Fluid Mechanics: Fundamentals and Applications, 4th edition

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Lecture slides by Mehmet Kanoglu

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Chapter 5

INTERNAL FLOW

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Internal flows through pipes, elbows, tees, valves, etc., as in this oil refinery, are found in nearly every industry.

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Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
- Understand various velocity and flow rate measurement techniques and learn their advantages and disadvantages

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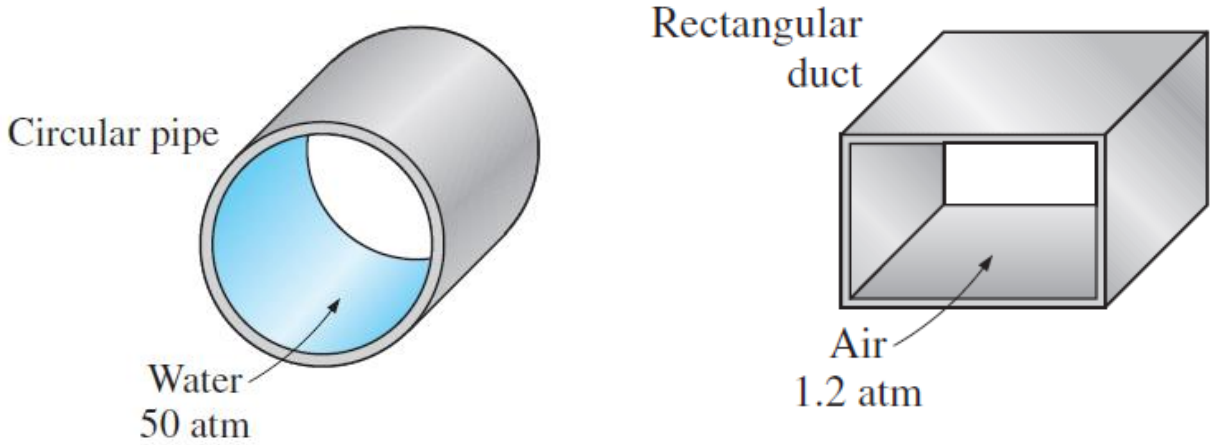
5–1 ■ INTRODUCTION

Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.

The fluid in such applications is usually forced to flow by a fan or pump through a flow section.

We pay particular attention to *friction (due to the viscosity)*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.

The pressure drop is then used to determine the *pumping power requirement*.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

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Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

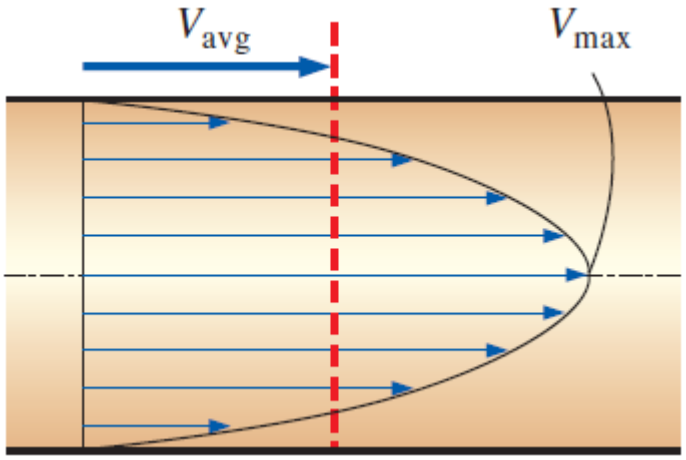
Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

$$\dot{m} = \rho V_{\text{avg}} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The value of the average velocity V_{avg} at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied

$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2}\int_0^R u(r) r\, dr$$

The average velocity for incompressible flow in a circular pipe of radius R

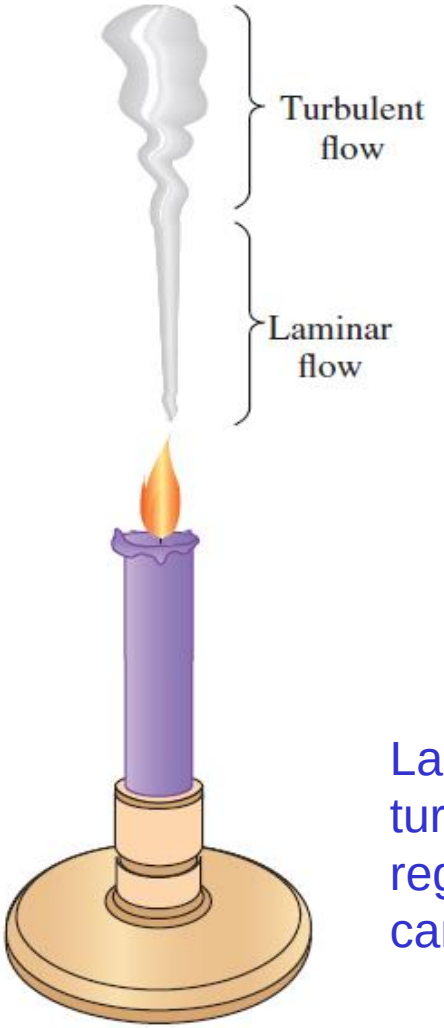


Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

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LAMINAR AND TURBULENT FLOWS

Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



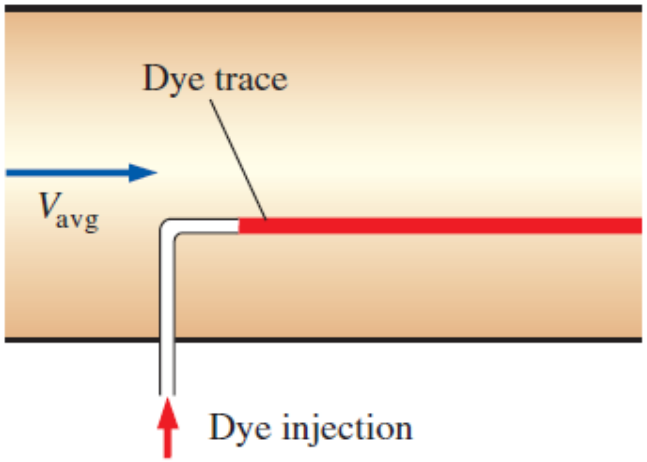
Laminar and turbulent flow regimes of candle smoke.

Laminar: Smooth streamlines and highly ordered motion.

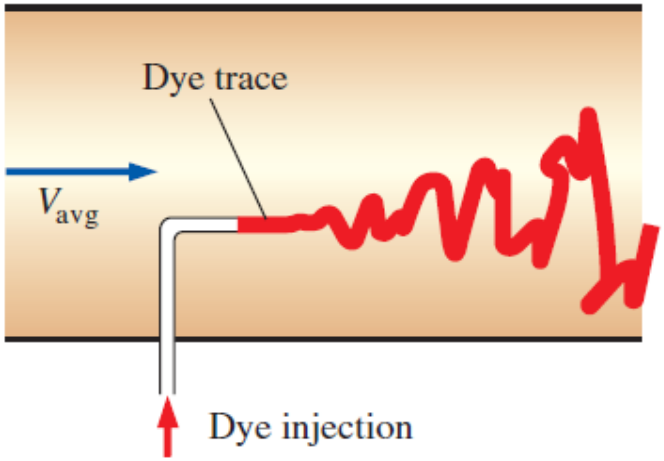
Turbulent: Velocity fluctuations and highly disordered motion.

Transition: The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.



(*a*) Laminar flow



(*b*) Turbulent flow

The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

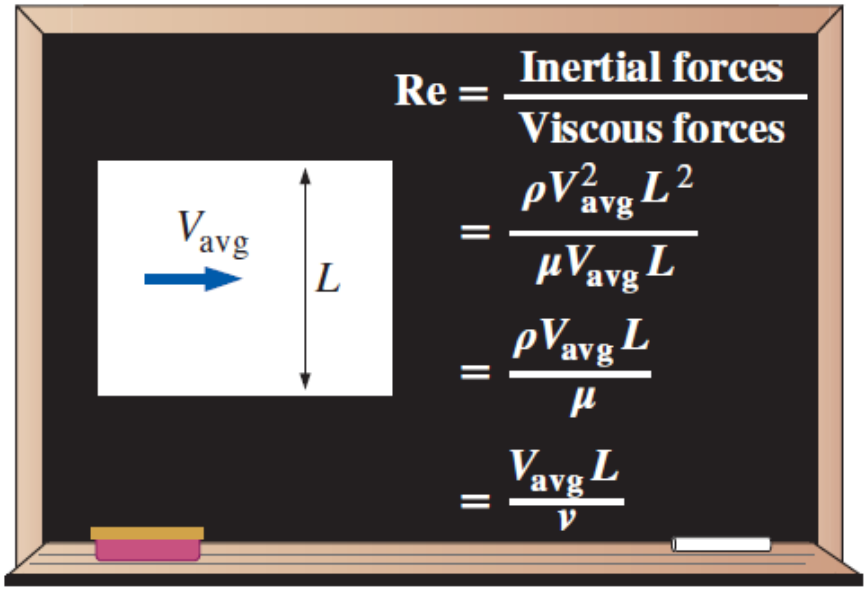
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Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (**Reynolds number**).

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{\text{avg}} D}{\nu} = \frac{\rho V_{\text{avg}} D}{\mu}$$



At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (**turbulent**). At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (**laminar**).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

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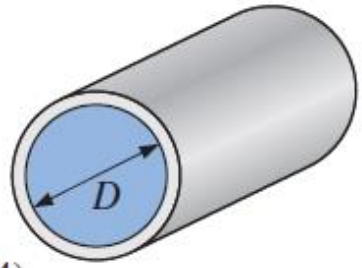
For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter**

Hydraulic diameter: $D_h = \dfrac{4A_c}{p}$

Circular pipes: $D_h = \dfrac{4A_c}{p} = \dfrac{4(\pi D^2/4)}{\pi D} = D$

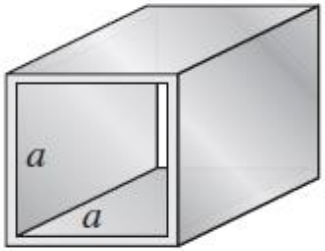
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

Circular tube:



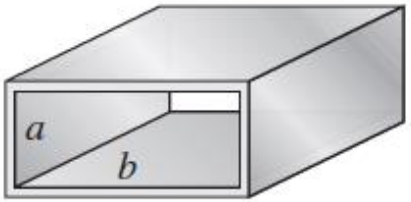
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



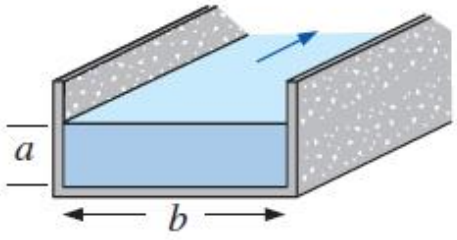
$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

Channel:

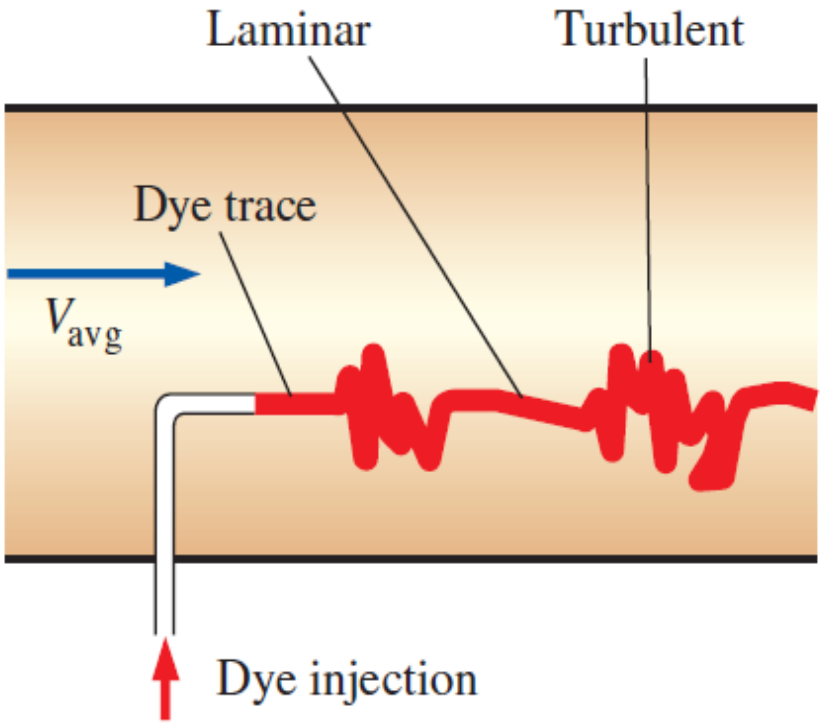


$$D_h = \frac{4ab}{2a+b}$$

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For flow in a circular pipe:

	Re ˆ 2300	laminar flow
2300 ˆ	Re ˆ 4000	transitional flow
	Re ‰4000	turbulent flow

