RSA Hardware Research Ideas

RSA Public Key Cryptosystem

- Developed in 1978, by Rivest, Shamir & Adleman
- Its security is based on the *integer factoring problem*
- The most popular method :-
  - simple to understand & implement
  - same algorithm for encryption & decryption
  - can also be used for digital signature

RSA Security

* Security depends on the key size.

larger key size → more secure system

RSA Implementations

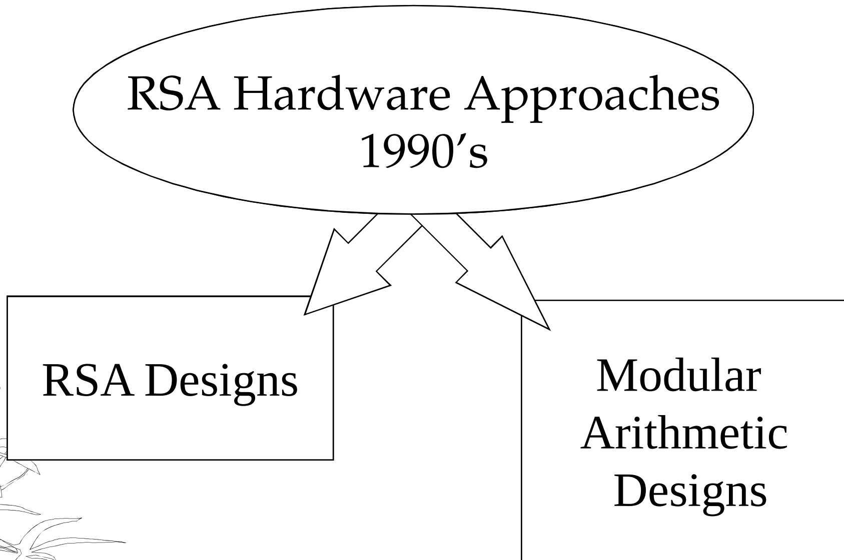
software
slow speed
Can be less secure

hardware

Modular Exponentiation
repeated squaring

&

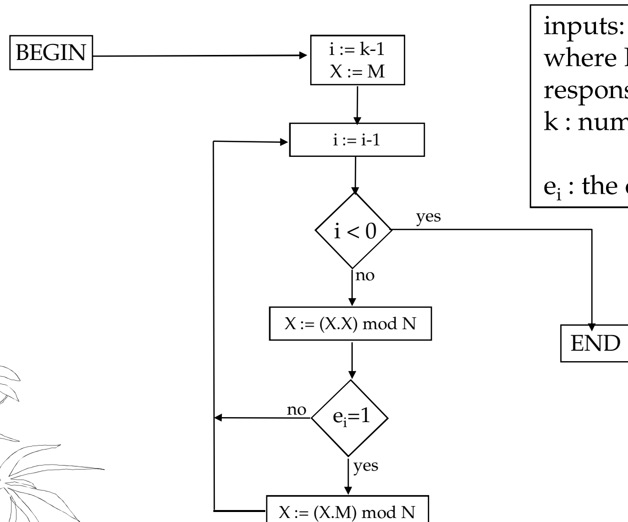
Modular Multiplication
- multiply/divide
- add/subtract
- logarithmic speed
- *Montgomery*



Modular Exponentiation

repeated squaring algorithm

inputs: E,M,N
where E>0 & 0<M<N
response: $X=M^E \bmod N$
k : number of digits in E
with 1 as MSB
e_i : the digit number i of E



Modular Exponentiation Example

repeated squaring algorithm

- Compute: $3^9 \bmod 7$
- $k = 9 = (1001)_2$; $i = 3$; $X = M = 3$
  - – $i = 2$; $X = 3.3 = 2$; $e_2=0 \Rightarrow X = 2$ (no change since $e_i = 0$)
  - – $i = 1$; $X = 2.2 = 4$; $e_1=0 \Rightarrow X = 4$ (no change since $e_i = 0$)
  - – $i = 0$; $X = 4.4 = 2$; $e_0=1 \Rightarrow X = 2.3 = 6$
- $3^9 \bmod 7 = X = 6$

