Umm Al-Qura University
College of Applied Science
Department of Physics
Time: 10.15 - 12.15

جامعة ام القرى
كلية العلوم التطبيقية
قسم الفيزياء
التاريخ: ١٧ ربيع الثاني ١٤٣٩

Final Exam
Academic Year 1438-1439 (1st Semester)

Program: Physics **Course:** Classical Mechanics (2) **Course code:** 403321-3

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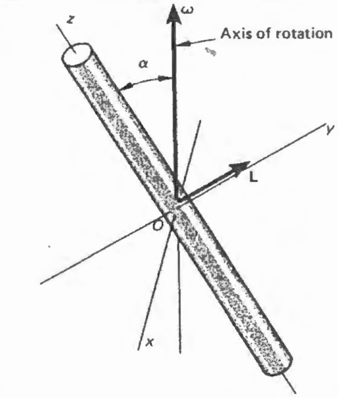
Question		**Mark**	**Signature**
Question 1	**(10 marks)**	9	Fatma
Question 2	**(10 marks)**	9	
Question 3	**(10 marks)**	6.5	
Question 4	**(10 marks)**	1	
Question 5	**(5 marks)**	4	
Question 6	**(5 marks)**	1	
Question 7	------		
Question 8	------		
Question 9	------		
Question 10	------		
Total Mark		30.5 / **50**	**Exam Committee** Dr. Doaa Abdallah Dr. Fatma El-Sayed

Question 1: (10 marks)

Find the product of inertia and the angular momentum vector $\vec{L}$ for a thin rod of length l and mass m which is constrained to rotate with constant angular velocity $\vec{\omega}$ about an axis passing through the center making an angle α with the rod.



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$$I_{xx} = \int (y^2+z^2)\,\rho\, ds = \int_{-l/2}^{l/2} (0+z^2)\rho\, ds \Rightarrow \rho \frac{z^3}{3}\Big|_{-l/2}^{l/2} \Rightarrow \frac{\rho}{3}\left(\frac{L^3}{8}\right)$$

$$= \frac{1}{3}\left(\frac{m}{L}\right)\frac{L^3}{8} \Rightarrow \frac{mL^2}{24} = \frac{mL^2}{12}$$

$$I_{yy} = \int (z^2+y^2)\,\rho\, ds = \int_{-l/2}^{l/2} (0+z^2)\rho\, ds = \rho \frac{x^3}{3}\Big|_{-l/2}^{l/2}$$

$$\frac{\rho}{3}\left(\frac{2L^3}{8}\right) \Rightarrow \frac{2L^3}{24}\left(\frac{m}{L}\right) = \frac{mL^2}{12}$$

$$I_{zz} = \int (x^2+y^2)\,\rho\, ds = 0$$

$$I_{xy} = -\int xy = 0, \quad I_{yz} = \int zy = 0, \quad I_{zx} = \int xz = 0$$

$$L = I\omega$$

$$L = \begin{bmatrix} \frac{1}{12}mL & 0 & 0 \\ 0 & \frac{mL}{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \sin\alpha \\ \cos\alpha \end{bmatrix}$$

$$L_x = 0$$

$$L_y = \frac{ma}{12}\sin\alpha$$

$$L_z = 0$$

$$L_y = \frac{ma}{12}\sin\alpha$$

$\alpha = 90$

Question 2: (10 marks)

(a) A thin square plate of side a rotates freely under zero torque. If the axis of rotation makes an angle of 45° with the symmetry axis of the plate. Find the period of the precession of the axis of rotation about the symmetry axis, and the period of the precession of the symmetry axis about the invariable line. **(5 marks)**

$$\Omega = \left(\frac{I_s}{I} - 1\right)\omega \cos\alpha \longrightarrow I_s = 2I$$

$$\Omega = \left(\frac{2I}{I} - 1\right)\omega \cos 45 = \frac{\sqrt{2}}{2}$$

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$$T = \frac{2\pi}{\Omega} = \frac{2\pi(2)}{(\sqrt{2})} = \sqrt{2} = 1.414\ s$$

$$\dot{\phi} = \omega\sqrt{1 + \left(\left(\frac{I_s}{I}\right)^2 - 1\right)\cos^2\alpha}$$

$$= \sqrt{1 + 3\cos 45} = 1.767$$

$$T = \frac{2\pi}{\dot{\phi}} = \frac{1}{1.767} = 0.56\ s$$

(b) A rigid body having an axis of symmetry rotates freely about a fixed point under no torques. If α is the angle between the axis of symmetry and the instantaneous axis of rotation, show that the angle between the axis of rotation and the invariable line is

$$tan^{-1}\left[\frac{(I_s - I)tan\alpha}{I_s + Itan^2\alpha}\right]$$

where I_s, the moment of inertia about the symmetry axis, is greater than I, the moment of inertia about an axis normal to the symmetry axis. **(5 marks)**

$$\tan(\alpha - \theta) = \frac{\tan\alpha - \tan\theta}{1 + \tan\alpha\tan\theta} \qquad \tan\theta = \frac{I}{I_s}\tan\alpha$$

$$= \frac{\tan\alpha - \frac{I}{I_s}\tan\alpha}{1 + \tan\alpha\left(\frac{I}{I_s}\tan\alpha\right)} \Rightarrow \frac{\tan\alpha\left(1 - \frac{I}{I_s}\right)}{1 + \frac{I}{I_s}\tan^2\alpha}$$

