1} \right) v'_2 - 2 v_1 = 0 \implies v'_2 = \frac{2 m_1}{m_1 + m_2} v_1$

$T_1 - T'_1 = \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m v'^2_1 = \frac{1}{2} m_2 v'^2_2 = \frac{1}{2} m_2 \left(\frac{2 m_1}{m_1 + m_2} v_1 \right)^2$

$= \frac{1}{2} m_2 \left(\frac{4 m_1^2}{(m_1 + m_2)^2} v_1^2 \right) = \frac{2 m_1^2 m_2}{(m_1 + m_2)} v_1^2$

$\because m = m_1 + m_2 \qquad \mu = \frac{m_1 m_2}{m_1 + m_2}$

$\Delta T = T_1 - T'_1 = \frac{2 m_1^2 m_2 v_1^2}{(m_1 + m_2)} = \frac{2 m \mu_1}{m} v_1^2$

$\frac{\Delta T}{T} = \frac{2 m_1 \mu}{m} v_1^2 \div \frac{1}{2} m_1 v_1^2 = \frac{2 m_1 \mu}{m} v_1^2 \times \frac{2}{m_1 v_1^2} = \frac{4 \mu}{m}$

Q13 & Q15 /

$$= \frac{\frac{V_p'}{\sqrt{2}}}{V_o - \frac{V_p'}{\sqrt{2}}} = \frac{\frac{V_p'}{\sqrt{2}}}{\frac{\sqrt{2}V_o - V_p'}{\sqrt{2}}}$$

$$\tan\phi = \frac{V_p'}{\sqrt{2}V_o - V_p'} = \frac{0.9288}{\sqrt{2} - 0.9288} \; 1.9134$$

$$\phi \; V_\alpha' \cos\phi = 0.086 V_o \,, \quad V_{\alpha y}' = -V_\alpha' \sin\phi = -0.164 V_o$$

Q14 /

$$\frac{1}{2} m_p U_o^2 = \frac{1}{2} m_p V_p'^2 + \frac{1}{2} 4 m_p V_\alpha'^2 + \frac{1}{4}\left(\frac{1}{2} m_p V_o^2\right)$$

$\times \frac{8}{m_p}$

$$4 V_o^2 = 4 V_o'^2 + 16 V_\alpha'^2 + V_o^2$$

$$16 V V_\alpha'^2 = 3V_o^2 - 4V_p'^2 \;\; --- \;(7)$$

$$16 V_\alpha'^2 = V_o^2 - \sqrt{2} V_o V_p' + V_p^2$$

$$V_o^2 - \sqrt{2} V_o V_p' + V_p'^2 - (3V_o^2 - 4U_p'^2) = 0$$

$$5 V_p'^2 - \sqrt{2} V_o V_p' = 0$$

using $x_{1,2} = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$V_p' = \frac{\sqrt{2} V_o \pm \sqrt{2V_o^2 + 40 V_o^2}}{2A} = \frac{V_o}{10}\left(\sqrt{2} \pm \sqrt{42}\right)$$

$V_p' > 0 \qquad U_p' = 0.7895 V_o$

$$16 V_\alpha'^2 = 3V_o^2 - 4V_p'^2 \qquad \boxed{\div 16}$$

$$V_\alpha'^2 = \frac{3}{16} V_o^2 - \frac{4}{16} V_p'^2$$

$$V_\alpha' = \sqrt{\frac{3}{16} V_o^2 - \frac{1}{4} U_p'^2} = \sqrt{\frac{3}{16} V_o^2 - \frac{(0.7895)^2}{4} V_o^2}$$

$$= \frac{V_o}{2} \sqrt{0.75 - (0.7895)^2} = 0.1780 V_o$$

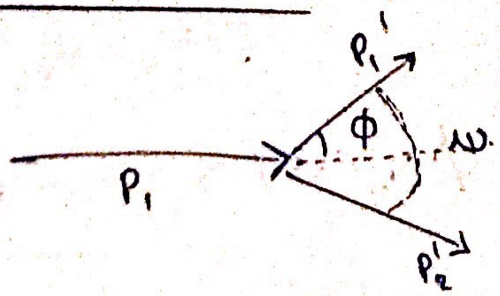
$$\tan\phi = \frac{\frac{V_p'}{\sqrt{2}}}{V_o - \frac{V_p}{\sqrt{2}}} = \frac{V_p'}{\sqrt{2} V_o - V_p} = \frac{0.7845}{\sqrt{2} - 0.7895} = 1.9$$