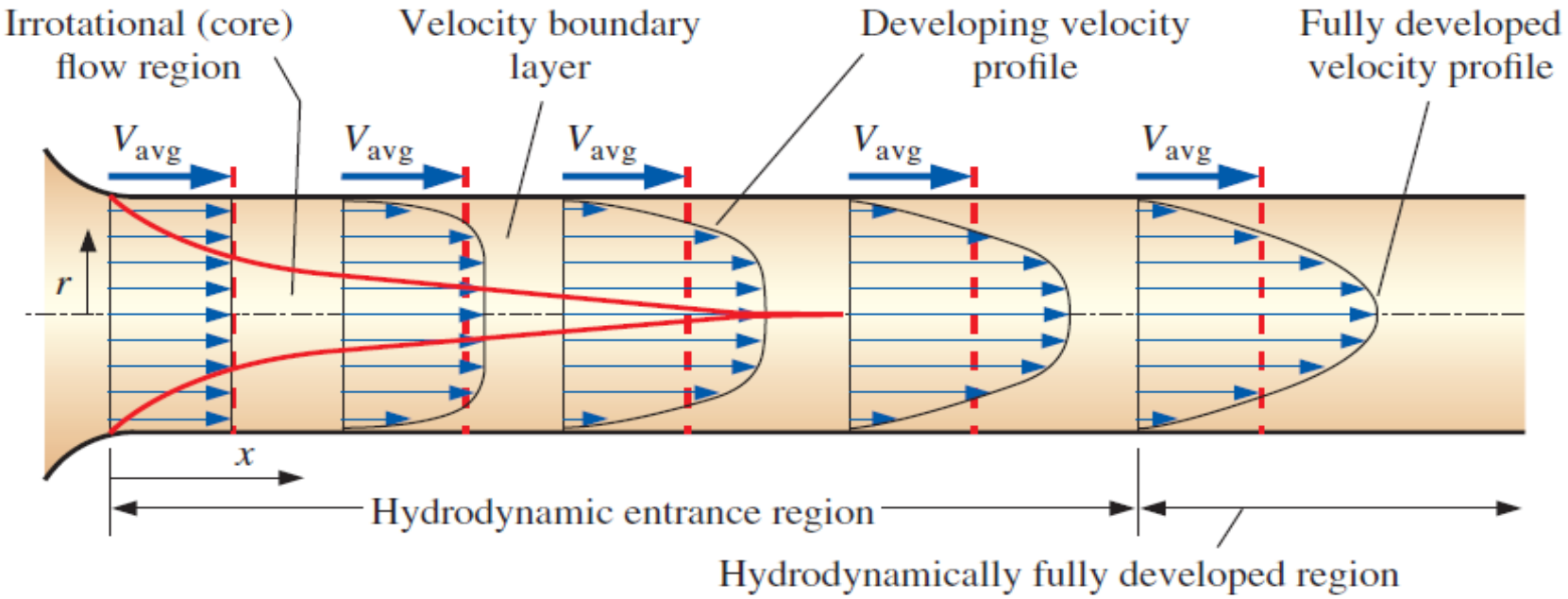


In the transitional flow region of $2300 \leq Re \leq 4000$, the flow switches between laminar and turbulent seemingly randomly.

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THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.
Boundary layer region: The viscous effects and the velocity changes are significant.
Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

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Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamic entry length L_h: The length of this region.

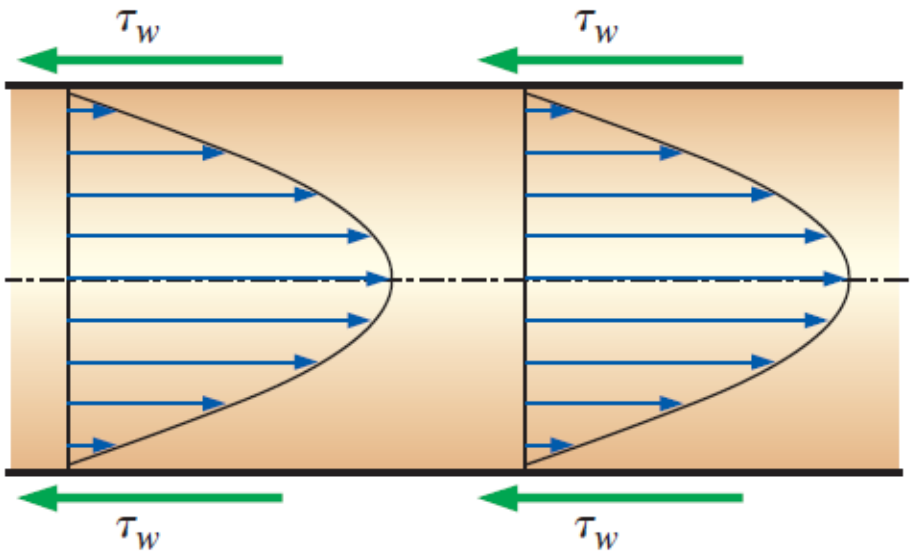
Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile the normalized temperature profile remain unchanged.

Hydrodynamically fully developed:

$$\frac{\partial u(r,x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$



In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

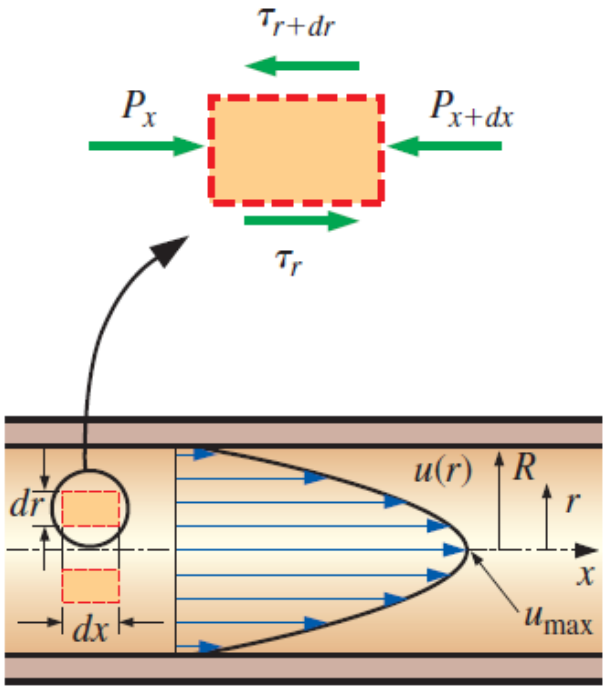
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LAMINAR FLOW IN PIPES

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile $u(r)$ remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.

$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$



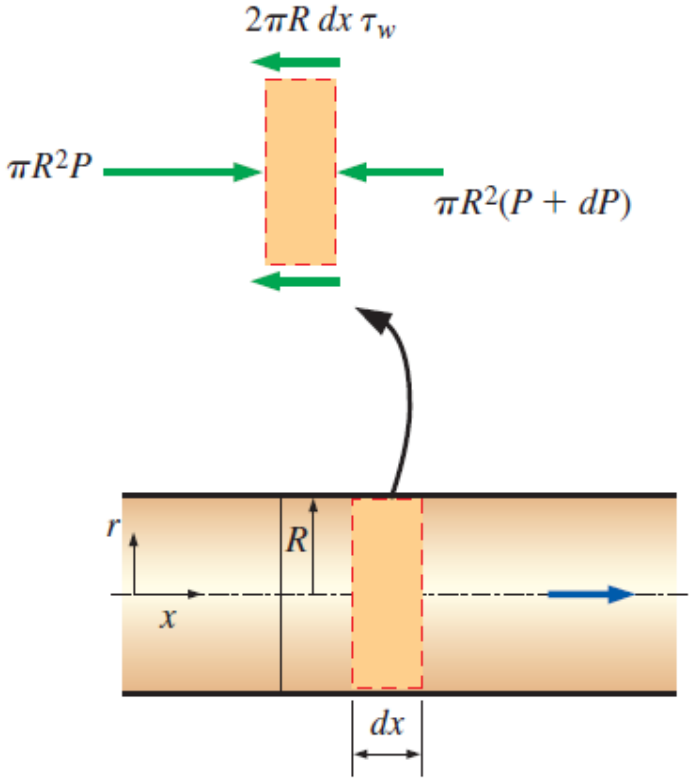
$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \tau = -\mu\left(du/dr\right)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Free-body diagram of a ring-shaped differential fluid element of radius r, thickness dr, and length dx oriented coaxially with a horizontal pipe in fully developed laminar flow.

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Force balance:

$$\pi R^2 P - \pi R^2 (P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$$\partial u / \partial r = 0 \text{ at } r = 0$$

$$u = 0 \text{ at } r = R$$

Boundary conditions

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Average velocity

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right)$$

Velocity profile

$$u_{\max} = 2V_{\text{avg}}$$

Maximim velocity at centerline

Free-body diagram of a fluid disk element of radius *R* and length *dx* in fully developed laminar flow in a horizontal pipe.

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Pressure Drop and Head Loss

$$\frac{P_1}{\rho_1 g}+\frac{V_1^2}{2g}+z_1+h_{\text{pump},u}=\frac{P_2}{\rho_2 g}+\frac{V_2^2}{2g}+z_2+h_{\text{turbine},e}+h_L$$

- $h_L=\dfrac{e_{\text{mech loss, piping}}}{g}=\dfrac{\dot{E}_{\text{mech loss, piping}}}{\dot{m}g}$ is the *irreversible head loss* between 1 and 2 due to all components of the piping system other than the pump or turbine

(**Major +Minor losses**).

$$\frac{dP}{dx}=\frac{P_2-P_1}{L}$$

Laminar flow: $\Delta P=P_1-P_2=\dfrac{8\mu L V_{\text{avg}}}{R^2}=\dfrac{32\mu L V_{\text{avg}}}{D^2}$

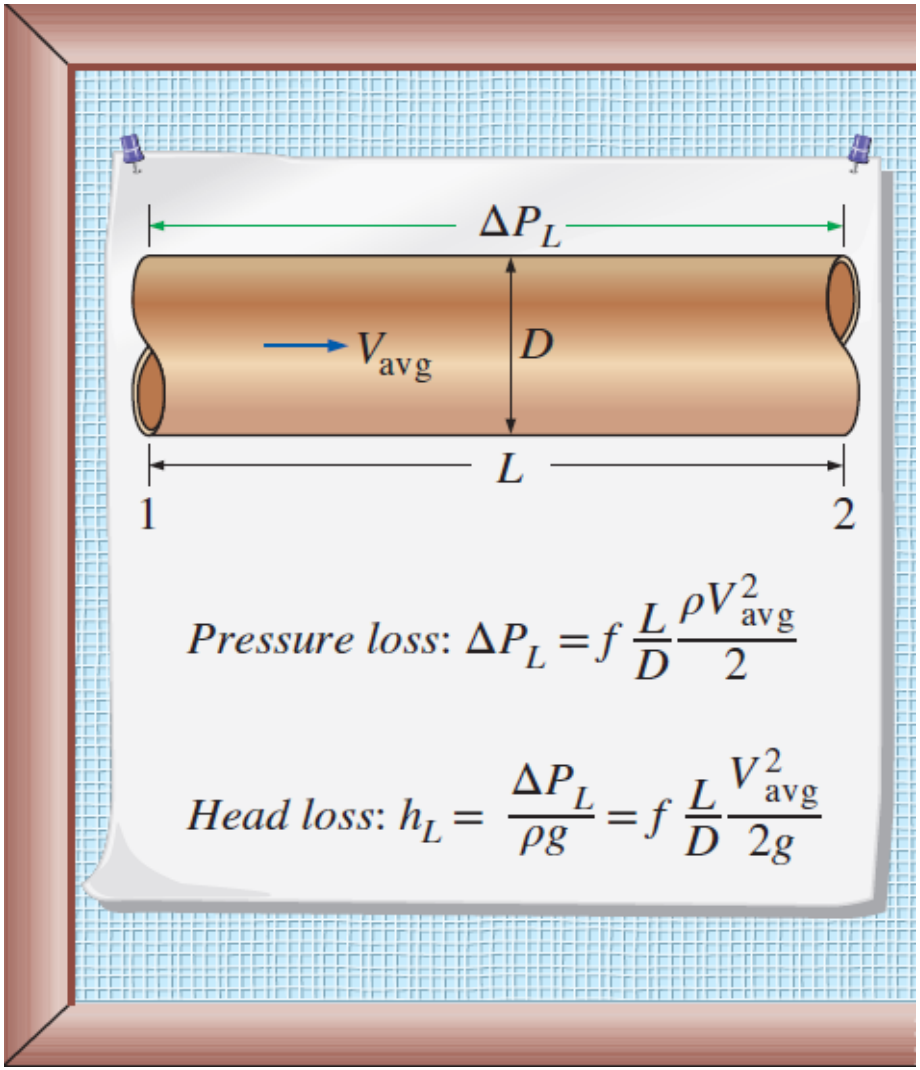
A pressure drop due to **viscous effects (Major losses)** represents an irreversible pressure loss, and it is called **pressure loss ΔP_L**.

