Umm Al-Qura Universtiy, Makkah

Department of Electrical Engineering

Control (802331)

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Solution Final Exam

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Max Marks: 80 Section 1 or 2



**Q1.** [4, 6] Consider the system shown. Find:

1. The damping ratio and undamped natural frequency (for above figure).
2. To improve the damping, the system is changed into (figure below). Find the value of K such that the new damping ratio is 1/2.

**Solution:**

1. $\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{10}{s\left(s+1\right)}}{1+\frac{10}{s\left(s+1\right)}}=\frac{10}{s\left(s+1\right)+10}=\frac{10}{s^{2}+s+10}=\frac{\left(\sqrt{10}\right)^{2}}{s^{2}+2\left(\frac{1}{2\sqrt{10}}\right)\sqrt{10}s+\left(\sqrt{10}\right)^{2}}$

Hence $ω\_{n}=\sqrt{10}=3.16 {rad}/{s} and ξ=\frac{1}{2\sqrt{10}}=0.158$

1. Consider the inner loop first: $\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{10}{s+1}}{1+\frac{10K}{s+1}}=\frac{10}{s+1+10K}$

Now consider the outer loop:

$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{10}{s\left(s+1+10K\right)}}{1+\frac{10}{s\left(s+1+10K\right)}}=\frac{10}{s\left(s+1+10K\right)+10}=\frac{10}{s^{2}+\left(1+10K\right)s+10}=\frac{\left(\sqrt{10}\right)^{2}}{s^{2}+2\left(\frac{1+10K}{2\sqrt{10}}\right)\sqrt{10} s+\left(\sqrt{10}\right)^{2}}$

Hence $ξ=\frac{1+10K}{2\sqrt{10}}=0.5 ⇒ K=\frac{\sqrt{10} -1}{10}=0.216$

**Q2.** [1, 3, 6] Consider an unstable system $G\left(s\right)=\frac{1}{s^{2}-1}$ . Your goal is to design a controller such that the undamped natural frequency be $ω\_{n}=\sqrt{2} {rad}/{s}$ and damping ration be $ξ=1/\sqrt{2}$.

1. Find the required pole location.
2. Can this goal be achieved by a simple controller $G\_{c}\left(s\right)=K$?
3. Design a PD controller $G\_{c}\left(s\right)=K\left(s-z\right)$ to achieve this goal.

**Solution:**

1. Required pole location: $-ξω\_{n}\pm jω\_{n}\sqrt{1-ξ^{2}}=-\frac{1}{\sqrt{2}}\left(\sqrt{2}\right)\pm j\sqrt{2}\sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^{2}}=-1\pm j$
2. Assume $G\_{c}\left(s\right)=K$. The root locus does not pass through the required poles. So, this goal cannot be achieved by simple adjustment of gain K.
3. Let the controller be

 $G\_{c}\left(s\right)=K\left(s-z\right)$.

$$180=∠G\_{c}\left(s\right) G\left(s\right)H\left(s\right)=∠\left[\frac{K\left(s-z\right)}{s^{2}-1}\right]=∠K+∠\left(s-z\right)-∠\left(s-1\right)-∠\left(s+1\right)=∠\left(-1+j-z\right)-∠\left(-1+j-1\right)-∠\left(-1+j+1\right)=tan^{-1}\frac{1}{-1-z}-153.43-90$$

$$tan^{-1}\frac{1}{-1-z}=180+90+153.43=423.43≡63.43$$

$$z=-\frac{1}{\tan(63.43)}-1=-1.5$$

Hence $G\_{c}\left(s\right)=K\left(s+1.5\right)$

Now we will find the value of K which will achieve the goal.

$$\left|G\_{c}\left(s\right) G\left(s\right)H\left(s\right)\right|=1 ⇒ \left|\frac{K\left(s+1.5\right)}{\left(s+1\right)s\left(s-1\right)}\right|=1 $$

$$⇒ K=\left|\frac{\left(s+1\right)\left(s-1\right)}{s+1.5}\right|=\left|\frac{\left(-1+j+1\right)\left(-1+j-1\right)}{-1+j+1.5}\right|=\left|\frac{\left(j\right)\left(-2+j\right)}{0.5+j}\right|=\frac{\sqrt{2^{2}+1^{2}}}{\sqrt{0.5^{2}+1^{2}}}=2$$