Q14 تج / $\phi = \tan^{-1}(1.2638) = 51.65^\circ$

$V'_{\alpha x} = V'_\alpha \cos\phi = 0.110\, V_0$, $V'_{\alpha y} = V'_\alpha \sin\phi = -0.140\, V_0$.

Q17 /

$$Q = \frac{P_1' P_2'}{m} \cos\psi$$



$$P_1 = P_1' \cos\phi + P_2' \cos(\psi - \phi)$$

$$0 = P_1' \sin\phi - P_2' \sin(\psi - \phi)$$

$$P_1 = P_1' \cos\phi + P_2' (\cos\psi \cos\phi + \sin\psi \sin\phi) \quad \text{---} ①$$

$$0 = P_1' \sin\phi + P_2' (\sin\psi \cos\phi - \cos\psi \sin\phi) \quad \text{----} ②$$

$$P_1^2 = P_1'^2 \cos^2\phi + P_2'^2 (\cos^2\psi \cos^2\phi + 2\cos\psi \cos\phi \sin\psi \sin\phi + \sin^2\psi \sin^2\phi + 2P_1' P_2' (\cos^2\phi \cos\psi + \cos\phi \sin\psi \sin\phi) \quad \text{..} ③$$

$$0 = P_1^2 \sin^2\phi + P_2'^2 (\sin^2\psi \cos^2\phi - 2\sin\psi \cos\phi \cos\psi \sin\psi + \cos^2\psi \sin^2\phi) - 2P_1' P_2' (\sin\phi \sin\psi \cos\phi - \cos\psi \sin^2\phi) \quad \text{..} ④$$

add eq(3) to eq(4):

$$P_1^2 = P_1'^2 + P_2'^2 + 2P_1' P_2' \cos\psi \quad \text{---} ⑤$$

$$\frac{P_1^2}{2m} = \frac{P_1'^2}{2m} + \frac{P_2'^2}{2m} + Q$$

$$Q = \frac{1}{2m} (P_1^2 - P_1'^2 - P_2'^2) \quad \text{---} ⑥$$

$$Q = \frac{1}{2m} (2P_1' P' \cos\psi) = \frac{P_1' P_2' \cos\psi}{m}$$

Q10 / $m = m_1 + m_2$ $\qquad \mu = \dfrac{m_1 m_2}{m_1 + m_2}$

$$\Delta T = T_1 - T_1' = \frac{2 m_1^2 m_2 v_1^2}{(m_1+m_2)^2} = \frac{2 m_1 \mu}{m} v_1^2$$

$$\frac{\Delta T}{T} = \frac{2 m_1 \mu}{m} v_1^2 \div \frac{1}{2} m_1 v^2 = \frac{2 m_1 \mu}{m} v_1^2 \times \frac{2}{m_1 v_1^2} = \frac{2\mu}{m}$$

Q11 / $\vec{r}_{cm} \times m\vec{v}_{cm} + R \times \mu \vec{v}$

$$L = \vec{r}_{cm} \times m\vec{v}_{cm} + \sum_{i=1}^{n=2} r_i \times m_i \vec{v}_i$$

$\because \vec{r}_{cm} = 0$

$$m_1 \vec{r}_1 + m_2 \vec{r}_2 = 0$$

$$\vec{r}_2 = \frac{-m}{m_2} \vec{r}_1$$

$$\vec{R} = r_1 - r_2 = r_1 + \frac{m_1}{m_2} \vec{r}_1$$

$$\vec{R} = r_1 \left(1 + \frac{m_1}{m_2}\right) = r_1 \left(\frac{m_1 + m_2}{m_2}\right) = \frac{m_1}{\mu} \vec{r}_1$$