Pressure loss: $\Delta P_L=f\dfrac{L}{D}\dfrac{\rho V_{\text{avg}}^2}{2}$ $\qquad f=\dfrac{8\tau_w}{\rho V_{\text{avg}}^2}$ Darcy friction factor

Circular pipe, laminar: $f=\dfrac{64\mu}{\rho D V_{\text{avg}}}=\dfrac{64}{\text{Re}}$ $\qquad \rho V_{\text{avg}}^2/2$ dynamic pressure

Head loss: $h_L=\dfrac{\Delta P_L}{\rho g}=f\dfrac{L}{D}\dfrac{V_{\text{avg}}^2}{2g}$

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The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.

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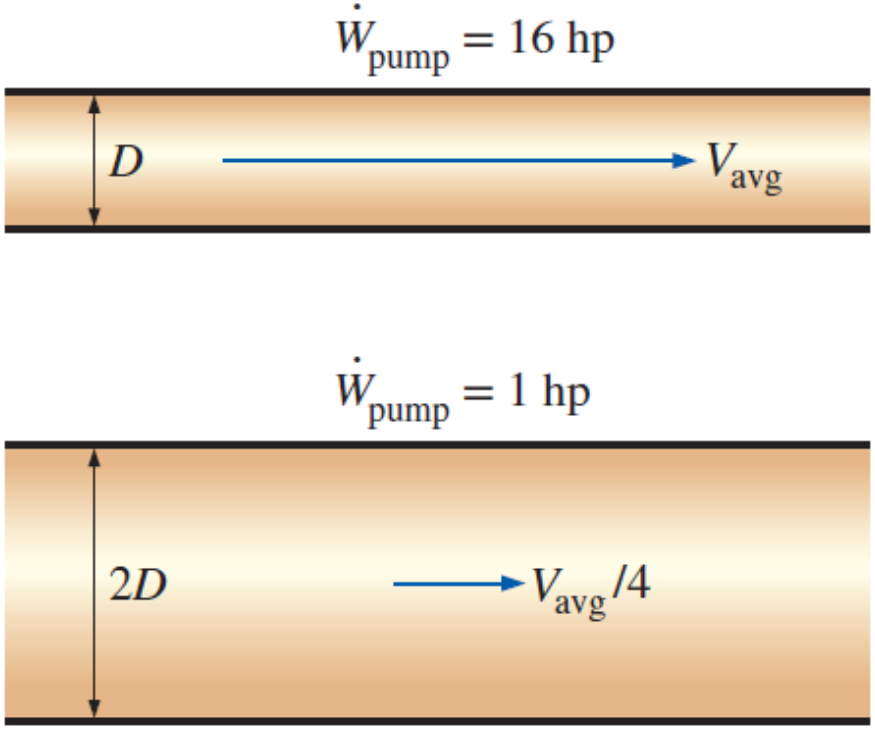
$$\dot{W}_{\text{pump, }L} = \dot{V}\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

Horizontal pipe:

$$V_{\text{avg}} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P D^2}{32\mu L}$$

$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P \pi D^4}{128\mu L}$$

Poiseuille's law



For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.

The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

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The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, }u}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, }e}+h_L$$

$$P_1-P_2=\rho(\alpha_2V_2^2-\alpha_1V_1^2)/2+\rho g[(z_2-z_1)+h_{\text{turbine, }e}-h_{\text{pump, }u}+h_L]$$

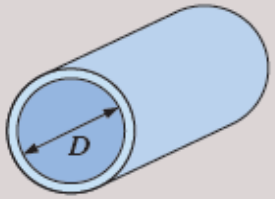
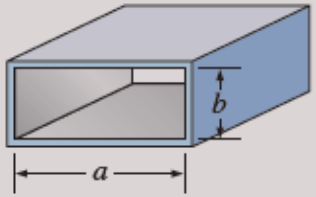
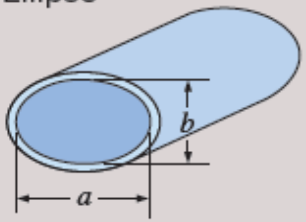
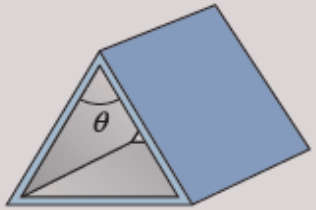
Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump, }u} = h_{\text{turbine, }e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

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Laminar Flow in Noncircular Pipes

The friction factor f relations are given in Table 5–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\mathrm{Re} = V_{avg} D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

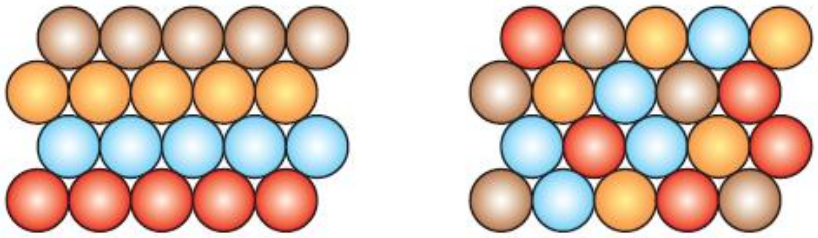
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TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.



(*a*) Before turbulence

(*b*) After turbulence

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients

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(*a*)



(*c*)

Photos by Alex Wouden.



(*b*)

Water exiting a tube: (*a) laminar flow* at low flow rate, (*b) turbulent flow at* high flow rate, and (*c) same as (b)* but with a short shutter exposure to capture individual eddies.

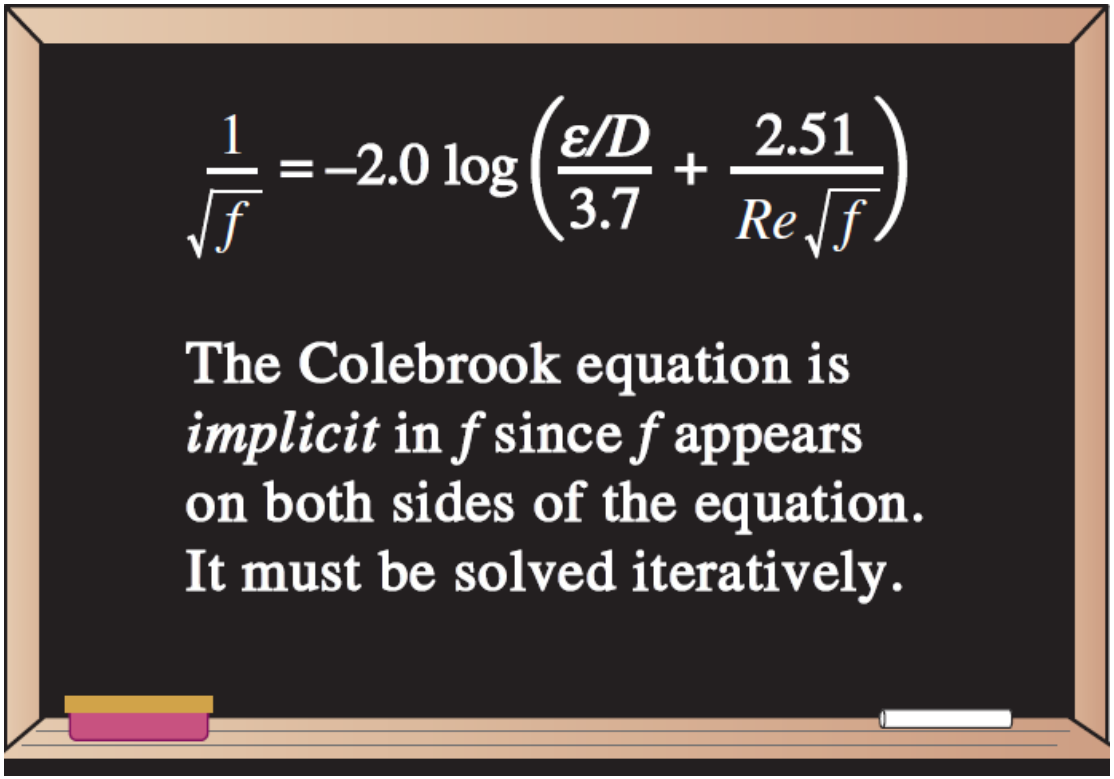
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The Moody Chart and Its Associated Equations

Colebrook equation (for smooth and rough pipes)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \quad \text{(turbulent flow)}$$

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the **relative roughness ε/D**.



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Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (Smooth)	0 (Smooth)
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

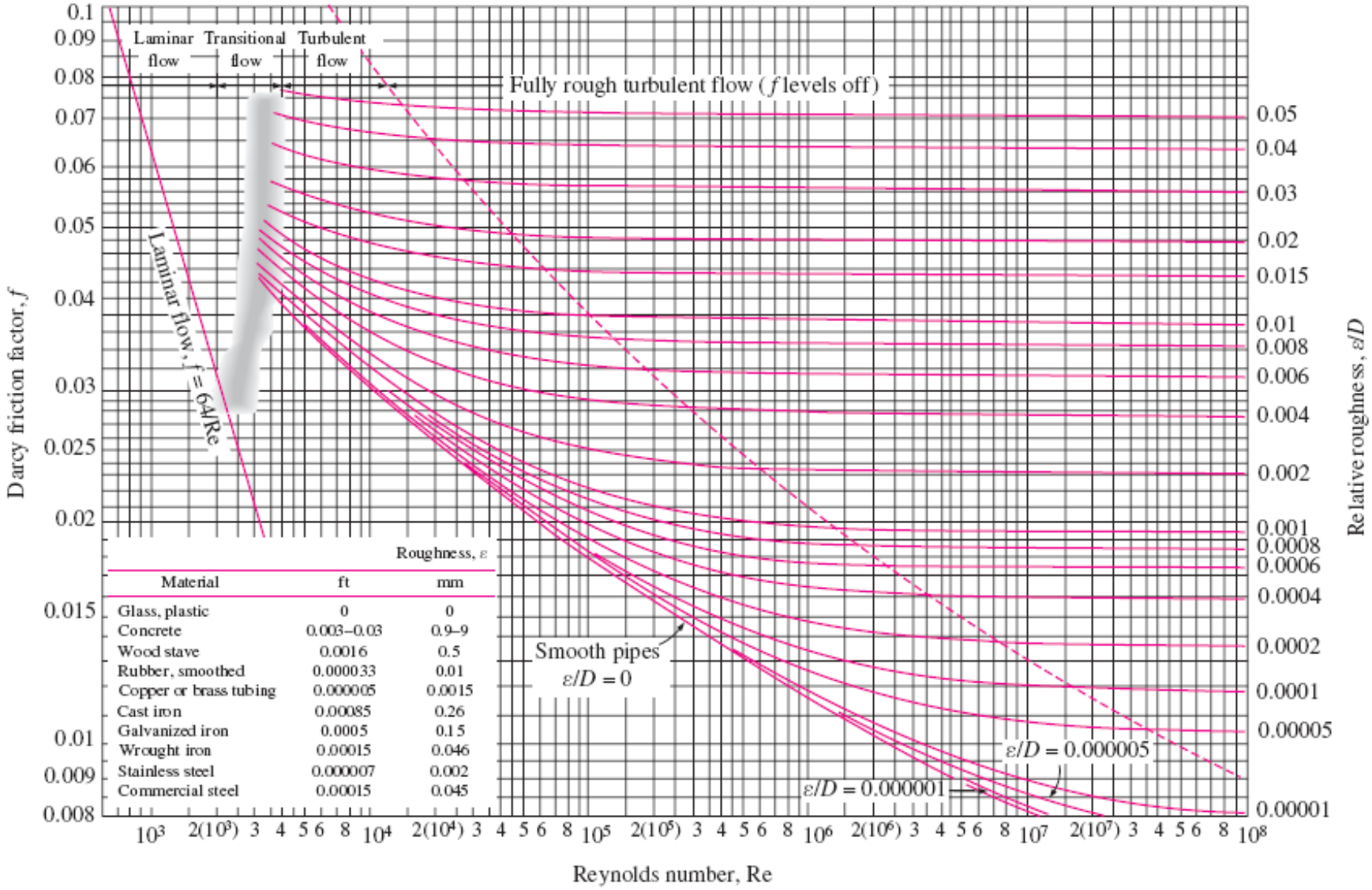
* The uncertainty in these values can be as much as ±60 percent.

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

* Smooth surface. All values are for Re = 10^6 and are calculated from the Colebrook equation.

The friction factor is minimum for a smooth pipe and increases with roughness.

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Material	Roughness, ε ft	mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The Moody Chart

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Observations from the Moody chart

For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.

The friction factor is a minimum for a smooth pipe and increases with roughness. The Colebrook equation in this case ($\varepsilon = 0$) reduces to the **Prandtl equation**.