Hence $G\_{c}\left(s\right)=2\left(s+1.5\right)$

The root locus for this controller



**Q3.** [6, 2, 1] Consider the liquid flow system shown. The transfer function from in-flow-rate $q\_{i}(t)$ to out-flow-rate $q\_{o}(t)$ is found to be

$$\frac{Q\_{o}(s)}{Q\_{i}(s)}=\frac{1}{R\_{1}C\_{1}R\_{2}C\_{2}s^{2}+\left(R\_{1}C\_{1}+R\_{2}C\_{2}+R\_{2}C\_{1}\right)s+1}$$

Assume $R\_{1}=R\_{2}=1 {s}/{m^{2}}, C\_{1}=1 m^{2}, C\_{2}=0.25 m^{2}$. The in-flow-rate of $1 {m^{3}}/{s}$ started at t=0. Find

1. Find the out-flow-rate as a function of time.
2. Find the out-flow-rate after 2 sec.
3. Find the steady-state value of the out-flow-rate.

**Solution:**

1. $\frac{Q\_{o}(s)}{Q\_{i}(s)}=\frac{1}{R\_{1}C\_{1}R\_{2}C\_{2}s^{2}+\left(R\_{1}C\_{1}+R\_{2}C\_{2}+R\_{2}C\_{1}\right)s+1}=\frac{1}{0.25 s^{2}+2.25s+1}=\frac{4}{ s^{2}+9s+4}$ . Hence $ξ=2.25>1$

$Q\_{o}\left(s\right)=\frac{4}{ s^{2}+9s+4}Q\_{i}\left(s\right)=\frac{4}{ s(s+8.531)(s+0.469)}$

Using the table to find inverse Laplace transform:

|  |  |
| --- | --- |
| $$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$$ | $$\frac{1}{s(s+a)(s+b)}$$ |

$q\_{o}\left(t\right)=\frac{4}{\left(8.531\right)\left(0.469\right)}\left[1+\frac{1}{8.531-0.469}\left(0.469e^{-8.531t}-8.531e^{-0.469t}\right)\right]$

$=1+0.0582e^{-8.531t}-1.0582e^{-0.469t}$

1. $q\_{o}\left(10\right)=1+0.0582e^{-8.531(2)}-1.0582e^{-0.469\left(2\right)}=0.586 {m^{3}}/{s}$
2. $q\_{o}\left(\infty \right)=1 {m^{3}}/{s}$

**Q4.** [3, 3, 4] Consider the system shown. Plot the root locus. Find all important parameters.

**Solution:**

$$G\left(s\right)H\left(s\right)=\frac{s+1}{s\left(s^{2}+2s+6\right)}\left(\frac{1}{s+1}\right)=\frac{1}{s\left(s^{2}+2s+6\right)}=\frac{1}{s\left(s+1+j\sqrt{5}\right)\left(s+1-j\sqrt{5}\right)}$$

We have 3 poles and no zeros. So 3 asymptotes.

Center of asymptotes: $σ\_{c}=\frac{\sum\_{}^{}p-\sum\_{}^{}z}{n-m}=\frac{\left(0-1+j\sqrt{5}-1+j\sqrt{5}\right)-(0)}{3-0}=-\frac{2}{3}$

Angles of asymptotes: $β=180, \pm 60$

Angle of departure: $θ\_{D}=180+∠GH=180+∠\left[\frac{1}{s\left(s+1+j\sqrt{5}\right)\left(s+1-j\sqrt{5}\right)}\right]$

$$=180-∠s-∠\left(s+1+j\sqrt{5}\right)-∠\left(s+1-j\sqrt{5}\right)=180-∠\left(-1+j\sqrt{5}\right)-∠\left(-1+j\sqrt{5}+1+j\sqrt{5}\right)-∠\left(-1+j\sqrt{5}+1-j\sqrt{5}\right)=180-114.09-90=-24.09$$



**Q5.** [10] Again consider the system of Q4. Using Routh’s stability test, find the range of K for the system to be stable.