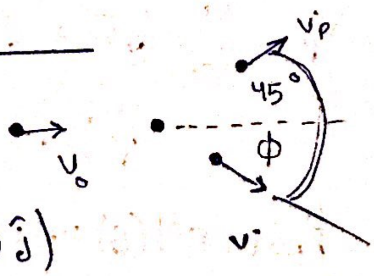
$$\vec{r}_1 = \frac{\mu R}{m_1}$$

$$\vec{r}_2 = \frac{-m_1 \vec{r}_1}{m_2} = \frac{\cancel{m_1}}{m_2}\left(\frac{\mu \vec{R}}{\cancel{m_1}}\right) = \frac{-\mu \vec{R}}{m_2}$$

$$\therefore \sum_i \vec{r}_i \times m \vec{v}_i = \frac{\mu}{m_1} \vec{R} \times m_1 \vec{v}_1 + \left(-\frac{\mu}{m_2}\right) \vec{R} \times m_2 \vec{v}_2 = \mu \vec{R} \times (\vec{v}_1 - \vec{v}_2) = \vec{R} \times \mu \vec{v}$$

$$\vec{L} = \vec{r}_{cm} \times m\vec{v}_{cm} + \sum_i \vec{r}_i \times m \vec{v}_i = \vec{r}_{cm} \times m v_{cm} + \vec{R} \times \mu \vec{v}$$

Q13 / $m_p \vec{V}_0 = m_p \vec{V}_p' + 4 m_p \vec{V}_\alpha'$



$$\vec{V}_0 = \vec{V}_p' + 4 \vec{V}_\alpha'$$

$$v_0 \hat{i} = v_p' \cos 45° \hat{i} + v_p' \sin 45° \hat{j} + 4(v_\alpha' \cos\phi \, \hat{i} - v_\alpha' \sin\phi \, \hat{j})$$

$$v_0 = v_p' \cos 45 + 4 v_\alpha' \cos\phi$$

$$4 v_\alpha' \cos\phi = v_0 - \frac{v_p}{\sqrt{2}} \quad \text{---} \; 1$$

$$v_p' \sin 45 - 4 v_\alpha' \sin\phi = 0$$

$$4 v_\alpha' \sin\phi = \frac{v_p'}{\sqrt{2}} \quad \text{---} \; 2$$

Q13 (2.6) / $m_p \vec{V}_o = m_p \vec{v}'_p + 4 m_p \vec{V}'_\alpha$

square eq (1) and (2) then add them :

$$(4 v'_\alpha \cos\phi)^2 + (4 v'_\alpha \sin\phi)^2 = \left[V_o - \frac{v'_p}{\sqrt{2}}\right]^2 + \left(\frac{v'_p}{\sqrt{2}}\right)^2$$

$$16 v'^2_\alpha = V_o^2 - \frac{2 v'_p V_o}{\sqrt{2}} + \frac{2 v'^2_p}{2}$$

$$16 v'_\alpha = v_o^2 - \frac{2 v'_p v_o}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + v'^2_p$$

$$16 v'^2_\alpha = V_o^2 - \sqrt{2}\, v'_p V_o + v'^2_p \quad ---- (3)$$

$$\frac{1}{2} m_p V_o^2 = \frac{1}{2} m_p v'^2_p + \frac{1}{2} 4 m_p v'^2_\alpha$$