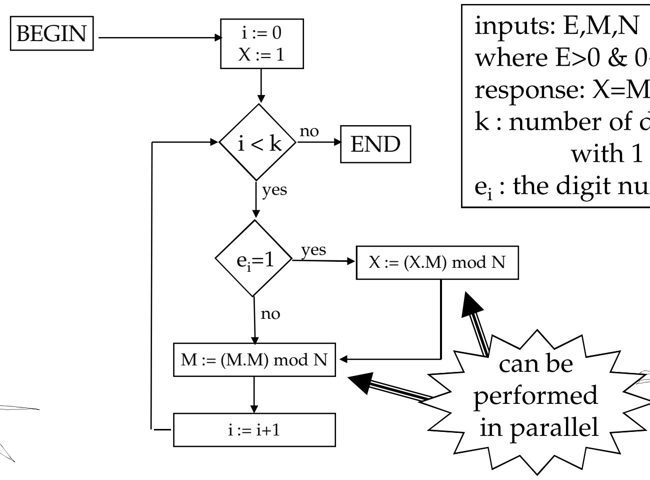
Modular Exponentiation Example

repeated squaring algorithm

- Compute: $7^{27} \bmod 10 = 3$
- $k = 27 = (11011)_2$; $i = 5$; $X = M = 7$
  - – $i = 3$; $X = 7.7 = 9$; $e_3=1 \Rightarrow X = 7.9 = 3$
  - – $i = 2$; $X = 3.3 = 9$; $e_2=0 \Rightarrow X = 9$ (no change since $e_i = 0$)
  - – $i = 1$; $X = 9.9 = 1$; $e_1=1 \Rightarrow X = 1.7 = 7$
  - – $i = 0$; $X = 7.7 = 9$; $e_0=1 \Rightarrow X = 9.7 = 3$

Modular Exponentiation

improved repeated squaring



inputs: E,M,N
where E>0 & 0<M<N
response: X=M^E mod N
k : number of digits in E with 1 as MSB
e_i : the digit number i of E

Modular Exponentiation Example

improved repeated squaring algorithm

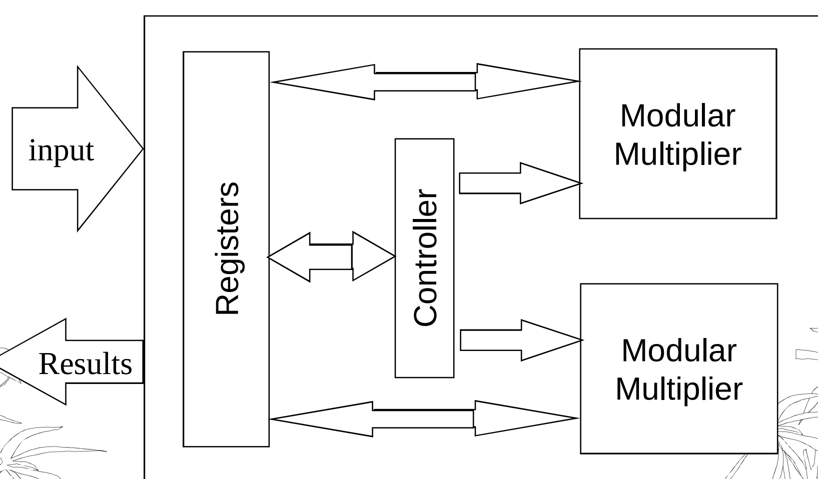
- Compute: 3^9 mod 7
- k = 9 = $(1001)_2$; i = 0→3 ; X = 1; M = 3
  - i = 0 ; e_0=1⇒ X = 1.3 = 3; M = 3.3 = 2
  - i = 1 ; e_1=0⇒X = 3 (no change since e_i = 0) ; M = 2.2=4
  - i = 2 ; e_2=0⇒X = 3 (no change since e_i = 0) ; M = 4.4=2
  - i = 3 ; e_3=1⇒X = 3.2 = 6; M = 2.2=4
- 3^9 mod 7 = X = 6