**Solution:**

The close-loop transfer function will be

$$\frac{C(s)}{R(s)}=\frac{G(s)}{1+G(s)H(s)}=\frac{\frac{K(s+1)}{s\left(s^{2}+2s+6\right)}}{1+\frac{K(s+1)}{s\left(s^{2}+2s+6\right)}\left(\frac{1}{s+1}\right)}=\frac{\frac{K(s+1)}{s\left(s^{2}+2s+6\right)}}{1+\frac{K}{s\left(s^{2}+2s+6\right)}}=\frac{K(s+1)}{s\left(s^{2}+2s+6\right)+K}=\frac{K(s+1)}{s^{3}+2s^{2}+6s+K}$$

|  |  |  |
| --- | --- | --- |
| $$s^{3}$$ | 1 | 6 |
| $$s^{2}$$ | 2 | K |
| $$s^{1}$$ | 6-K/2 |  |
| $$s^{0}$$ | K |  |

Hence K>0 and 6-K/2>0 or 12>K

0<K<12

**Q6.** [6, 4] Consider a PID controller with $k\_{p}=2, k\_{I}=1 and k\_{D}=3$. The error signal is a combination of step and ramp: $e\left(t\right)=2 u\left(t\right)+t u(t)$. Find the control signal generated by the PID controller.

**Solution:**

Transfer function of the PID controller: $\frac{W\left(s\right)}{E\left(s\right)}=k\_{P}+\frac{k\_{I}}{s}+k\_{D}s=2+\frac{1}{s}+3s$

$E\left(s\right)=\frac{2}{s}+\frac{1}{s^{2}}$ . $W\left(s\right)=\left(2+\frac{1}{s}+3s\right)\left(\frac{2}{s}+\frac{1}{s^{2}}\right)=6+\frac{7}{s}+\frac{4}{s^{2}}+\frac{1}{s^{3}}$ .

Control signal: $w\left(t\right)=6 δ\left(t\right)+\left(7+4t+0.5 t^{2}\right)u\left(t\right)$

**Q7.** [4, 2, 4] Consider

1. What will be the output of the following MATLAB commands?

n=[1 0 1];

d=[1 2 3 0];

k=0:10;

y=step(n, d, k);

y

1. What is the difference between following two MATLAB commands?
2. n=[1 0 1];
3. n=[1 0 1]
4. Consider the following system:

$$G\left(s\right)=\frac{s+2}{s\left(s+1\right)^{3}} , H\left(s\right)=1$$

Write MATLAB command to plot the root locus of this system.

**Solution:**

1. First 2 lines will define the numerator and denominator of a transfer function: $G(s)=\frac{s^{2}+1}{s^{3}+2s^{2}+3s}$

Third line will generate a vector k=[0 1 2 3 4 5 6 7 8 9 10].

Forth line will compute the step response of the system G(s) at times t=0, 1, … , 10 and store the output in ‘y’.

Last line will print the calculated values of the step response on the screen. No plot will be generated.

1. Command in the first line will make a vector n=[1 0 1] but do not print it on the screen.

Command in the second line will make a vector n=[1 0 1] and echo it on the screen.

1. Here

$$G\left(s\right)H(s)=\frac{s+2}{s\left(s+1\right)^{3}}=\frac{s+2}{s\left(s^{3}+3s^{2}+3s+1\right)} =\frac{s+2}{s^{4}+3s^{3}+3s^{2}+s}$$

Hence the following command will do the job:

rlocus([1 2], [1 3 3 1 0]);

**Q8a.** [5] Compare open-loop system with close-loop system. Give at least 3 points.

**Ans.**

|  |  |
| --- | --- |
| Open loop control system | Close loop control system |
| CheaperSimpleLess accurateEasier/cheaper to maintainNo issue of stability | ExpensiveComplicatedMore accurateDifficult/expensive to maintainStability is an important issue |