$$V_o^2 = v'^2_p + 4 v'^2_\alpha \quad --- (4) \qquad \times eq (4) \times 4$$

$$16 v'^2_\alpha = 4 V_o^2 - 4 v'^2_p \quad --- (5)$$

$$4 V_o^2 - 4 v_p'^2 = V_o^2 - \sqrt{2}\, v'_p V_o + v'^2_p$$

$$V_o^2 - \sqrt{2} V_o v'_p + v'_p - (4 V_o^2 - 4 v'^2_o) = 0$$

$$5 v'^2_p - \sqrt{2}\, v'_p V_o - 3 V_o^2 = 0$$

using $\quad x_{1,2} = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

$$v'_p = \frac{\sqrt{2} V_o \pm \sqrt{2 V_o^2 - 4(5) \cdot (-3 V_o)^2}}{2(5)}$$

$$= \frac{\sqrt{2} V_o \pm \sqrt{2 V_o^2 - 6(5) V_o^2}}{10} = \frac{\sqrt{2} V_o \pm V_o \sqrt{62}}{10}$$

$$v'_p = \frac{V_o}{10}\left(\sqrt{2} \pm \sqrt{62}\right)$$

$$v'_p > 0 \qquad v'_p = 0.9288 V_o \quad --- (6)$$

from eq (6) in eq (4)

$$4 v'^2_\alpha = V_o^2 - v'^2_p$$

$$v'^2_\alpha = \frac{1}{4}(V_o^2 - v_p'^2)$$

$$v'_\alpha = \frac{1}{2}\sqrt{V_o^2 - v'^2_p} = \frac{1}{2}\sqrt{V_o^2 - (0.9288 V_o)^2}$$

$$\frac{V_o}{2}\sqrt{1 - (0.9288)^2} = 0.1853 V_o$$

$$\tan\phi = \frac{\sin\phi}{\cos\phi}$$

Q8.11 $\alpha = 45^\circ$ $I_s = 2I$

$$\Omega = (2-1)\omega\cos(45) = \frac{\omega}{\sqrt{2}}$$

$$T = \frac{2\pi}{\Omega} = \frac{2\pi}{\omega}\sqrt{2} = 1.414s$$

$$\dot\varphi = \omega\sqrt{1+(4-1)\cos^2(45)} = \omega\sqrt{1+\frac{3}{2}}$$

$$= \sqrt{\frac{5}{2}}\,\omega$$

$$T = \frac{2\pi}{\dot\varphi} = \frac{2\pi}{\omega}\sqrt{\frac{2}{5}} = 0.632s$$

Q8.12 $\tan^{-1}\left[\frac{(I_s - I)\tan\alpha}{I_s + I\tan^2\alpha}\right]$

$I_s > I$

$\tan^{-1}\left(\frac{1}{\sqrt{5}}\right)$

$\tan\theta = \frac{I}{I_s}\tan\alpha$ $I_s > I$ $\theta < \alpha$

$$\tan(\alpha - \theta) = \frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta}$$

$$= \frac{\tan\alpha - I/I_s\tan\alpha}{I_s + I\tan^2\alpha} \quad \times \frac{I}{I_s}$$

$$= \frac{(I_s - I)\tan\alpha}{I_s + I\tan^2\alpha}$$

$$\alpha - \theta = \tan^{-1}\left(\frac{(I_s - I)\tan\alpha}{I_s + I\tan^2\alpha}\right)$$

$$\frac{I_s}{I} = 2$$

$$\tan(\alpha - \theta) = \frac{2I - I\tan\alpha}{2I + I\tan^2\alpha} = \frac{\tan\alpha}{2\tan^2\alpha}$$

$$= \frac{d[\tan(\alpha - \theta)]}{d \tan\alpha} = 0$$

$$= \frac{(2+\tan^2\alpha)(1) - (\tan\alpha)(2\tan\alpha)}{(2+\tan^2\alpha)} = 0$$

$$\frac{2+\tan^2\alpha}{(2+\tan^2\alpha)^2} - \frac{2\tan^2\alpha}{(2+\tan^2\alpha)^2} = 0$$

$$\tan(\alpha - \theta) = \frac{\sqrt{2}}{2+2} = \frac{\sqrt{2}}{4} = \sqrt{\frac{2}{16}} = \sqrt{\frac{1}{8}}$$