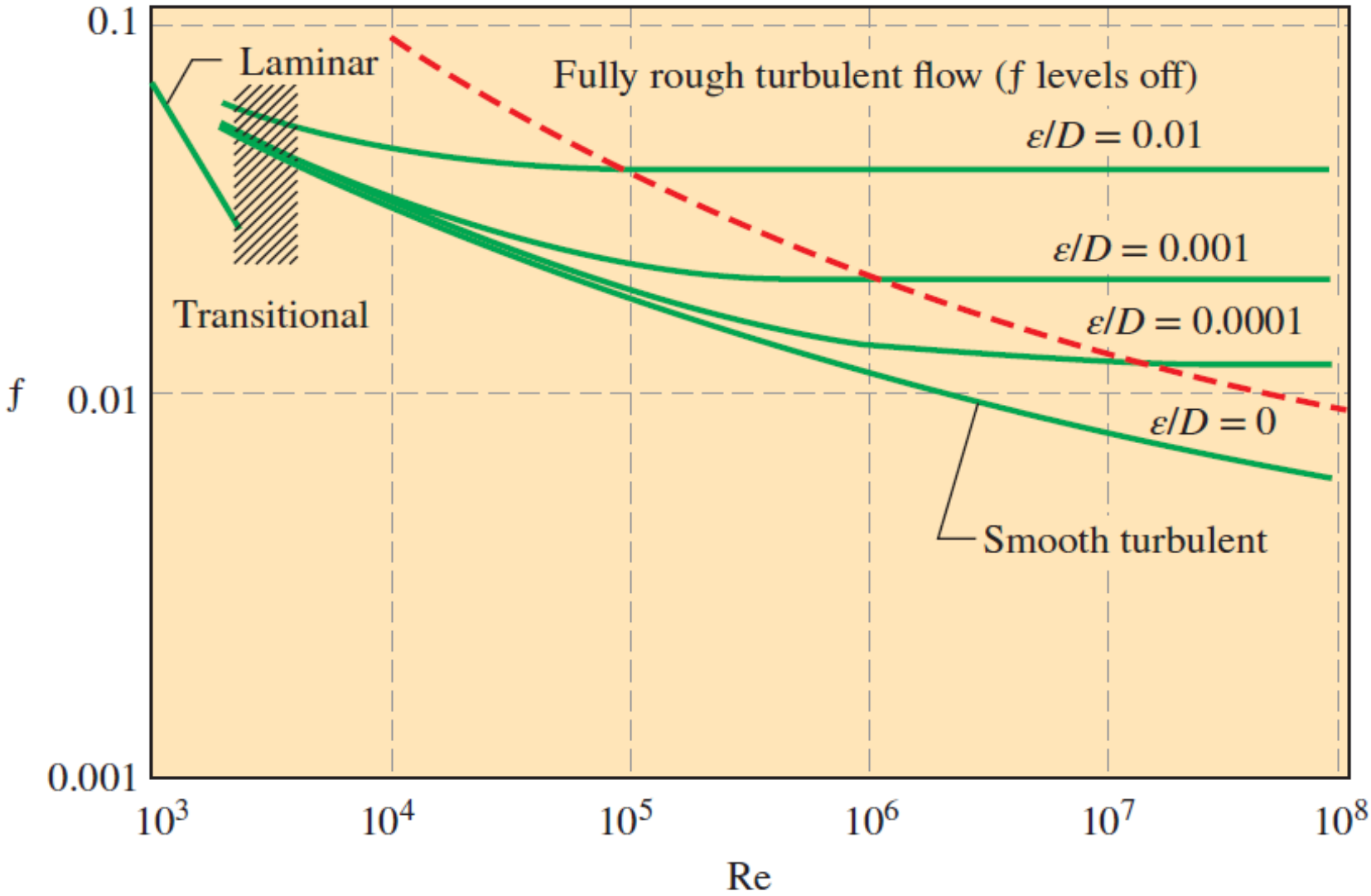
$$1/\sqrt{f} = 2.0\log(\mathrm{Re}\sqrt{f}) - 0.8$$

The transition region from the laminar to turbulent regime is indicated by the shaded area in the Moody chart. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.

At very large Reynolds numbers (to the right of the dashed line on the Moody chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called *fully rough turbulent flow* or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The Colebrook equation in the *fully rough* zone reduces to the **von Kármán equation**.

$$1/\sqrt{f} = -2.0\,\log[(\varepsilon/D)/3.7]$$

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At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–12 for a full-page moody chart.

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Types of Fluid Flow Problems

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

The three types of problems encountered in pipe flow.

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MINOR LOSSES

The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.

These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

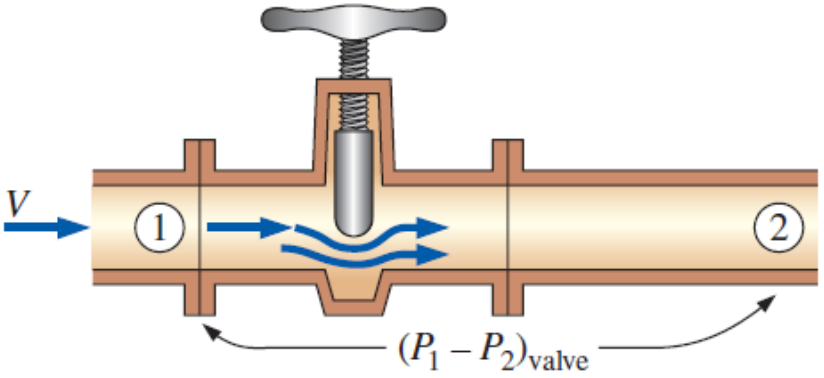
In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the **major losses**) and are called **minor losses**.

Minor losses are usually expressed in terms of the **loss coefficient K_L**.

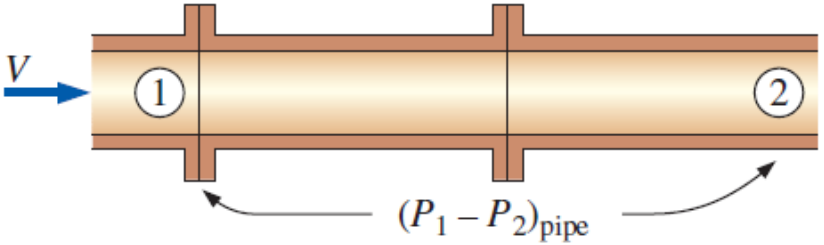
$$\textit{Loss coefficient}: \quad K_L = \frac{h_L}{V^2/(2g)}$$

$$h_L = \Delta P_L/\rho g$$ Head loss due to component

Pipe section with valve:



Pipe section without valve:



$$\Delta P_L = (P_1 - P_2)_{valve} - (P_1 - P_2)_{pipe}$$

For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

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When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

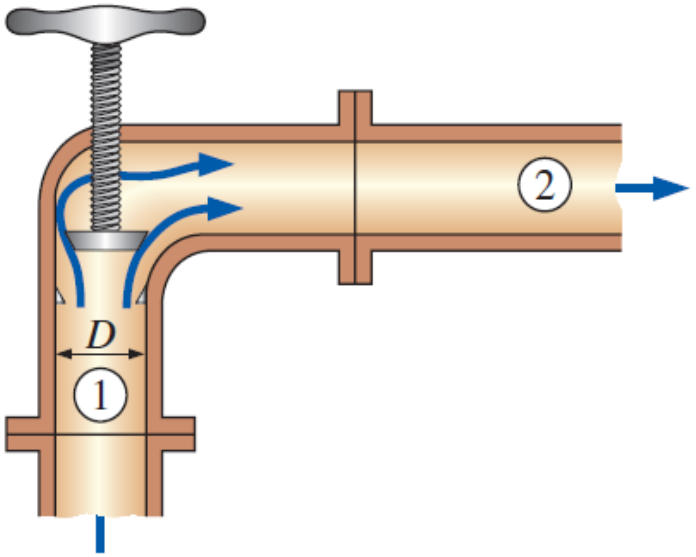
$$K_L = \Delta P_L / (\rho V^2/2)$$

When the loss coefficient for a component is available, the head loss for that component is

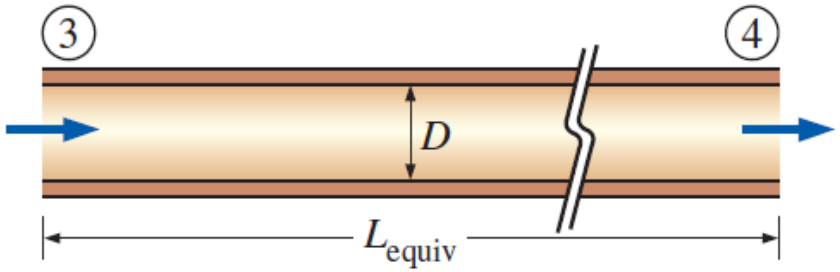
$$\textit{Minor loss}: \quad h_L = K_L \frac{V^2}{2g}$$

Minor losses are also expressed in terms of the **equivalent length L_{equiv}**

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{equiv}}{D} \frac{V^2}{2g} \quad \rightarrow \quad L_{equiv} = \frac{D}{f} K_L$$



$$\Delta P = P_1 - P_2 = P_3 - P_4$$



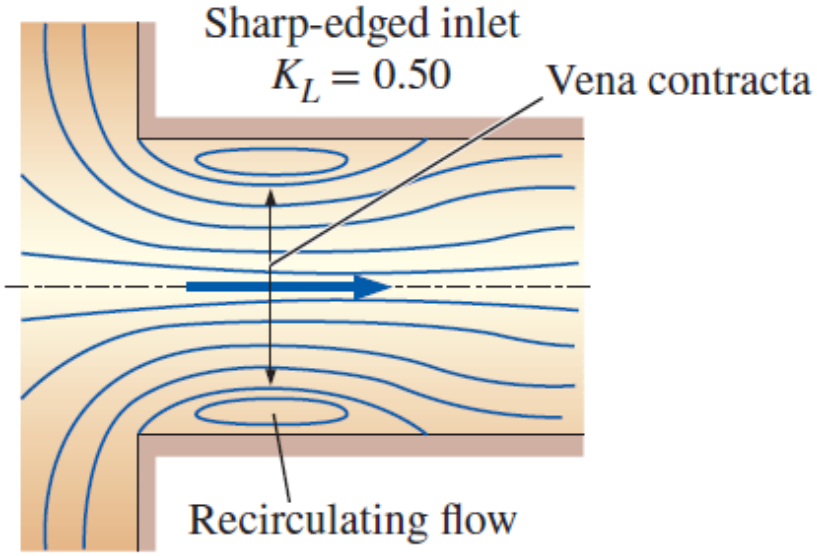
The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

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Total head loss (general)

$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}}$$

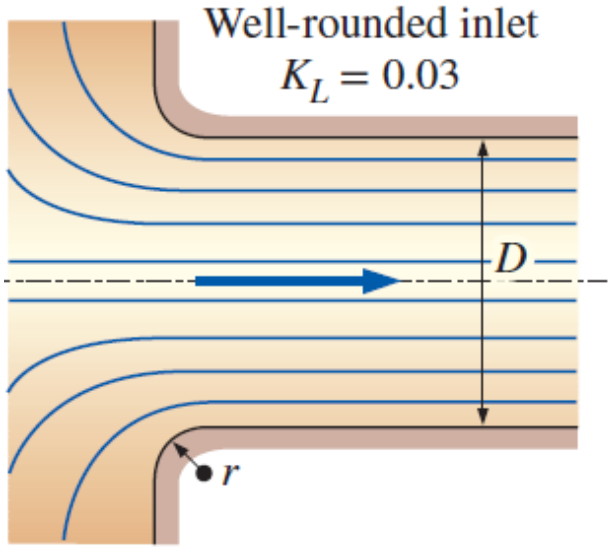
$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,\,j}\frac{V_j^2}{2g}$$



Total head loss (D = constant)

$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L \right)\frac{V^2}{2g}$$

The head loss at the inlet of a pipe is almost negligible for well-rounded inlets (K_L = 0.03 for $r/D > 0.2$) but increases to about 0.50 for sharp-edged inlets.