**Q8b.** [5] Give at least 3 examples of open-loop systems and close-loop systems.

**Ans.**

|  |  |
| --- | --- |
| Open loop control system | Close loop control system |
| Speed control of DC motor without velocity feedback.Open loop system for position control.ToasterWashing machine | Speed control of DC motor using velocity feedback.Position control of DC motor using position feedback.Position control of DC motor using position and velocity feedback.Temperature control using thermostat. |

Laplace transform

|  |  |
| --- | --- |
| $$f\left(t\right)$$ | $$F\left(s\right)$$ |
| Unit impulse: $δ\left(t\right)$ | 1 |
| Unit step: 1 | $$\frac{1}{s}$$ |
| Unit ramp: $t$ | $$\frac{1}{s^{2}}$$ |
| $$\frac{t^{n-1}}{\left(n-1\right)!} (n=1, 2, 3, \cdots )$$ | $$\frac{1}{s^{n}}$$ |
| $$t^{n} (n=1, 2, 3, \cdots )$$ | $$\frac{n!}{s^{n+1}}$$ |
| $$e^{-at}$$ | $$\frac{1}{s+a}$$ |
| $$\frac{1}{a}\left(1-e^{-at}\right)$$ | $$\frac{1}{s\left(s+a\right)}$$ |
| $$te^{-at}$$ | $$\frac{1}{(s+a)^{2}}$$ |
| $$\frac{1}{a^{2}}\left(1-e^{-at}-ate^{-at}\right)$$ | $$\frac{1}{s(s+a)^{2}}$$ |
| $$\sin(ωt)$$ | $$\frac{ω}{s^{2}+ω^{2}}$$ |
| $$\cos(ωt)$$ | $$\frac{s}{s^{2}+ω^{2}}$$ |
| $$1-\cos(ωt)$$ | $$\frac{ω\_{n}^{2}}{s\left(s^{2}+ω\_{n}^{2}\right)}$$ |
| $$1-\frac{e^{-ξω\_{n}t}}{\sqrt{1-ξ^{2}}}\sin(\left(ω\_{n}\sqrt{1-ξ^{2}}t+cos^{-1}ξ\right)) (0<ξ<1)$$ | $$\frac{ω\_{n}^{2}}{s\left(s^{2}+2ξω\_{n}s+ω\_{n}^{2}\right)}$$ |
| $$\frac{1}{a^{2}}\left(1-e^{-at}-ate^{-at}\right)$$ | $$\frac{1}{s(s+a)^{2}}$$ |
| $$\frac{1}{ab}\left[1+\frac{1}{a-b}\left(be^{-at}-ae^{-bt}\right)\right]$$ | $$\frac{1}{s(s+a)(s+b)}$$ |
| $$\sin(ω\_{n}t)$$ | $$\frac{ω\_{n}}{s^{2}+ω\_{n}^{2}}$$ |
| $$\frac{ω\_{n}}{\sqrt{1-ξ^{2}}}e^{-ξω\_{n}t}\sin(\left(ω\_{n}\sqrt{1-ξ^{2}}t\right)) (0<ξ<1)$$ | $$\frac{ω\_{n}^{2}}{s^{2}+2ξω\_{n}s+ω\_{n}^{2}}$$ |
| $$te^{-at}$$ | $$\frac{1}{(s+a)^{2}}$$ |
| $$\frac{1}{b-a}\left(e^{-at}-e^{-bt}\right)$$ | $$\frac{1}{(s+a)(s+b)}$$ |

* Close loop system with feedback: $\frac{C(s)}{R(s)}=\frac{G}{1+GH}$
* For root locus:
	+ Center of asymptotes: $σ\_{c}=\frac{\sum\_{i=1}^{n}p\_{i}-\sum\_{i=1}^{m}z\_{i}}{n-m}$ Angles of asymptotes: $β=\frac{180+360k}{n-m}$
	+ Break-away/break-in point: $\sum\_{i=1}^{n}\frac{1}{σ\_{b}-p\_{i}}=\sum\_{i=1}^{m}\frac{1}{σ\_{b}-z\_{i}}$
	+ Departure/arrival angles: $θ\_{D}=180+∠GH$; $θ\_{A}=180-∠GH$