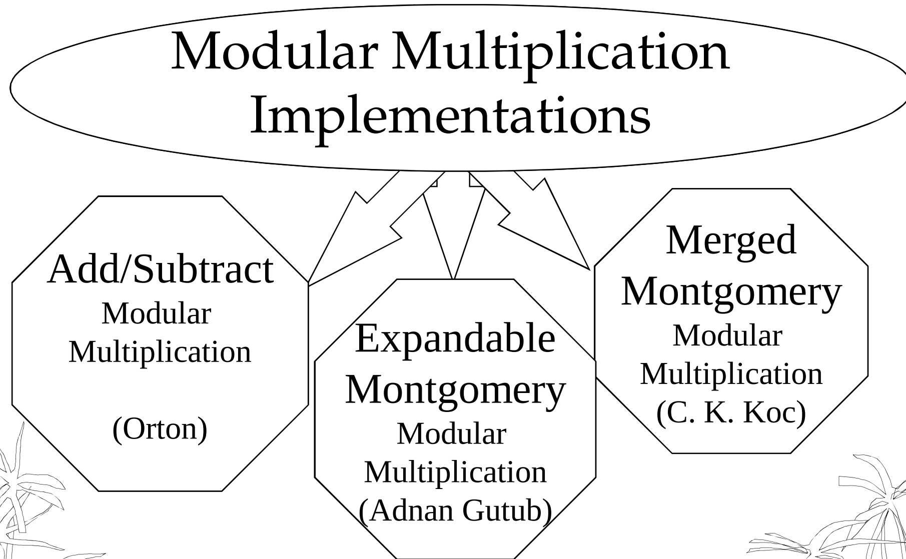
Modular Exponentiation Example

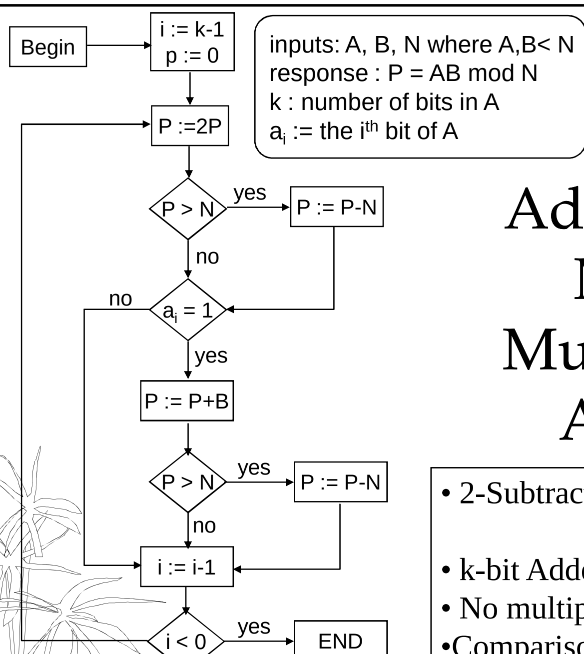
improved repeated squaring algorithm

- Compute: $7^{10} \bmod 10 = 9$
- $k = 10 = (1010)_2$; $i = 0 \rightarrow 3$; $X = 1$; $M = 7$
  - $i = 0$; $e_0=0 \Rightarrow X = 1$ (no change since $e_i = 0$) ; $M = 7.7 = 9$
  - $i = 1$; $e_1=1 \Rightarrow X = 1.9 = 9$; $M = 9.9 = 1$
  - $i = 2$; $e_0=0 \Rightarrow X = 9$ (no change since $e_i = 0$) ; $M = 1.1 = 1$
  - $i = 3$; $e_0=1 \Rightarrow X = 9.1 = 9$; $M = 1.1 = 1$