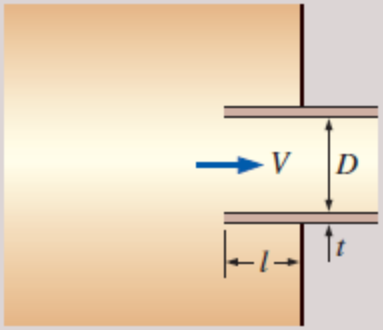


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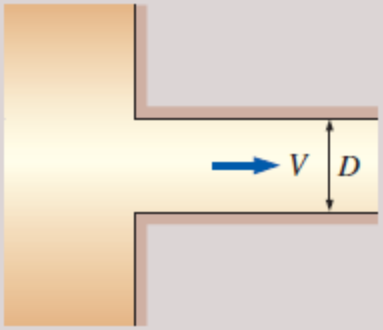
Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where V is the average velocity in the pipe that contains the component)*

Pipe Inlet

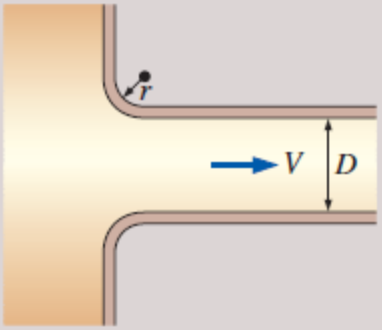
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)



Sharp-edged: $K_L = 0.50$

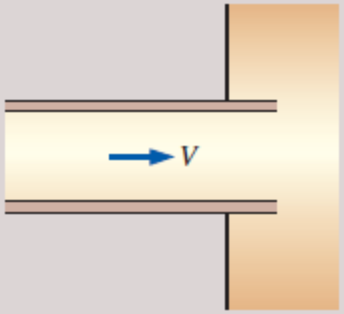


Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–40)

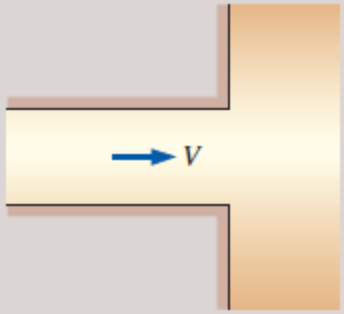


Pipe Exit

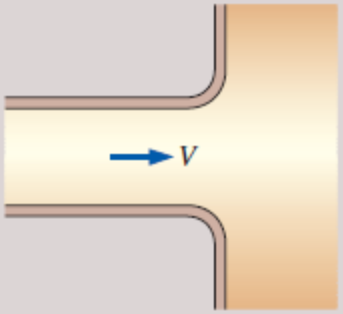
Reentrant: $K_L = \alpha$



Sharp-edged: $K_L = \alpha$



Rounded: $K_L = \alpha$

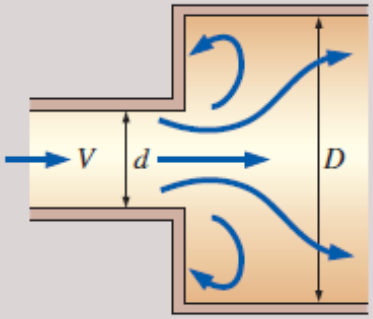


Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

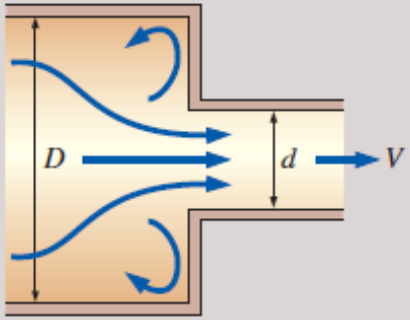
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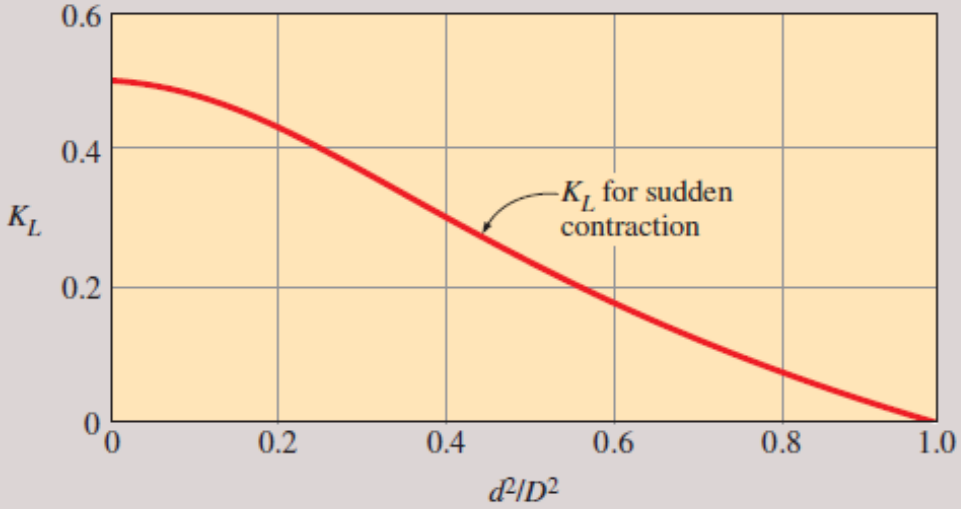
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$



Sudden contraction: See chart.

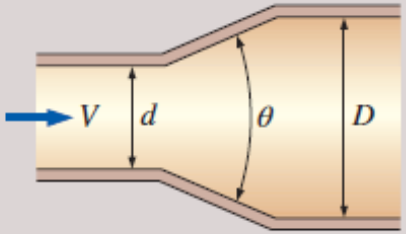




Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

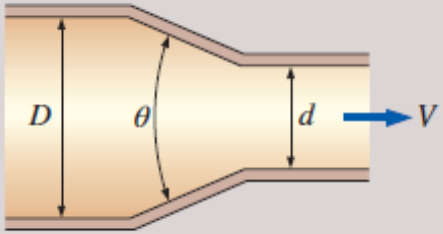
Expansion (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$
$K_L = 0.25$ for $d/D = 0.4$
$K_L = 0.15$ for $d/D = 0.6$
$K_L = 0.10$ for $d/D = 0.8$



Contraction:

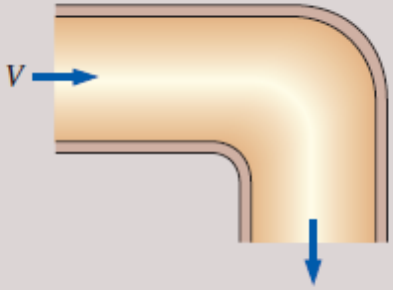
$K_L = 0.02$ for $\theta = 30°$
$K_L = 0.04$ for $\theta = 45°$
$K_L = 0.07$ for $\theta = 60°$



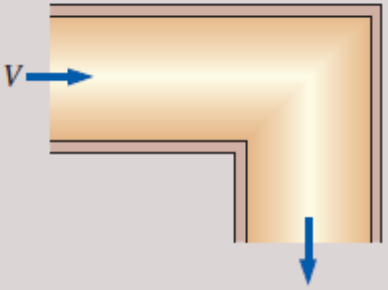
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Bends and Branches

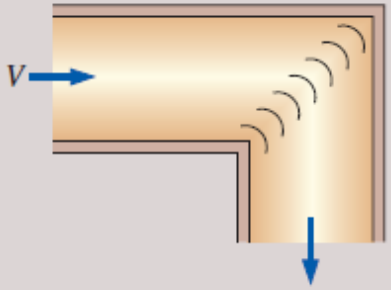
90° *smooth bend:*
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



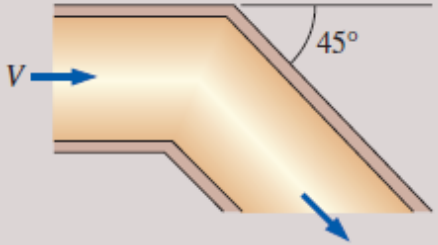
90° *miter bend*
(without vanes): $K_L = 1.1$



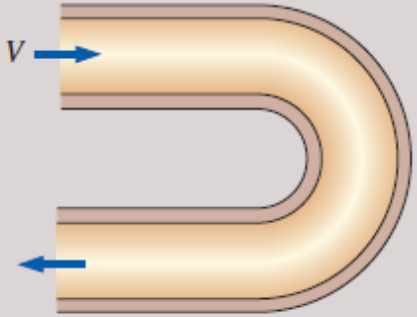
90° *miter bend*
(with vanes): $K_L = 0.2$



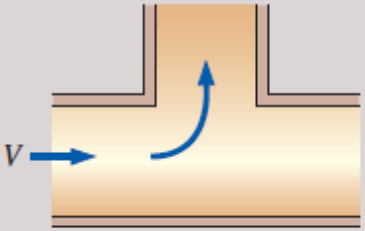
45° *threaded elbow:*
$K_L = 0.4$



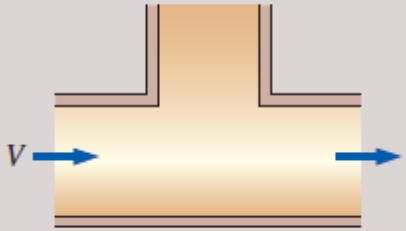
180° *return bend:*
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



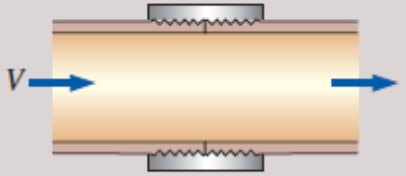
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$



Valves

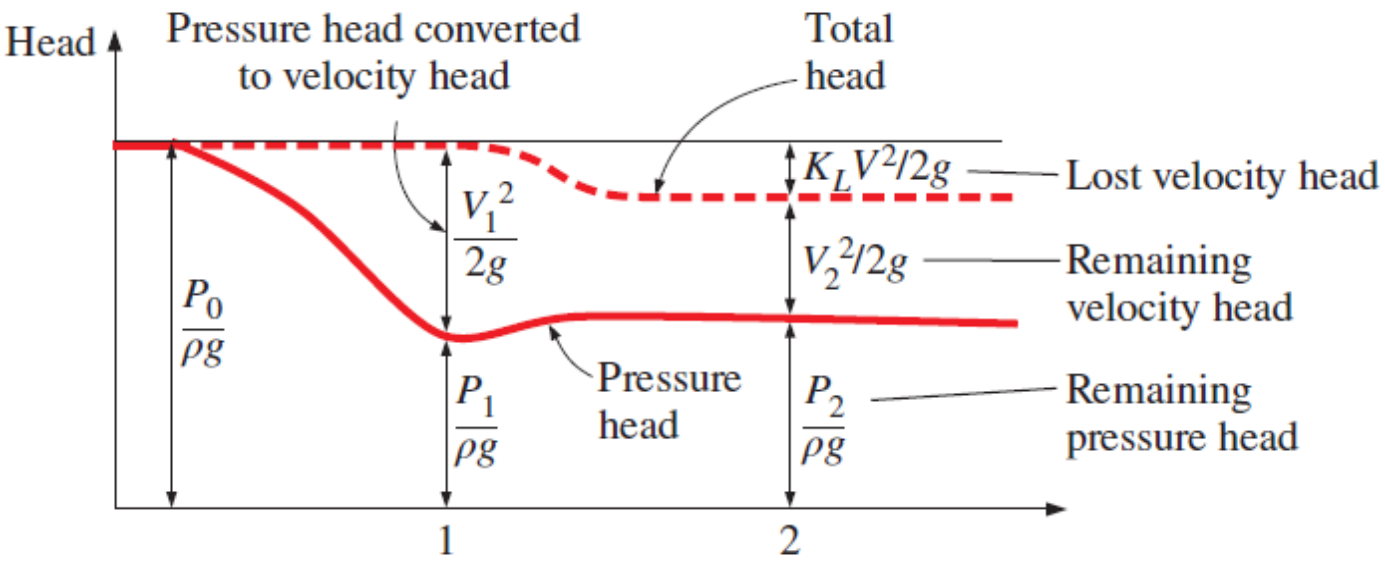
Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

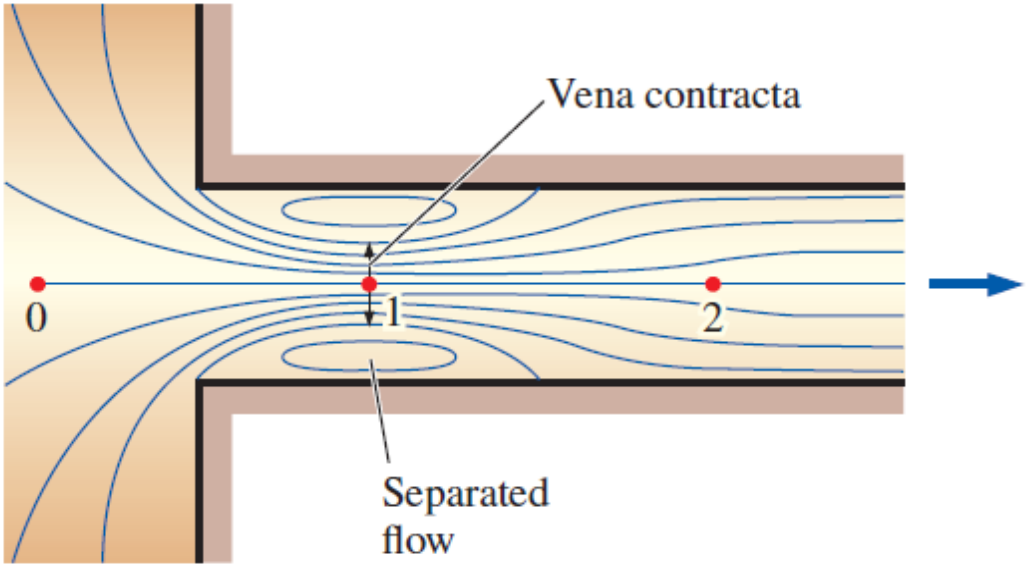
Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$

* These are representative values for loss coefficients. Actual values strongly depend on the design and manufacture of the components and may differ from the given values considerably (especially for valves). Actual manufacturer's data should be used in the final design.