$$(\alpha - \theta) \leq \tan^{-1}\left(\frac{1}{\sqrt{8}}\right)$$

Q 8.10

$$0 = I_1\dot{\omega}_1 + \omega_2\omega_3(I_3 - I_2)$$

$$0 = I_2\dot{\omega}_2 + \omega_1\omega_3(I_1 - I_3)$$

$$I_3 = I_1 + I_2$$

$$0 = I_1\dot{\omega}_1 + \omega_2\omega_3 I_1$$

$$0 = I_2\dot{\omega}_2 - \omega_1\omega_3 I_2$$

$$0 = \omega_1\dot{\omega}_1 + \omega_1\omega_2\omega_3 \qquad \times\frac{\omega_1}{I_1}, \times\frac{\omega_2}{I_2}$$

$$0 = \omega_2\dot{\omega}_2 - \omega_1\omega_2\omega_3$$

$$0 = \omega_1\dot{\omega}_1 + \omega_2\dot{\omega}_2 = \frac{1}{2}\frac{d}{dt}(\omega_1^2 + \omega_2^2)$$

$$\omega_1^2 + \omega_2^2 = \text{constant}$$

If $I_1 = I_2$, $\quad I_3\dot{\omega}_3 = 0$

$$\omega_3 = \text{constant}$$

9.2/ $\frac{\partial}{\partial t}\left(\frac{dL}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$

$L = T - V$ Potiutial Energy

$T = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} I\omega^2$

$= \frac{1}{2} m\dot{x}^2 + \frac{1}{2}\left(\frac{2}{5} ma^2\right)$

$= \frac{1}{2} m\dot{x}^2 + \frac{1}{5} m\dot{x}^2 = \left(\frac{5+2}{10}\right) m\dot{x}^2$

$T = \frac{7}{10} m\dot{x}^2$

$V = -mgx\sin\theta$

$L = \frac{7}{10} m\dot{x}^2 + mgx\sin\theta$

$\frac{\partial L}{\partial \dot{x}} = \frac{7}{5} m\dot{x}$

$\frac{dL}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{dt}\left(\frac{7}{5} m\dot{x}\right) = \frac{7}{5} m\ddot{x}$

$\frac{dL}{dx} = mg\sin\theta$

$$\boxed{\ddot{x} = \frac{5}{7} g\sin\theta}$$

Q9.41 $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot q}\right) = \frac{dL}{dq}$

$v^2 = \dot R^2 + R^2\dot\varphi^2 + \dot z^2$

$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot R^2 + R^2\dot\varphi^2 + \dot z^2)$

$V = V(R, \varphi, z)$

$L = T - V = \frac{1}{2}m(\dot R^2 + R^2\dot\varphi^2 + \dot z^2) - V$

$\frac{dL}{d\dot R} = m\dot R$

$\frac{d}{dt}\left(\frac{dL}{d\dot R}\right) = \frac{\partial}{\partial t}(m\dot R) = m\ddot R$

$\frac{dL}{dR} = mR\dot\varphi^2 - \frac{dV}{dR}$

$m\ddot R = mR\dot\varphi^2 - \frac{dV}{dR}$

$m\ddot R - mR\dot\varphi^2 = -\frac{dV}{dR} = QR = f_R$ $\boxed{f_R = (\ddot R - R\dot\varphi^2)}$

$\frac{dL}{d\dot\varphi} = mR^2\dot\varphi$

$\frac{d}{dt}\left(\frac{dL}{d\dot\varphi}\right) = \frac{d}{dt}(mR^2\dot\varphi) = m(2R\dot R\dot\varphi + R^2\ddot\varphi)$

$\frac{dL}{d\varphi} = -\frac{dV}{d\varphi}$

$mR(2\dot R\dot\varphi + R\ddot\varphi) = -\frac{dV}{\partial\varphi} = Q_\varphi = Rf_\varphi$