Modular Exponentiation Hardware



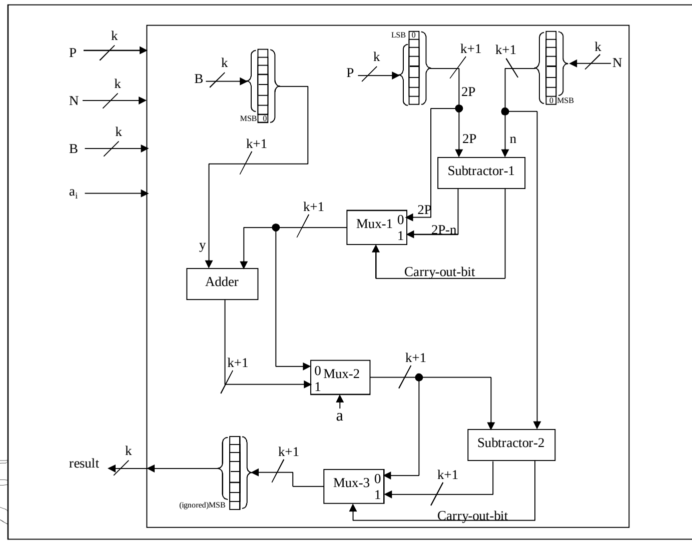




Add/Subtract Modular Multiplication Algorithm

- 2-Subtractors & Adder
  => Max. speed
- k-bit Adder
- No multiplier
- Comparison => sign-bit (subtractor)

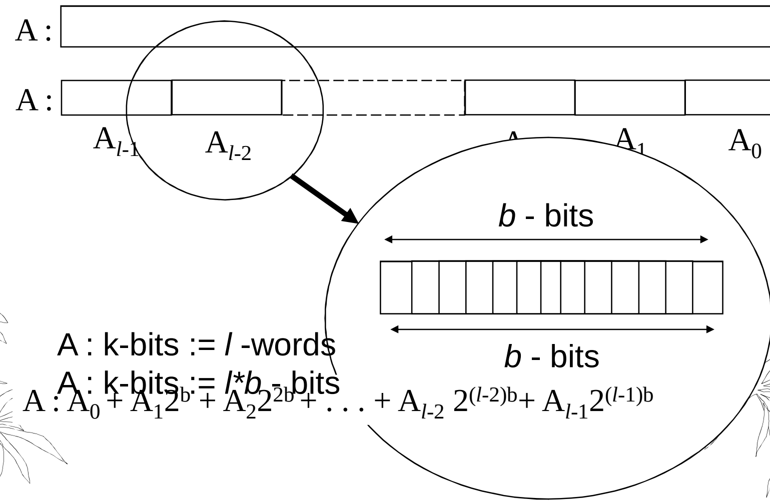
Add/Subtract Modular Multiplication Algorithm Data-path



Expandable RSA Hardware

- Crypto Hardware is faster & more secure compared to software
- However, it is limited by its design parameters and capability.
- If more security is needed & more bits are needed to be computed, a new hardware is to be designed and the old one cannot be utilized.
- Expandable hardware will be flexible to solve this problem by adding extra chips whenever needed.