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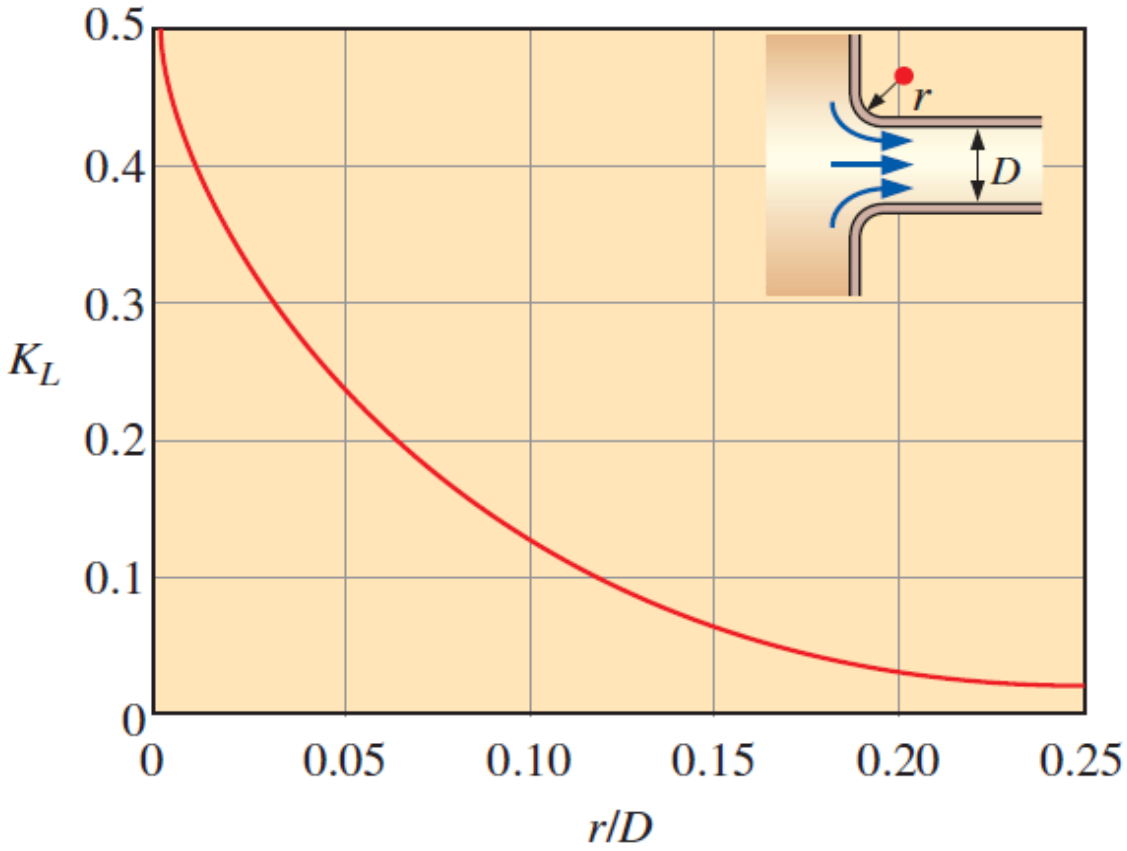
$$K_L = \alpha\left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 \qquad \text{(sudden expansion)}$$





Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

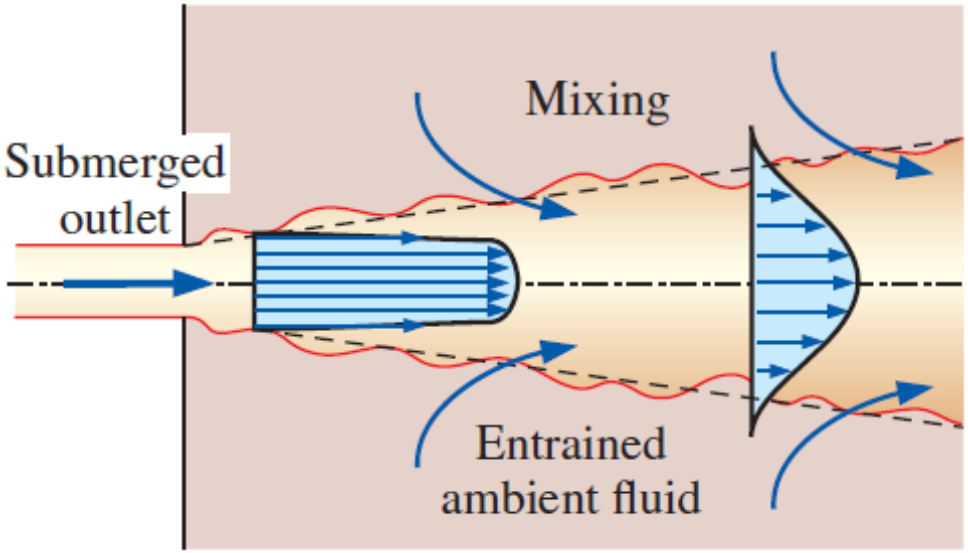
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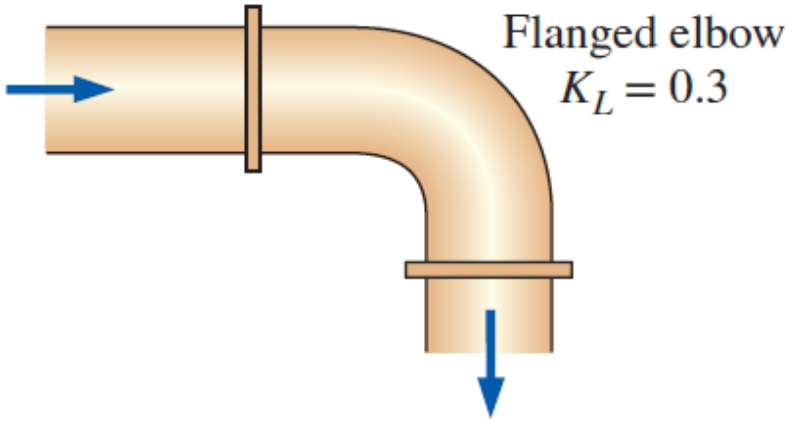
Data from ASHRAE Handbook of Fundamentals.

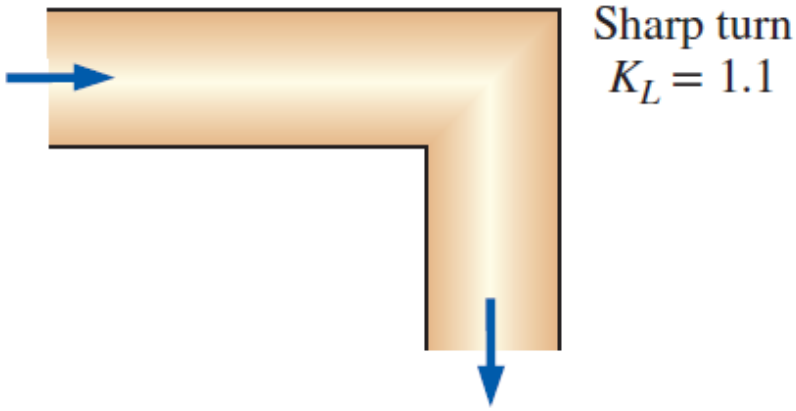
The effect of rounding of a pipe inlet on the loss coefficient.

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All the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.

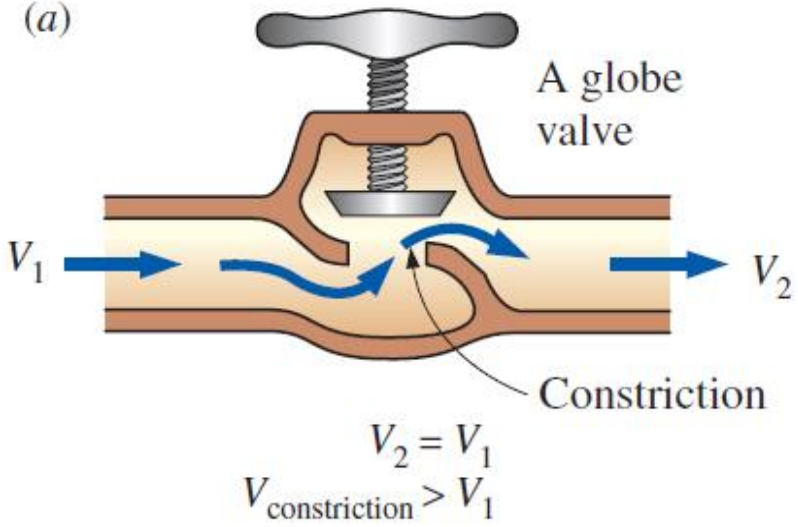




The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.

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(a)



(b)



Photo by John M. Cimbala.

(*a*) The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.
(*b*) The head loss through a fully-open ball valve, on the other hand, is quite small.

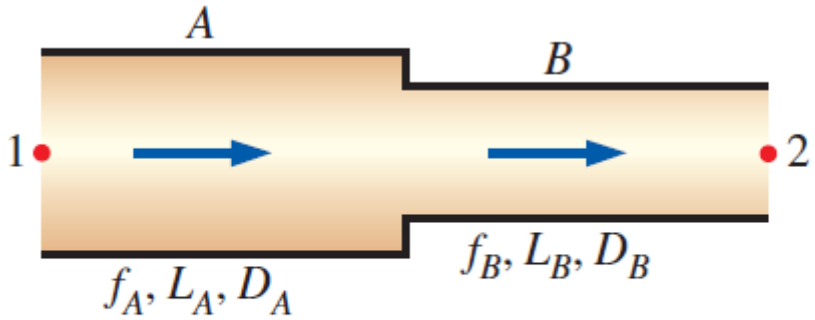
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PIPING NETWORKS AND PUMP SELECTION



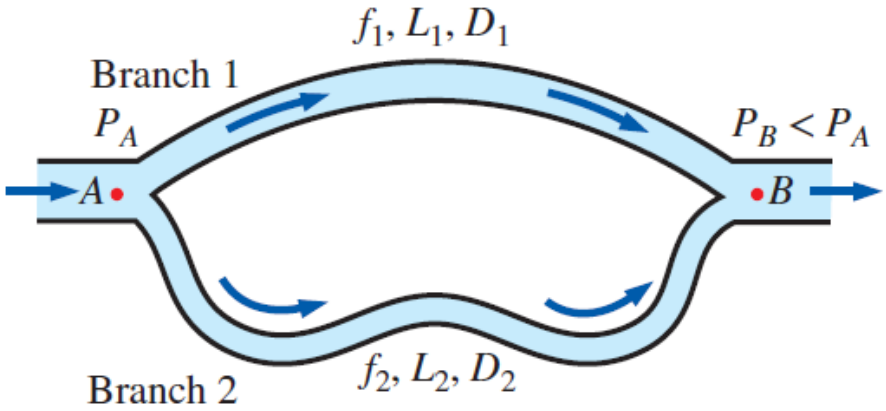
© 123RF

A piping network in an industrial facility.



$$\dot{V}_A = \dot{V}_B$$

$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B} + h_{L,\,\text{sudden contraction}}$$



$$h_{L,\,1} = h_{L,\,2}$$

$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.

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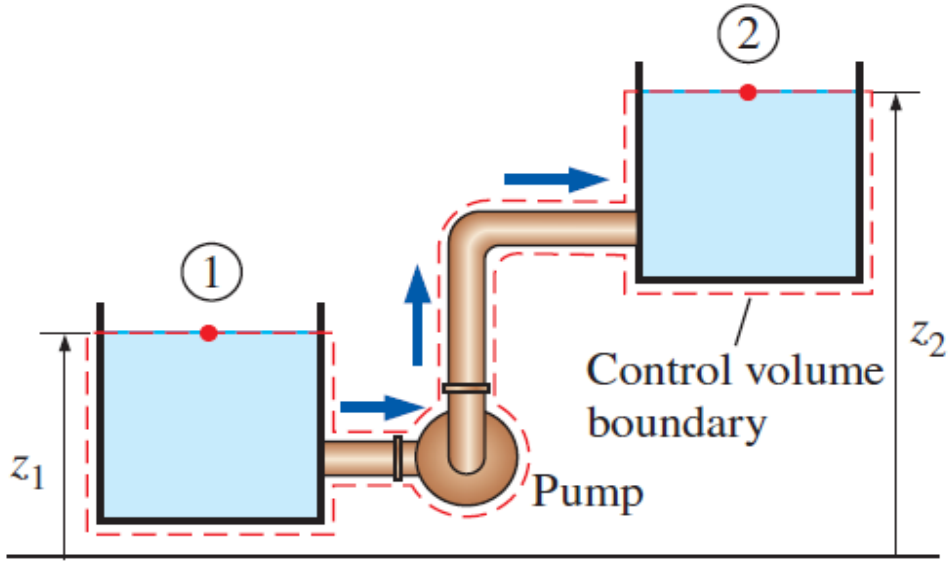
Piping Systems with Pumps and Turbines