$\boxed{f_\varphi = m(2\dot R\dot\varphi + R\ddot\varphi)}$

Q7.4$ | 7.6

$$\frac{dL}{d\dot{z}} = m\dot{z}$$

$$\frac{d}{dt} = \left(\frac{dL}{d\dot{z}}\right) = \frac{d}{dt}(m\dot{z}) = m\ddot{z}$$

$$\frac{dL}{dz} = -\frac{dU}{dz}$$

$$m\ddot{z} = -\frac{dU}{dz} = Q_z = f_z$$

$$\boxed{f_z = m\ddot{z}}$$ *

Q9.5 | $L = T - U$

$$\frac{dL}{dt}\left(\frac{dL}{d\dot{q}}\right) = \frac{dL}{\partial q}$$

$$v^2 = \dot{r}^2 - r^2\dot{\theta}^2 + r^2\sin\theta\dot{\varphi}^2$$

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m(\dot{r}^2 - r^2\dot{\theta}^2 + r^2\sin\theta\dot{\varphi}^2)$$

$$L = T - V = \frac{1}{2}m(\dot{r}^2 - r^2\dot{\theta}^2 + r^2\sin\theta\,\dot{\varphi}^2) - V(r)$$

$$\frac{dL}{d\dot{r}} = m\dot{r}$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{r}}\right) = \frac{d}{dt}(m\dot{r}) = m\ddot{r}$$

$$\frac{dL}{dr} = mr\dot{\theta}^2 + mr\sin^2\theta\dot{\varphi}^2 - \frac{dV}{dr}$$

$$m\ddot{r} = mr\dot{\theta}^2 + mr\sin^2\theta\dot{\varphi}^2 - \frac{dV}{dr}$$

$$m(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\varphi}^2) = -\frac{dV}{dr} = Q_r = f_r$$

$$\boxed{f_r = m(\ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\varphi}^2)}$$

$$\frac{dL}{d\dot{\theta}} = mr^2\dot{\theta}$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\theta}}\right) = \frac{d}{dt}(mr^2\dot{\theta}) = 2mr\dot{r}\dot{\theta} + mr^2\ddot{\theta}$$

$$\frac{dL}{d\theta} = mr^2\sin\theta\cos\theta\dot{\varphi}^2 - \frac{dV}{d\theta}$$

$$m(2r\dot{r}\dot{\theta} - r^2\ddot{\theta}) = mr^2\sin\theta\cos\theta\dot{\varphi}^2 - \frac{dV}{d\theta}$$

$$m(2r\dot{r}\dot{\theta} + r^2\ddot{\theta} - r^2\sin\theta\cos\theta\dot{\varphi}^2) = -\frac{dV}{d\theta} = Q_\theta = rf_\theta$$

Q9.5 $\boxed{f_\theta = m(2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\,\dot{\phi}^2)}$

$$\frac{dL}{d\dot{\phi}} = mr^2\sin^2\theta\,\dot{\phi}$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{\phi}}\right) = m(2r\dot{r}\sin^2\theta\,\dot{\phi} + 2r^2\sin\theta\cos\theta\,\dot{\theta}\dot{\phi} + r^2\sin\theta\,\ddot{\phi})$$

$$\frac{dL}{d\phi} = -\frac{dV}{d\phi}$$

$$m(2r\dot{r}\sin^2\theta\,\dot{\phi} + 2r^2\sin\theta\cos\theta\,\dot{\theta}\dot{\phi} + r^2\sin^2\theta\,\ddot{\phi}) - V(r,\phi,\theta)$$

$$m(2r\dot{r}\sin^2\theta\,\phi + 2r^2\sin\theta\cos\theta\,\dot{\theta}\dot{\phi} + r^2\sin^2\theta\,\ddot{\phi}) = -\frac{dV}{d\phi} = Q_\phi = f_\phi$$

$$\boxed{f_\phi = m(2\dot{r}\dot{\phi}\sin\theta + 2r\cos\theta\,\dot{\theta}\dot{\phi} + r\sin\theta\,\ddot{\phi})}$$