Numbers Representation

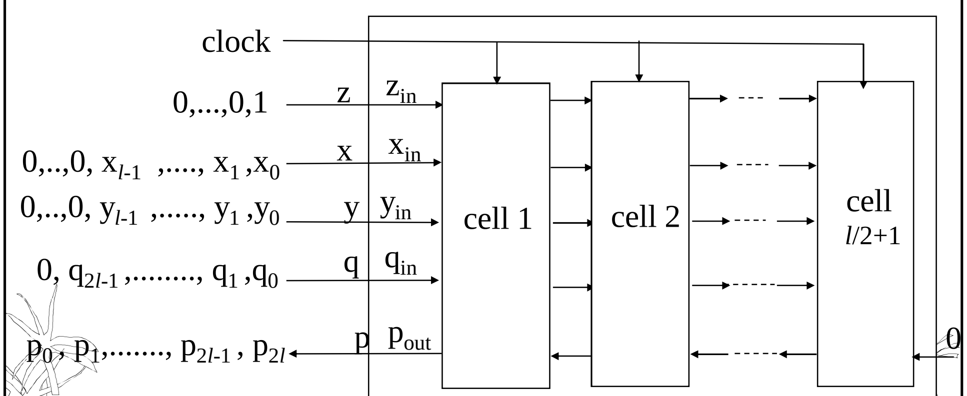


A : k-bits := *l* -words

A : k-bits := *l***b* - bits

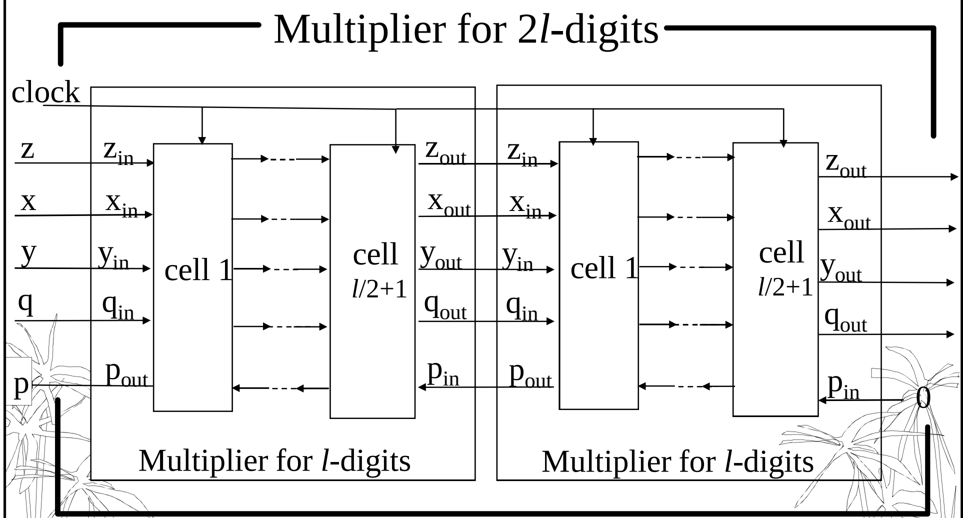
$A : A_0 + A_1 2^{b} + A_2 2^{2b} + \ldots + A_{l-2}\, 2^{(l-2)b} + A_{l-1} 2^{(l-1)b}$

Building the Systolic Multiplier



• (l/2 + 1) cells required for l-digit multiplication

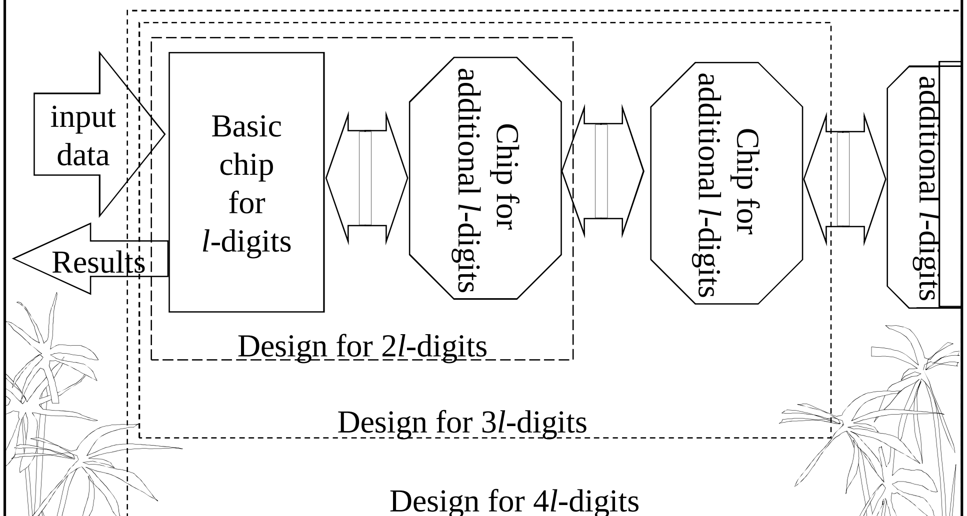
Expandable Systolic Multiplier



For Expandability

- Allow input data to have more digits
- Allow systolic multiplier to be expandable
- Allow registers to be expandable
- Multiplexing

The Expandable RSA Design



Expandable RSA Hardware Ideas Summary

- The expandable hardware has the best speed while its area is the largest.
- The add/subtract model has the best area while its speed is the slowest.
- The cost (Area*Time^2) of the merged design is the best followed by the expandable design, and then the add/subtract model.