$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 + w_{\text{pump, } u} = \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 + w_{\text{turbine, } e} + gh_L$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L$$

$$h_{\text{pump, } u} = (z_2 - z_1) + h_L$$

the steady-flow energy equation



$$h_{\text{pump, } u} = (z_2 - z_1) + h_L$$

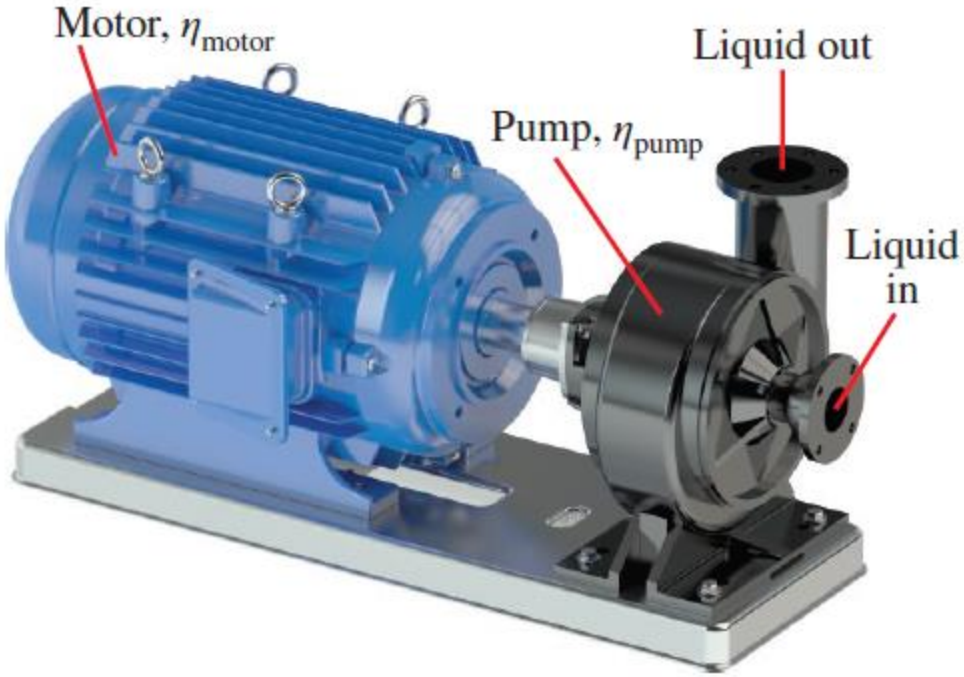
$$\dot{W}_{\text{pump, } u} = \rho \dot{V} g h_{\text{pump, } u}$$

When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

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$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump, } u}}{\eta_{\text{pump}}}$$

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump, } u}}{\eta_{\text{pump–motor}}}$$

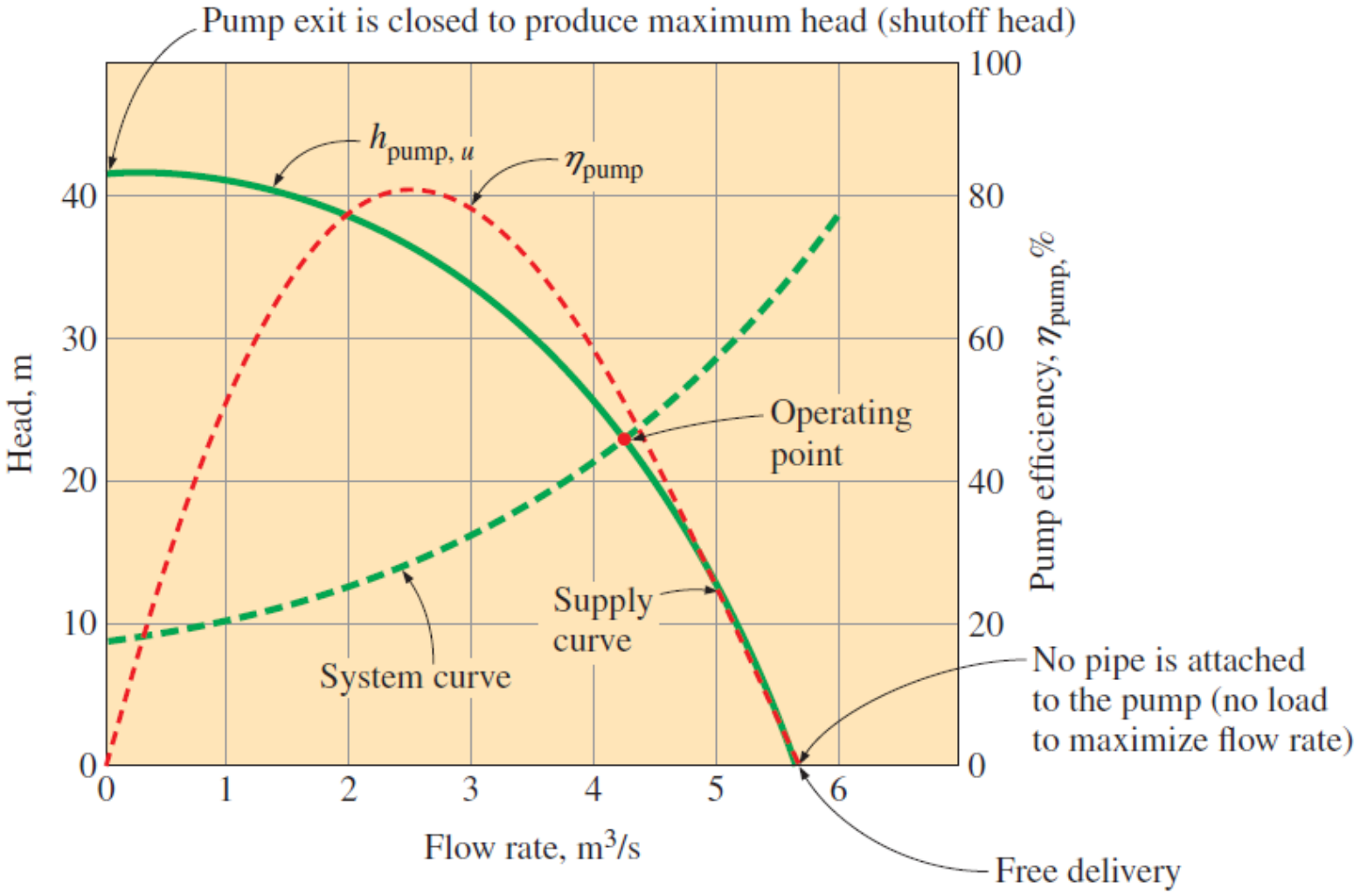


$$\eta_{\text{pump–motor}} = \eta_{\text{pump}} \eta_{\text{motor}}$$

The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.

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Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

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FLOW RATE AND VELOCITY MEASUREMENT

A major application area of fluid mechanics is the determination of the flow rate of fluids, and numerous devices have been developed over the years for the purpose of flow metering.

Flowmeters range widely in their level of sophistication, size, cost, accuracy, versatility, capacity, pressure drop, and the operating principle.

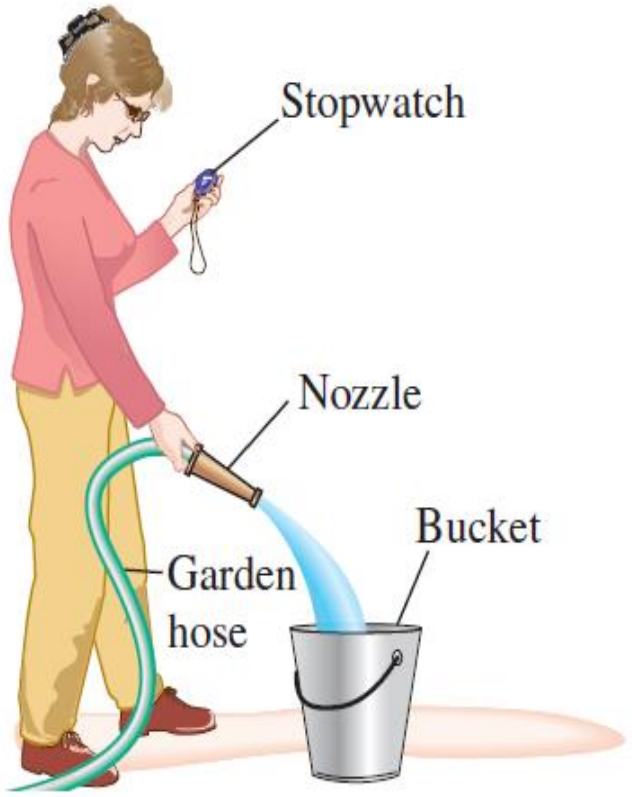
We give an overview of the meters commonly used to measure the flow rate of liquids and gases flowing through pipes or ducts.

We limit our consideration to incompressible flow.

$$\dot{V} = VA_c$$

Measuring the flow rate is usually done by measuring flow velocity, and many flowmeters are simply velocimeters used for the purpose of metering flow.

A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.

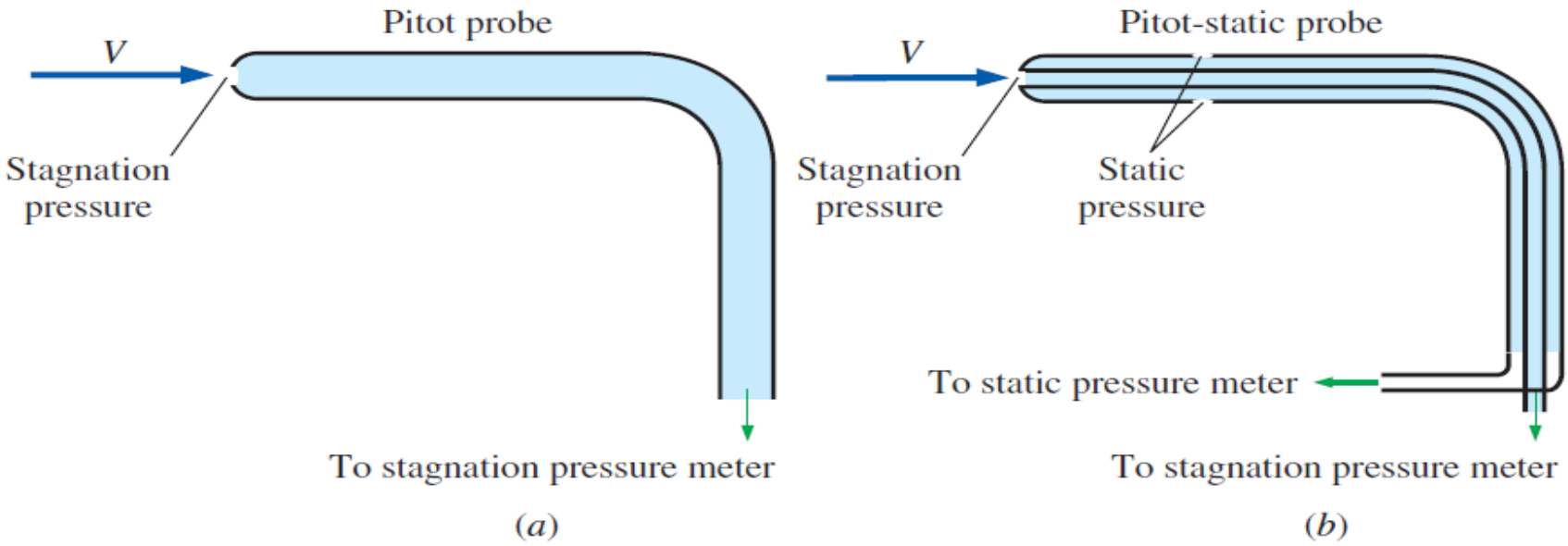


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Pitot and Pitot-Static Probes

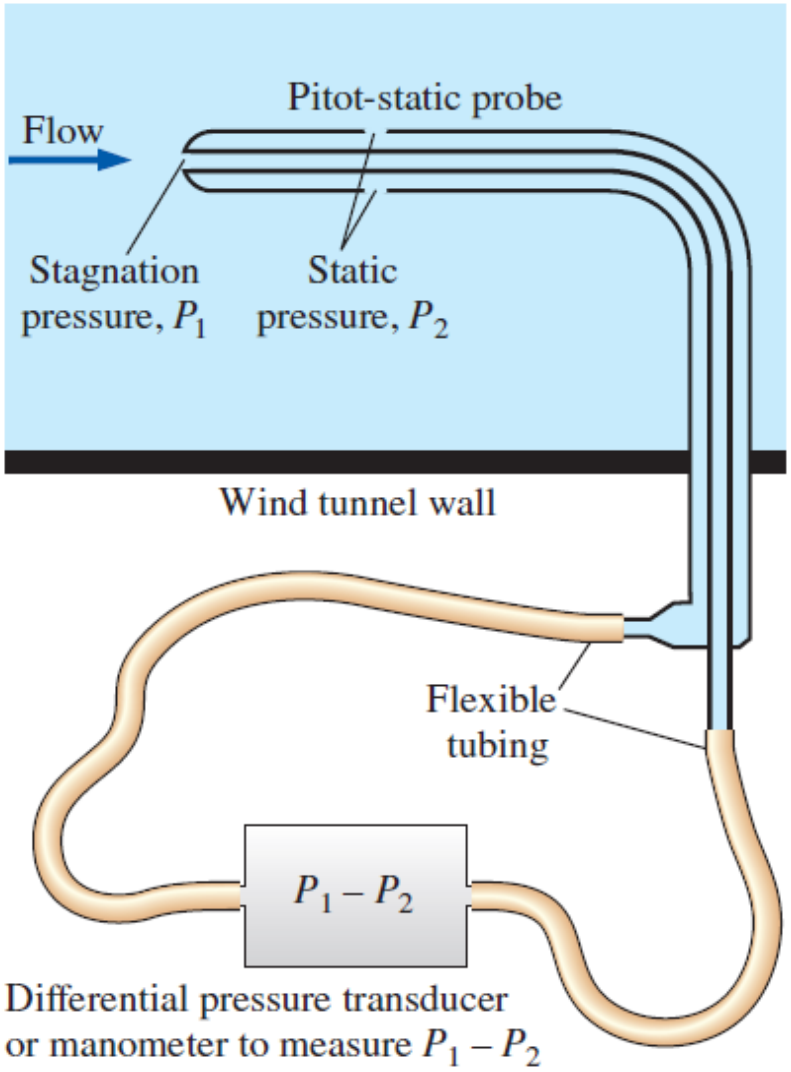
Pitot probes (also called *Pitot tubes*) and **Pitot-static probes** are widely used for flow speed measurement.