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$$= \frac{\tan\alpha\left(\frac{I_s - I}{I_s}\right)}{\frac{I_s + I\tan^2\alpha}{I_s}} \Rightarrow \frac{\tan\alpha(I_s - I)}{\tan^2\alpha\, I + I_s}$$

Question 3: (10 marks)

(a) Find the acceleration of a solid uniform sphere rolling down a perfectly rough fixed inclined plane. **(5 marks)**

$$L = \cancel{T - U}$$

$$T = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + \sin^2\theta \, \dot{\phi}^2 \right)$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{r}}\right) = \frac{d}{dt}(m\dot{r}) = m\ddot{r} \,, \quad \frac{dL}{dr} = r\dot{\theta}^2$$

$$m\ddot{r} = r\dot{\theta}^2$$

$$\ddot{r} = \frac{r\dot{\theta}^2}{m}$$

(b) Find the acceleration of an Atwood's machine system which consists of two weights of mass m_1 and m_2, connected by a light inextensible of length l which passes over a pulley. **(5 marks)**

$$L = T - U$$

$$T = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \dot{x}^2 + \frac{1}{2} I \frac{\dot{x}^2}{a} \Rightarrow \frac{1}{2}\left(m_1 + m_2 + I/a\right)\dot{x}^2$$

$$U = -mgx - mg(l - x) = gx(-m + m) - mgl$$

$$\frac{d}{dt}\left(\frac{dL}{d\dot{x}}\right) = \frac{dL}{dx}$$

$$\frac{d}{dt}\left(\dot{x}\left(m_1 + m_2 + \frac{I}{a}\right)\right) \Rightarrow \ddot{x}\left(m_1 + m_2 + \frac{I}{a}\right)$$

$$\frac{dL}{dx} = g(-m + m)$$

4.5

$$\ddot{x}\left(m_1 + m_2 + \frac{I}{a}\right) = g(-m + m)$$

$$\ddot{x} = \frac{g(-m + m)}{m_1 + m_2 + I/a}$$

Question 4: (10 marks)

Find the Hamiltonian equations of motion for:

1- A one-dimensional harmonic oscillator.

2- A particle in a central field.

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$L = T - V$

$T = \frac{1}{2} \dot{x}^2 m$

$V = \frac{1}{2} k x^2$

$L = \frac{1}{2} \dot{x}^2 m - \frac{1}{2} k x^2$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$

$\frac{d}{dt}(\dot{x} m) = m \ddot{x}$

$\frac{\partial L}{\partial x} = -kx$

$F = -C\dot{x}$

$m\ddot{x} = -kx - C\dot{x}$

$m\ddot{x} + kx + C\dot{x} = 0$ غير مطلوب

Question 5: (5 marks)

Deduce the relation between Impulse $\hat{p}$ and coefficient of restitution ε.

$m v_0 + m v_1 = \cancel{P_c}$

$m v_1' + m v_2 = \cancel{P_c}$

$m v_1' + m v_0 = \cancel{P_r}$

$m v_2' + m v_0 = \cancel{P_r}$

$m_1 u_0 + m v_1 = m_2 v_2 + m v_1$

$v_0 (m_1 + m_2) = m v_2 - m v_1$

u

$\left(v_0 = \frac{m v_2 - m v_1}{m_1 + m_2} \right)$

$m \left(\frac{m v_2 - m v_1}{m_1 + m_2} \right) + m v_1 = P_c$

$m (m v_2 - m v_1) + m v_1 = P_c (m_1 + m_2)$

$m m v_2 - m m v_1 + m v_1 = P_c (m_1 + m_2)$

$m_1 m_2 (\cancel{v_2 - v_1}) + m v_1 = P_c \cancel{(m_1 + m_2)}$ —— (1)

$m v_1' + m v_0 = m_2 v_2' + m v_0$

$v_0 (m_1 + m_2)$

$v_0 = \left(\frac{m v_2' - m v_1'}{(m_1 + m_2)} \right)$

$m v_1' + m \left(\frac{m v_2' - m v_1'}{m_1 + m_2} \right) = P_r$

$m v_1' + m (m v_2' - m v_1') = P_r (m_1 + m_2)$

$m m (v_2' - v_1') + m v_1' = P_r (m_1 + m_2)$ —— (2)

$\frac{P_r}{P_c} = \frac{v_2' - v_1'}{\cancel{v_2 - v_1}} = \varepsilon$

Question 6: (5 marks)

Prove that the position of the center of oscillation of the physical pendulum relative to the center of mass is $\ell' = \frac{k_{cm}^2}{\ell}$.

$$N = r \times F \Rightarrow r \cdot F \sin\theta$$ غلط

$$N = \frac{\omega}{I} = \frac{\ddot{\theta}}{I} = mg \sin\theta$$

$$\frac{\ddot{\theta}}{I} = mg \sin\theta$$

$$\ddot{\theta} = mgI \sin\theta$$

$$\omega = \sqrt{\frac{mgl}{l'}}$$

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$$f = \frac{1}{2\pi}\left(\sqrt{\frac{mgl}{l'}}\right)$$

$$T = \frac{1}{f} = 2\pi \sqrt{\frac{mg\,k_{cm}^2}{l}}$$

$$l' = \frac{k_{cm}^2}{l}$$