A Pitot probe is just a tube with a pressure tap at the stagnation point that measures stagnation pressure, while a Pitot-static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures



(*a*) A Pitot probe measures stagnation pressure at the nose of the probe, while (*b*) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed is calculated.

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Measuring flow velocity with a Pitotstatic probe. (A manometer may be used in place of the differential pressure transducer.)

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Pitot formula: $$V = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$



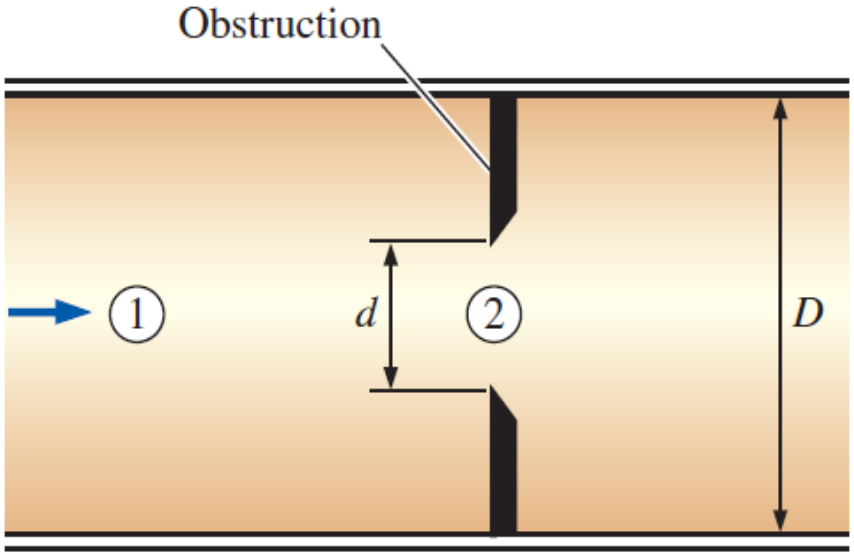
Photo by Po-Ya Abel Chuang. Used by permission.

Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

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Obstruction Flowmeters: Orifice, Venturi, and Nozzle Meters

Flowmeters based on this principle are called **obstruction flowmeters** and are widely used to measure flow rates of gases and liquids.



Flow through a constriction in a pipe.

Mass balance: $\dot{V} = A_1 V_1 = A_2 V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2 V_2$

Bernoulli equation $(z_1 = z_2)$: $\dfrac{P_1}{\rho g} + \dfrac{V_1^2}{2g} = \dfrac{P_2}{\rho g} + \dfrac{V_2^2}{2g}$

Obstruction (with no loss): $V_2 = \sqrt{\dfrac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$

$\beta = d/D \qquad \dot{V} = A_2 V_2 = (\pi d^2/4) V_2$

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The losses can be accounted for by incorporating a correction factor called the **discharge coefficient C_d** whose value (which is less than 1) is determined experimentally.

Obstruction flowmeters:

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

$$A_0 = A_2 = \pi d^2/4 \qquad \beta = d/D$$

The value of C_d depends on both b and the Reynolds number, and charts and curve-fit correlations for C_d are available for various types of obstruction meters.

Orifice meters:

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}}$$

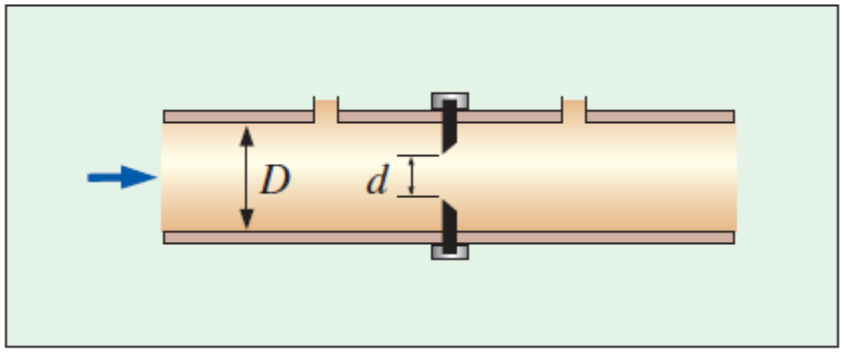
Nozzle meters:

$$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\mathrm{Re}^{0.5}}$$

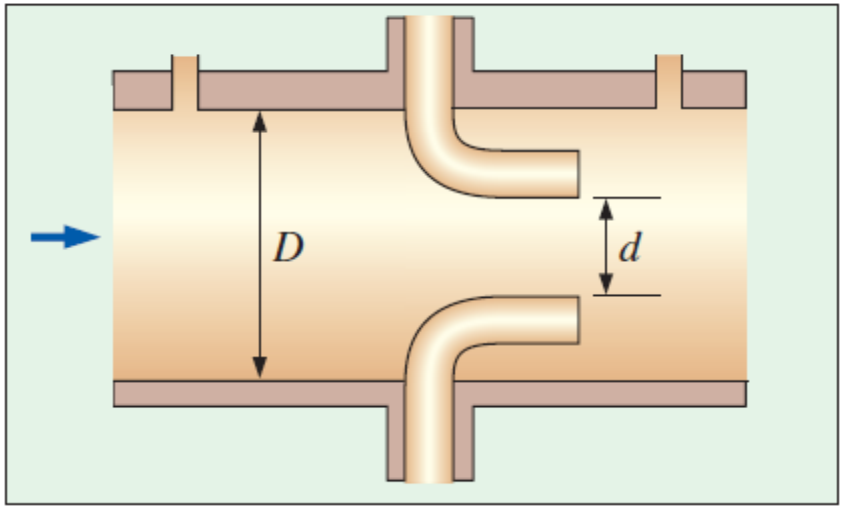
$$0.25 < \beta < 0.75 \text{ and } 10^4 < Re < 10^7$$

For flows with high Reynolds numbers (Re > 30,000), the value of C_d can be taken to be 0.96 for flow nozzles and 0.61 for orifices.

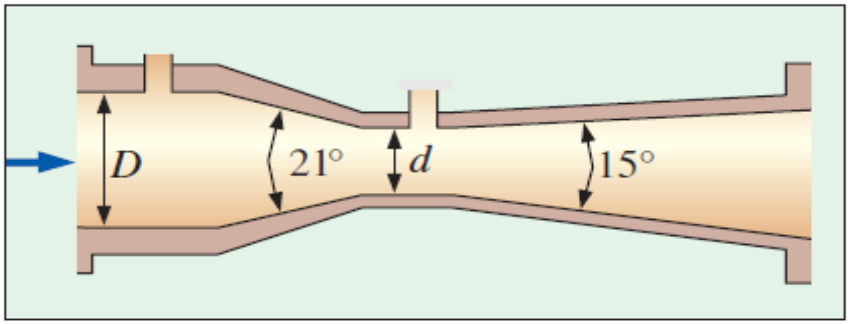
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(*a*) Orifice meter



(*b*) Flow nozzle

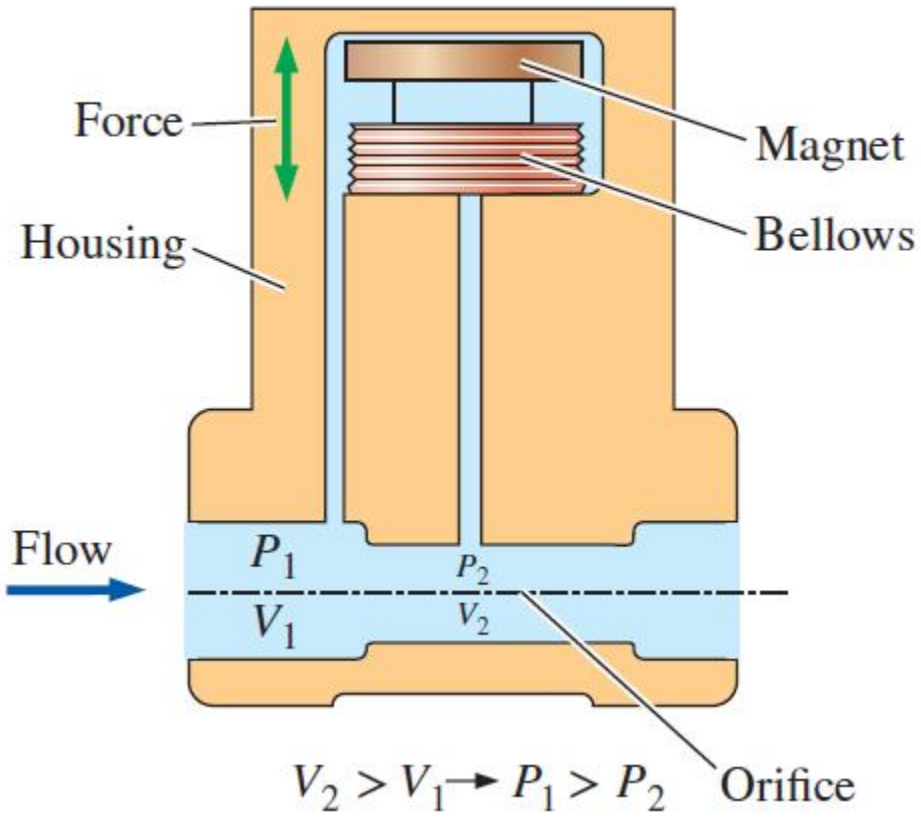


(*c*) Venturi meter

Common types of obstruction meters.

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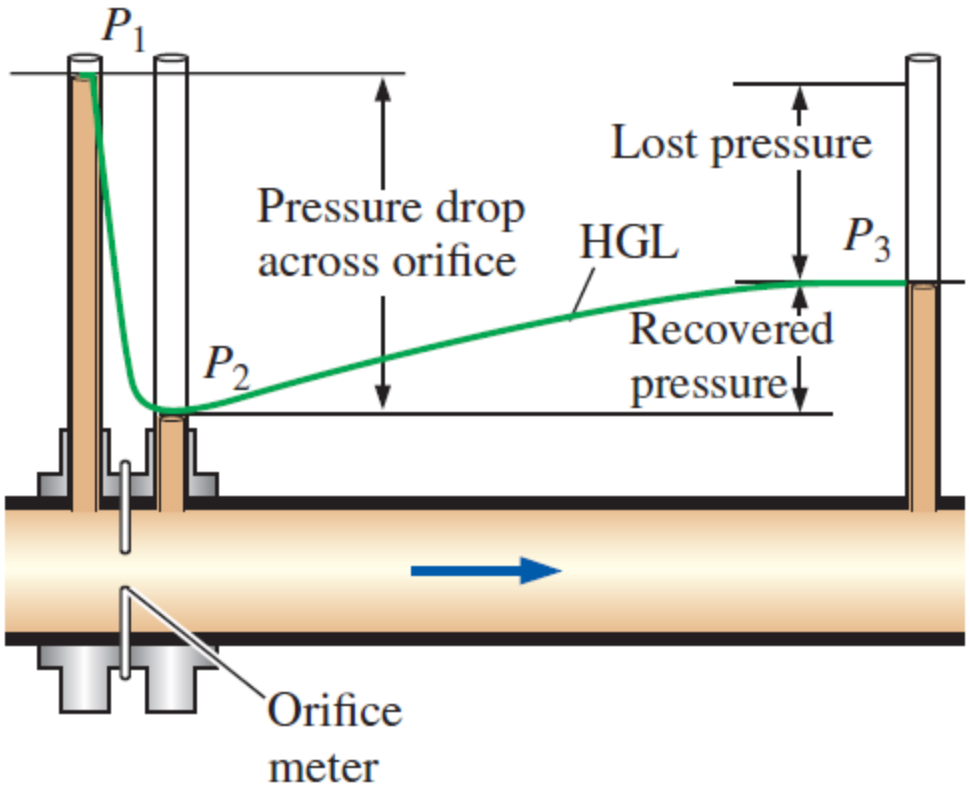




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An orifice meter and schematic showing its built-in pressure transducer and digital readout.

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The variation of pressure along a flow section with an orifice meter as measured with piezometer tubes; the lost pressure and the pressure recovery are shown.

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